Illiquidity and stock returns: A revisit

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1. Introduction

I am honored that the *Critical Finance Review* has commissioned two groups of scholars, Jozef Drienko, Tom Smith, and Anna von Reibnitza (2018) and Larry Harris and Andrea Amato (2018), to replicate and extend my 2002 study. I thank the authors of these studies for agreeing to undertake this task and for their excellent analysis.

My 2002 article is titled "Cross-section and time-series analysis." The results in the cross-section analysis, which show a positive cross-stock effect of illiquidity on expected returns, are not new. This has already been shown by Amihud and Mendelson (1986) and supported by evidence in Brenan and Subrahmanyam (1996), Brennan, Chordia, and Subrahmanyam (1998), Datar, Naik, and Radcliff (1998) and other studies, reviewed in Amihud, Mendelson, and Pedersen (2005, 2013). The objective of this part is to show that the illiquidity measure *ILLIQ*, the average ratio of absolute returns to the dollar trading volume, positively affects stock expected returns. This enables the use of *ILLIQ* in the subsequent time-series analysis that presents new results on the negative relation between market illiquidity shocks and realized stock returns, which is stronger for smaller, less liquid stocks. This part proposes that, given the persistence of illiquidity, a positive illiquidity shock raises expected returns and thus lowers stock prices for a given level of cash flows.[1] This argument follows the analysis of the effect of volatility shocks on stock returns in Merton (1980) and French, Schwert, and Stambaugh (1987) and is based on findings on the effects of changes in illiquidity on stock prices in Amihud, Mendelson, and Wood (1990), Amihud, Mendelson, and Lauterbach (1997), and Amihud, Mendelson, and Uno (1999).

I then propose (p. 53) that "the greater sensitivity of small stocks to illiquidity means that these stocks are subject to greater illiquidity risk which, if priced, should result in higher illiquidity risk premium." Illiquidity risk—the covariance of return and illiquidity shocks—is shown to be positively priced in Pastor and Stambaugh (2003) and Acharya and Pedersen (2005).

The illiquidity measure *ILLIQ* was developed out of necessity to enable the time-series analysis over a long period. I could calculate *ILLIQ* easily for 34 years from daily data on return and volume that was then available since 1963 in the Center for Research in Security Prices (CRSP) database, whereas the calculation of finer measures required intraday data, which were

[1] The assumption is that illiquidity shocks do not affect cash flow. This condition can be weaker, see Acharya and Pedersen, 2005.

available for only 12 years.[2] The variable *ILLIQ* is a coarse, low-frequency measure of illiquidity, which I show to be positively correlated with Kyle's (1985) λ—a measure of price impact cost—and with the fixed cost of trading.[3] Having shown that *ILLIQ* is priced across stocks, I could use it for the time-series analyses.

Another measure of liquidity is trading volume or turnover whose effect on expected return in cross section tests is negative and significant (Brennan et al., 1998, Datar et al., 1998). Turnover is also included in Amihud (2000) and there too its effect on expected return is negative and significant, in addition to the positive and significant effect of *ILLIQ*. However, the time series of aggregate market volume sometimes gives an incorrect reading of market liquidity. During the October 19, 1987 stock market crash, while illiquidity rose, as documented in the Brady report[4] and in Amihud et al. (1990), trading volume rose too.[5] And, during the recent financial crisis in September–October of 2008 market liquidity worsened, *ILLIQ* rose sharply as did other measures of illiquidity (bid–ask spread and Kyle's λ), yet aggregate market volume remained flat.[6] In Amihud (2000) I replicate the time series tests using market turnover series (the cross-section average of stock turnover) and find that while the results are qualitatively similar to those using market *ILLIQ*, when including both market *ILLIQ* and market turnover in the time series model, "only illiquidity is statistically significant whereas turnover is not" (p. 28). My objective in 2002 was to have a single measure of liquidity for both cross-section and time-series analysis that could be (easily) obtained from widely available databases for long periods. *ILLIQ* satisfied this requirement, but other measures are not excluded.

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[2] I state (p. 32), "There are finer and better measures of illiquidity…. These measures, however, require a lot of microstructure data that are not available in many stock markets. And, even when available, the data do not cover very long periods of time."
[3] I regress *ILLIQ* on λ and ψ, the latter being the fixed-cost component related to the bid–ask spread, estimated by the method of Glosten and Harris (1988) and available for 1984. (Michael Bennan and Avanidhar Subrahmnaym kindly provided me with their estimates of these parameters.) The coefficients of λ and ψ are positive and highly significant. Goyenko et al. (2009) and Hasbrouck (2009) show that *ILLIQ* performs best among low-frequency measures of λ.
[4] The Report of the Presidential Task Force on Market Mechanisms, January 1988. Available at https://archive.org/stream/reportofpresiden01unit/reportofpresiden01unit_djvu.txt.
[5] On October 19, 1987, the trading volume of Standard & Poor's (S&P) 500 stocks was 604.3 million shares, compared with the 141.9 million shares that traded a week before, on October 12, 1987. The S&P 500 index levels were 224.84 and 309.39 on October 19 and 12, 1987, respectively, meaning that the dollar volume increased as well on October 19, 1987. Source: finance.yahoo.com.
[6] Amihud and Noh (2018) discuss the inconsistency between the behavior of aggregate market volume and observed market liquidity. The pattern of a rise in market volume when illiquidity rises is also noted by Pastor and Stambaugh (2003).

In what follows, I use *ILLIQ* to present new analyses on the cross-sectional and time-series effects of illiquidity on stock returns.

2. Construction of the illiquid-minus-liquid (*IML*) factor

I present evidence on the illiquidity premium across stocks, using a return factor denoted *IML*, the differential return on illiquid-minus-liquid stock portfolios.[7] The illiquidity of stock *j* on day *d* is measured by $ILLIQ_{j,d} = |return_{j,d}|/dollar\ volume_{j,d}$ and is averaged over a 12-month period that ends in November of each year *y*. The variable $ILLIQ_{j,y}$ is used to analyze stock returns in year *y* + 1, as in Amihud (2002). In calculating annual $ILLIQ_{j,y}$ values, I delete stock–days with a negative price,[8] a trading volume of less than 100 shares, or a return of less than -100% and I delete the highest daily value of $ILLIQ_{j,d}$ in each year. A stock is included if, during the 12-month period, its price is between $5 and $1000 and it has more than 200 days of valid return and volume data. Finally, the sample in each year *y* excludes stocks whose $ILLIQ_{j,y}$ values are in the top 1%, since they are potential outliers. In addition, $SD_{j,y}$ is the standard deviation of the daily returns of stock *j* over the same 12 months. I employ New York Stock Exchange (NYSE)/American Stock Exchange (AMEX) common stocks (codes 10 and 11).

Portfolios are formed in each month *t* (January through December) in year *y* for stocks that satisfy the above criteria and exist at the end of the preceding month. Stocks are sorted on $SD_{j,y-1}$ into three portfolios and, within each volatility portfolio, they are sorted by $ILLIQ_{j,y-1}$ into five portfolios, resulting in 15 (3×5) portfolios. I do the double sorting[9] because these two variables are positively correlated (Stoll, 1978; Amihud, 2002), each having its own effect on expected returns. In my 2002 cross-section analysis I control for volatility by including $SD_{j,y-1}$ among the explanatory variables and finds that its effect on expected returns is negative, as also found in Ang, Hodrick, Xing, and Zhang (2006, 2009).[10] I then calculate the monthly weighted average return for each portfolio using the capitalizations of the previous month as weights.

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[7] This factor is used in Amihud, Mendelson, and Pedersen (2013), Amihud, Hameed, Kang, and Zhang (2015), and Amihud and Noh (2017).
[8] This indicates that the price is the mid-point between the quoted bid and ask prices rather than a transaction price.
[9] This procedure follows that of Fama and French (1993), where *HML* is constructed by double-sorting stocks into size and book-to-market portfolios so as not to confound the effects of these two stock characteristics.
[10] Levy (1978) and Merton (1987) propose that expected stock return is *positively* related to idiosyncratic (and total) volatility because of limited diversification by risk-averse investors.

Stock returns are adjusted by Shumway's (1997) method to correct for the delisting bias.[11] Finally, IML_t is the average of the returns of month *t* of the highest *ILLIQ* quintile portfolios across the three corresponding *SD* portfolios minus the average returns on the lowest *ILLIQ* quintile portfolios across the three corresponding *SD* portfolios.

3. The risk-adjusted premium on *IML*

INSERT TABLE 1

Table 1 presents estimated statistics of *IML* for two periods, where Period I, 1964 to 1997 (408 months), is the period studied in my 2002 article and Period II, 1998 to 2017 (240 months), extends the analysis.

Panel A of Table 1 presents the mean, median, and proportion of months with positive values of *IML*. The mean *IML* value is positive and significant, at 0.635 (t = 4.47) and 0.430 (t = 2.14) in Periods I and II, respectively. The respective medians are 0.615 and 0.218, indicating positive skewness in Period II. In both periods, the proportion of months with *IML* > 0 is significantly greater than 0.50, the chance result.

Panel B of Table 1 presents the risk-adjusted mean *IML* values measured by *alpha*, the intercept from a regression of IML_t on the risk factors of Fama and French (1993) and Carhart (1997) (FFC):

$$IML_t = alpha + \beta_{RMrf}*RMrf_t + \beta_{SMB}*SMB_t + \beta_{HML}*HML_t + \beta_{UMD}*UMD_t + \varepsilon_t\,. \qquad (1)$$

where *RMrf*, *SMB*, *HML*, and *UMD* are, respectively, the market excess return over the T-bill rate and the returns on small-minus-big firms (size factor), high-minus-low book-to-market (BE/ME) ratio firms (value-growth factor), and winner-minus-loser stocks (momentum factor). Panel B1 presents the results using only the factor *RMrf*, because firm size, used to construct *SMB*, is considered a measure of liquidity and the book-to-market ratio used in constructing *HML* is affected by stock liquidity (Fang, Noe, and Tice, 2009). Panel B2 includes all four FFC factors.

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[11] The last month's return of a delisted stock is either the last return available from the CRSP database, RET, or the delisting return DLRET, if available. If both are available, the calculated last-month return proposed by the CRSP is (1 + RET)*(1 + DLRET) - 1. If neither the last return nor the delisting return is available and the deletion code is in the 500s—which includes 500 (reason unavailable), 520 (became traded over the counter), 551–573 and 580 (various reasons), 574 (bankruptcy), 580 (various reasons), and 584 (does not meet exchange financial guidelines)—the delisting return is assigned to be -30%.

In Panels B1 and B2 of Table 1, *alpha* is positive and highly significant for both periods, indicating a positive illiquidity premium adjusted for risk. In Panel B2, *alpha* is 0.372% with t = 3.80 for Period I and 0.403% with t = 3.12 for Period II. The positive and highly significant coefficient of *SMB* reflects the well-known positive relation between illiquidity and small size. The positive slope coefficient of *HML* indicates the greater illiquidity of the stocks with high book-to-market ratio which Fama and French (1993) suggest reflect financial distress, and is also consistent with the evidence of Fang et al. (2009) on the negative effect of illiquidity on the market-to-book ratio. The momentum factor is insignificant.

Panel C of Table 1 presents the estimation results of Model (1) with an added dummy variable, JAN_t, which equals one in the month of January and zero otherwise. This model tests if the illiquidity effect is confined to the month of January, as is the case with the small firm effect. The results show that, in Period I, the January effect is positive and insignificant but, in Period II, it becomes negative and significant. Thus, in the recent 20-year period, the positive illiquidity premium is confined to the 11 months from February to December.

Panel D of the table presents estimates of one-month-ahead out-of-sample estimates of $alpha_t$ of IML_t. I first estimate the coefficients of Model (1) over a 60-month rolling window that ends in month t - 1. Then, the estimated coefficients $\beta_{K,t-1}$ for K = *RMrf, SMB, HML,* and *UMD* are used to calculate $alpha_t$, conditional on the realized factor returns in month t:

$$alpha_t = IML_t - [\beta_{RMrf,t-1}*RMrf_t + \beta_{SMB,t-1}*SMB_t + \beta_{HML,t-1}*HML_t + \beta_{UMD,t-1}*UMD_t] \ .$$

This procedure is repeated by rolling forward the 60-month estimation window one month at a time. Table 1 presents statistics of the series $alpha_t$ for January 1964 through December 2017. The mean out-of-sample $alpha_t$ for Period I is 0.523%, with t = 5.73, the median is 0.480%, which is close to the mean, and the fraction of $alpha_t > 0$ is 0.627, significantly greater than 0.50, which is the chance result. For Period II—after Amihud's (2002) analysis period—the mean $alpha_t$ is 0.333%, with t = 2.58, the median is 0.363%, again close to the mean, and the fraction of $alpha_t > 0$ is 0.579, which is significantly higher than 0.50.

Figure 1 plots the 12-month moving average of $alpha_t$. The series is mostly in positive territory, including in recent years. Its most negative value is in 2000, the year when the dot-com bubble burst.

INSERT FIGURE 1

Panel E of Table 1 presents the test results of the January effect on $alpha_t$. In Period I, the January effect is positive and in Period II it is negative. For the month of January alone, the means $alpha_t$ in Period I (n = 34) and Period II (n = 20) are, respectively, 0.901% (t = 1.76) and -0.834% (t = -2.25). The flip in the sign of the mean in January is a puzzle. For the 11 months from February to December, the means of $alpha_t$ are positive and significant in both periods and of similar magnitude.

4. Effect of market illiquidity shocks on *IML* over time

Amihud (2002) finds a negative and significant relation between market illiquidity shocks and realized stock returns. A positive shock to illiquidity which is highly persistent raises expected illiquidity and makes investors demand higher expected returns on stocks. Consequently, stock prices fall to raise expected returns, assuming that cash flows are unaffected by illiquidity shocks. It follows that the market return, which is the sum of expected and unexpected returns, is negatively affected by contemporaneous illiquidity shocks and positively affected by lagged illiquidity, which is a proxy for expected illiquidity. The analysis follows Merton's (1980) analysis of expected market returns as an increasing function of market volatility, causing expected returns to change through time and the analysis of French, Schwert, and Stambaugh (1987), who find that the market return is a negative function of unexpected market volatility and a positive function of expected volatility.

Earlier studies show that stock prices are negatively impacted by changes in illiquidity. Amihud, Mendelson, and Wood (1990) study the stock market crash of October 19, 1997, when the S&P 500 share index fell by more than 20% and illiquidity sharply increased. The average quoted bid–ask spread (in dollars) increased by 63% relative to its average level in the first week of October and there was a sharp decline in market depth, the size of orders that can be exercised at the quoted bid and ask prices. This study proposes that the price decline occurred partly because of investors' recognition that illiquidity is hurt by program trading, which was prevalent at the time and not as high as previously thought. The finding is that, across firms on the day of the crash, stock returns were negatively related to changes in illiquidity. Amihud et al. (1997) resolve the issue of causality with by presenting evidence that prices rise on stocks that undergo an exogenous increase in liquidity. The Tel Aviv Stock Exchange gradually transferred stocks from trading in a once-a-day call auction session to more continuous trading sessions, which improved liquidity. Measures of stock liquidity—*Amivest* and trading volume— rose. This led to a sharp rise in the price of the transferred stocks. Muscarella and Piwowar (2001) find similar results for the Paris Bourse where prices increased for stocks whose liquidity improved when they were transferred from call trading to continuous trading. In their study of the Japanese market, Amihud et al. (1999) find an increase in stock prices after companies reduced the minimum order size in their stocks, thus facilitating trading by small retail investors, who are viewed as uninformed liquidity traders. Stock price appreciation was an increasing function of the liquidity improvement that resulted from this change.[12]

In addition to the price decline because of the rise in expected returns when expected illiquidity rises, I suggested that illiquid stocks suffer further price declines because of the flight to liquidity, where investors substitute away from illiquid into liquid assets when expected illiquidity rises.[13] For liquid stocks that become more attractive as lliquidity rises, the two effects work in opposite directions and, therefore, the negative effect of illiquidity shocks on prices of liquid stocks is weaker. The differential negative impact of illiquidity shocks on illiquid stocks, defined as illiquidity systematic risk (or illiquidity *beta*), is used in the pricing of stocks and bonds by Pastor and Stambaugh (2003), Acharya and Pedersen (2005), Lin, Wang, and Wu (2011), and Bongaerts, de Jong, and Driessen (2018), among others.

In my 2002 paper, I studied the differential effect of illiquidity shocks on stock returns across five size-based stock portfolios. I now estimate the differential effect of illiquidity shocks on *IML*, the high-minus-low illiquidity quintile portfolio, with the following model:

$$IML_t = \mathrm{a0} + \mathrm{b1}{*}uMILLIQ_t + \mathrm{b2}{*}RMrf_t + \mathrm{b3}{*}JAN_t + u_t, \quad (2)$$

where $MILLIQ_t$ is the market liquidity, calculated as the value-weighted monthly average of stock illiquidity, $uMILLIQ_t$ is the one-month-ahead unexpected illiquidity using an AR(2) model that is estimated dynamically over a lagged rolling window of 60 months, and $RMrf_t$ is added as a control variable.

The monthly market illiquidity series $MILLIQ_t$ is constructed by averaging the market's daily average illiquidity over month *t*. For each day *d* in month *t*, I calculate a weighted average

[12] In a study of the Tel Aviv Stock Exchange, Amihud, Lauterbach, and Mendelson (2003) find a significant increase in stock prices subsequent to the anticipated exercise of warrants that significantly improved stock liquidity.
[13] Supporting evidence for such a pattern is presented by Huang (2010) who finds that mutual funds switch from illiquid to liquid holdings when they anticipate adverse market conditions. Acharya, Amihud, and Bharath (2015) find evidence of a flight to liquidity in corporate bonds that is distinct from flight to safety.

of the daily values of $ILLIQ_{j,d,t}$ using all stocks j that are employed in the construction of IML_t. Then, I calculate the illiquidity shocks as follows. Over a window of 60 months, I conduct a regression of the logarithm of $MILLIQ_t$ on its two lags (and a constant). I adjust the two autoregressive coefficients to account for small-sample bias using the bias correction method of Shaman and Stine (1988, 1989) and I adjust the intercept accordingly. Finally, I calculate $uMILLIQ_t$ for month 61 as the difference between $\log MILLIQ_t$ and its predicted value, using the estimated coefficients from the preceding 60 months. I then roll the window ahead by one month and repeat the procedure. Thus, there is no hindsight in the generation of $uMILLIQ_t$.

INSERT TABLE 2

Table 2 presents the estimation results of Model (2) for both sample periods. The effect of market illiquidity shocks on IML_t is negative in both periods, consistent with Amihud (2002), and it is statistically significant in both periods when $RMrf_t$ is included. Notably, when $RMrf_t$ is regressed on $uMILLIQ_t$, the slope coefficient is negative and significant in both periods. Similarly, in the regression of the component returns of IML_t—the returns on the highest and lowest illiquidity quintile portfolios—on $uMILLIQ_t$ the coefficients are negative and significant in both periods regardless of whether $RMrf_t$ is included in the model. As in the cross-section analysis, the January effect is positive in Period I and negative in Period II.

Adding $logMILLIQ_{t\text{-}1}$, an estimator of expected illiquidity, to Model (2), I find that the coefficients are insignificantly different from zero. In Period I, the slope coefficient of $logMILLIQ_{t\text{-}1}$ is positive as suggested in Amihud (2002) at 0.197 but this difference in the effect of lagged illiquidity on expected return of illiquid and liquid stocks is insignificant, t = 1.23. In Period II, the slope coefficient is -0.070 with t = -0.32, being practically zero.

In summary, there is evidence on a significant greater negative effect of illiquidity shocks on the returns of less liquid stocks, but no support for the hypothesis on the differential effect of expected illiquidity on the expected returns between illiquid and liquid stocks. The difficulty of finding in a time series a significant positive effect of expected illiquidity on expected returns is similar to the difficulty of finding a significant effect of expected risk on expected returns, noted in French et al. (1987). Guo and Whitelaw (2006) discuss the econometric problems in estimating the effect of expected volatility on ex ante return. Some of these problems apply for the estimation of expected illiquidity on expected return. Further study may be needed along the line of Guo and Whitelaw's (2006) study of Merton's (1980) prediction of the varying effect of expected volatility on expected returns, in which they estimate a conditional expectation model based on state variables. The analysis needs to account for not only the effect of macroeconomic state variables on expected illiquidity but also their effect on the price of illiquidity. Brunnermeier and Pedersen (2009) show theoretically that both the level and the pricing of market liquidity varies over time as a function of macroeconomic changes in funding liquidity.

5. Concluding remarks

Amihud and Mendelson (1986) propose that investors require a return premium to compensate for illiquidity costs. This premium differs by investors' holding-period clientele and exceeds expected illiquidity costs because of funding constraints and investor clienteles. This theory predicts a positive relation between expected returns and illiquidity costs. In my view, this is the important takeaway; the specific measure of illiquidity that is used in empirical tests is of secondary importance.

Illiquidity has a number of dimensions that are hard to capture in a single measure, including fixed costs, variable costs—price impact costs that increase in the traded quantity—and opportunity costs. The variable *ILLIQ*, which mainly reflects price impact costs, is one proxy, just as the bid–ask spread was a proxy in Amihud and Mendelson (1986). The variable *ILLIQ* has been shown to have a high positive correlation with high-frequency measures of illiquidity estimated from intraday data, but it does not capture everything about illiquidity. It is possible to combine *ILLIQ* and other low-frequency measures of illiquidity into one measure, using principal component analysis, as in Korajczyk and Sadka (2008) for high-frequency measures. Low-frequency measures of illiquidity are used in Roll (1984), Hasbrouck (2009), Goyenko et al. (2009), Holden (2009), Lesmond, Ogden, and Trzcinka (1999), Pastor and Stambaugh (2003), Liu (2006), Das and Hanouna (2010), Corwin and Schultz (2012), and Fong, Holden and Trzcinka (2017), among others. There is a group of measures that use volume and volatility. Amihud (2002, p. 34) points out that *ILLIQ* is "strongly related to the liquidity ratio known as the *Amivest* measure, the ratio of the sum of the daily volume to the sum of the absolute return," which is used in Amihud et al. (1997). The variable *Amivest* is defined as "a liquidity measure that calculates the dollar value of trading that would occur if prices changed

1 percent."[14] Harris and Amato (2018) test the pricing of a set of simple illiquidity measures that can be constructed using only daily return and volume data. They use $Amivest^{-1}$ and the invariance illiquidity measure of Kyle and Obizhaeva (2016), the third root of the ratio of the variance of returns to the average dollar volume, which can be approximated by the product of $Amivest^{-1/3}$ and the third root of the return standard deviation, which is closely related to the average absolute return. They find that these measures significantly predict expected stock returns.

The magnitude of the relation between expected stock returns and illiquidity costs varies over time. These variations are partly affected by institutional changes in the market which affects liquidity and trading and by the means developed to circumvent the costs of illiquidity. One such means is trading in a liquid security that represents a claim on a portfolio of illiquid assets. Unlike risk, illiquidity is not reduced when holding a portfolio of illiquid stocks. Whereas asset risk can be reduced by portfolio diversification, illiquidity is additive: buying and selling a portfolio of illiquid assets entails bearing the illiquidity costs of its components.[15] However, if the security that represents a claim on a portfolio of illiquid assets is liquid, its pricing will reflect a lower illiquidity premium, which will permeate to the underlying securities and reduce their illiquidity premium.

This has been suggested by Amihud and Mendelson (1988) in the context of the securitization of loans. Banks and financial firms pool and repackage individual loans, which are highly illiquid, into standard debt securities that are liquid traded assets. Competition between financial intermediaries passes the benefits of increased liquidity to the borrowers in the form of lower interest rates or a lower illiquidity premium.

Similarly, the market has developed exchange-traded funds (ETFs) that are often more liquid than their constituent securities that include illiquid stocks and bonds. As in Amihud and Mendelson's (1988) analysis of securitization, ETFs transfer the benefit of their higher liquidity to their constituent securities in the form of a lower illiquidity premium. The existence of liquid ETFs partly underscores the importance of illiquidity costs. Ben-Rephael, Wohl, and Kadan (2015) propose that the expansion of investment through ETFs explains the decline in the

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[14] See http://www.yourdictionary.com/amivest-liquidity-ratio, quoted from Webster New World Finance and Investment Dictionary.

[15] Amihud and Mendelson (2010)

illiquidity premium that they document. Mutual funds, especially index funds, also reduce trading costs and the illiquidity premium on the securities that they hold. At the end of a trading day, a typical fund offsets buy and sell orders for its units, thus transferring ownership of the underlying securities without having to trade these securities. In this way, the fund saves the cost that investors would have incurred if they had directly bought and sold the portfolio of these securities. The fund trades only to the extent required by the residual unmatched demand for its units.

However, there is no free lunch. Cost is partially shifted from trading costs to opportunity costs. Fund investors are restricted to portfolios that could deviate from what they consider optimal. They face a tradeoff: on the one hand, they have to pay higher trading costs when directly holding their optimal portfolio of securities; on the other hand, they bear the cost of deviating from optimality and losing flexibility when holding a liquid fund whose return is correlated with their optimal portfolio. In addition, a fund charges a continuous management fee, which effectively shifts part of the cost from trading costs to these fees. Thus, mutual funds, ETFs, and other such instruments reduce the cost of illiquidity, which is what they are designed to do, but do not eliminate them.[16]

Amihud and Mendelson (2010) also propose that illiquidity costs of asset portfolios can be reduced by *liquidity-motivated portfolio diversification*. This is particularly important for mutual funds that are subject to unexpected redemptions and need to liquidate their holdings. Following Kyle's (1985) model where the price change is an increasing linear function of trade size, the fund's liquidation cost is a convex function of the traded quantity of each security in its portfolio. This calls for a new motive for portfolio diversification in addition to the common risk-reducing motive. In a liquidity-induced diversification, a fund portfolio consists of more securities, each with a smal quantity, and the portfolio weight of each security is inversely related to its trading cost. Amihud and Mendelson (2010) also show that for a given number of securities in the fund portfolio, the optimal fund size is a decreasing function of the price impact (or the Kyle's λ) of its constituent securities. This implies, for example, that a fund of illiquid securities should typically be of a smaller size. In summary, Amihud and Mendelson (2010, p.

[16] Petajisto (2016) finds that deviations of ETF prices from their net asset value reflect the cost of redeeming and creating units of ETFs. These deviations are greater in funds of illiquid securities.

182) propose: "Asset managers should explicitly analyze the transaction cost structure of the assets they manage and take that into account when structuring their investment portfolios."

As before, there is no free lunch. Liquidity-induced diversification may require the construction of an investment portfolio that is suboptimal from a risk-return viewpoint. Also, in actively managed funds, adding securities to be tracked and managed by the fund is particularly expensive because of the costs of analysis and research. Yet, liquidity considerations dictate adding more securities to the fund portfolio in order to reduce liquidation costs. In addition, the liquidity-induced limit on the fund's size makes it forego economies of scale.

Recent developments in capital markets raise the value of liquidity for some investors. Liquid securities are in greater demand by high-frequency traders, consistent with Amihud and Mendelson's (1986) prediction that the demand for assets with different illiquidity costs is affected by investor clienteles that differ in their expected holding periods. Overall, an increase in the trading frequency makes liquidity more valuable. Another development is the rise of activist investors, who favor firms with liquid stock (Fos, 2017).

Ultimately, the evidence supports the proposition that illiquidity is priced, using a variety of proxies, and that there is a positive illiquidity premium which varies over time. Amihud, Mendelson, and Pedersen (2005, 2013) review evidence on the pricing of illiquidity and illiquidity risk. Drienko, Smith, and Reibnitz (2018) and Harris and Amato (2018) find that the cross-sectional effect of illiquidity is positive and significant for their entire estimation period. Worldwide, Amihud, Hameed, Kang, and Zhang (2015) find that stock illiquidity has a positive effect on expected returns.

There are now two major theories of asset pricing, one based on asset risk and the other on asset illiquidity. Both theories propose that these asset characteristics are not desirable by investors and therefore entail an expected return premium. Empirical evidence of the determinants of cross-stock expected return show that the positive pricing of illiquidity is at least as robust as the pricing of risk.[17] The existence of a positive risk premium is supported by

[17] Evidence on the significant pricing of stock systematic (β) risk is meager. However Bali and Engle (2010) find that the dynamically conditional covariance risk is positively priced. Evidence on a negative effect of risk, measured by return standard deviation, on the cross section of expected stock returns is presented in Amihud (2002), Drienko et al. (2018), and Harris and Amato (2018). Ang et al. (2006, 2009) find idiosyncratic risk to have a negative and significant effect on the cross section of expected returns. However, Ghysels, Santa-Clara and Valkanov (2005) find a significant positive relation between risk and expected return and Han and Lesmond (2011)

evidence on a positive and significant mean of *RMrf*, the excess return of the market portfolio over the risk-free rate. Similarly, there is a positive (risk-adjusted) mean of *IML*, the excess return of illiquid over liquid stocks. Although in some periods the realized market excess return is insignificantly different from zero,[18] we do not infer from this evidence that investors are not averse to risk nor do we dismiss the existence of risk premium. Similarly, subperiods with an insignificant relation between illiquidity and expected returns do not imply that investors are not averse to illiquidity cost or that illiquidity is not priced.

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suggest that the negative relation between idiosyncratic risk and expected return is due to idiosyncratic biases in estimated returns. Empirical evidence on the determinants of security returns is reviewed in Bodie, Kane and Markus (2018, Ch. 13).

[18] With hindsight, we observe periods of more than 10 years with a negative arithmetic mean of monthly *RMrf* and there is a period of 40 years, 1969 to 2008, where the arithmetic mean of *RMrf* is 0.338% with t = 1.61, insignificant. The geometric mean during this period is 0.23% with t = 1.08.

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Table 1: Estimates of risk-adjusted returns on an illiquid-minus-liquid (*IML*) portfolio

The variable IML_t is the return on an illiquid-minus-liquid portfolio for month *t*, the differential return between the highest and lowest quintile portfolios of stocks sorted on their illiquidity, measured by the average daily value of *ILLIQ* = |return|/dollar volume. In November of each year, stocks are sorted into three portfolios by *SD*, the standard deviation of their daily returns and, within each tercile portfolio, stocks are sorted into five portfolios by their *ILLIQ* value, producing 15 (3×5) portfolios. The variables *ILLIQ* and *SD* are calculated over 12 months. For each portfolio, the value-weighted average return is calculated for each month *t* from January to December of the following year, using *ILLIQ* and *SD* for the month of November of the previous year. The variable *IML* is the average return on the three highest *ILLIQ* quintile portfolios (across volatility portfolios) minus the average return on the three lowest *ILLIQ* quintile portfolios. We use NYSE/AMEX stocks and apply some filters. The returns are in monthly percentage points. Estimations are carried out for the period 1964–1997, as in Amihud (2002), and for the years that follow, 1998–2017. The *t*-statistics of the estimated coefficients employ robust standard errors (White, 1980). In parentheses next to "% positive" are values from a *z*-test approximation of the binomial test of the proportion against the null of 50%, the chance result.

Panel A shows the statistics for *IML*. **Panel B** presents *alpha* and the β coefficients of the FFC factors obtained from the regression model

$$IML_t = alpha_t + \beta_{RMrf}*RMrf_t + \beta_{SMB}*SMB_t + \beta_{HML}*HML_t + \beta_{UMD}*UMD_t + \varepsilon_t, \qquad (1)$$

where *RMrf* is the market excess return over the risk-free rate, *SMB* and *HML* are the Fama–French (1993) factors of size and the book-to-market (BE/ME) ratio, and *UMD* is the Carhart (1997) momentum factor. **Panel C** shows the estimations of Model (1) with an added variable, JAN_t, which equals one in the month of January and zero otherwise. **Panel D** presents the out-of-sample, one-month-ahead rolling $alpha_t$ values. Model (1) is estimated over a rolling window of 60 months beginning in January 1950. For month 61,

$$alpha_t = IML_t - [\beta_{RMrf,t-1}*RMrf_t + \beta_{SMB,t-1}*SMB_t + \beta_{HML,t-1}*HML_t + \beta_{UMD,t-1}*UMD_t],$$

using the β values estimated from the preceding 60-month window. **Panel E** shows the results from a regression of $alpha_t$ on a constant and JAN_t.

	Periods	
	I: 1964–1997	**II: 1998–2017**
Panel A: Statistics of *IML*		
Mean	0.635 (4.47)	0.430 (2.14)
Median	0.641	0.218
% positive	60.5% (2.37)	56.7% (2.06)
Panel B: Regression of *IML* on risk factors		
Panel B1: Regressions of *IML* on the market excess return		
Alpha	**0.714 (5.18)**	**0.552 (2.86)**
RMrf	-0.154 (-2.85)	-0.221 (-4.70)

Adj. R^2	0.053	0.097
Panel B2: Regressions of *IML* on Fama-French-Carhart factors		
Alpha	**0.372 (3.80)**	**0.403 (3.12)**
RMrf	-0.264 (-8.92)	-0.317 (-9.14)
SMB	0.687 (11.89)	0.676 (9.58)
HML	0.304 (7.44)	0.339 (6.62)
MOM	0.029 (1.01)	0.015 (0.51)
Adj. R^2	0.621	0.613
Panel C: Model (1) with JAN_t		
Alpha	0.398 (3.82)	0.513 (3.70)
Four factors included	Yes	Yes
JAN	0.442 (1.04)	-1.183 (-3.25)
Adj. R^2	0.624	0.616
Panel D: Statistics for rolling one-month-ahead $alpha_t$		
Mean	0.523 (5.73)	0.333 (2.58)
Median	0.480	0.363
% positive	62.7% (5.05)	57.9% (2.45)
N	408	240
Panel E: Regression of one-month-ahead $alpha_t$ on a constant and JAN_t		
Constant	0.489 (5.13)	0.439 (3.30)
JAN	0.412 (1.25)	-1.274 (2.76)

Table 2: Effect of illiquidity shocks on *IML*

This table presents estimation results of the model

$$IML_t = a0 + b1*uMILLIQ_t + b2*RMrf_t + b3*JAN_t + u_t, \quad (2)$$

where $uMILLIQ_t$ is the one-month-ahead unexpected market illiquidity *and* $MILLIQ_t$, market liquidity, is the value-weighted monthly average of stock illiquidity and. Unexpected illiquidity is the difference between log$MILLIQ_t$ and its predicted value using an AR(2) model that is estimated over a rolling window of 60 months up to month *t*-1, and whose coefficients are used to obtain a predicted value of log$MILLIQ_t$. The *t*-statistics in parentheses employ robust standard errors.

	Period I: 1964–1997		**Period II: 1998–2017**	
alpha	0.375 (2.86)	0.420 (3.43)	0.482 (2.26)	0.649 (3.21)
$uMILLIQ_t$	-2.063 (-2.88)	-4.717 (-5.64)	-0.996 (-1.15)	-2.547 (3.06)
$RMrf_t$		-0.292 (-5.21)		-0.259 (5.26)
JAN_t	2.736 (3.41)	3.471 (4.47)	-0.622 (1.09)	-0.906 (1.68)
Adj. R^2	0.082	0.230	0.000	0.124
N	408		240	

Figure 1: 12-month moving average of one-month-ahead rolling *alpha*$_t$

This figure plots a 12-month moving average of the monthly one-month-ahead $alpha_t$, calculated as

$$alpha_t = IML_t - [\beta_{RMrf}*RMrf_t + \beta_{SMB}*SMB_t + \beta_{HML}*HML_t + \beta_{UMD}*UMD_t],$$

where IML_t is the monthly return on an illiquid-minus-liquid portfolio (see Table 1 for details) and the β values are estimated over 60 months preceding month t from the regression Model (1). The sample period is 1964 through 2017. The numbers on the y-axis are monthly returns as a percentage.