Stock Prices Still Move Too Much For Dividends But Less So: A Reappraisal of Shiller 1981

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\section*{ABSTRACT}
In this paper\textsuperscript{1}, we revisit and extend the analysis in Shiller, 1981 to an updated sample. The main puzzling result of the paper is that the fundamental present value model of stock prices predicts a volatility at odds with the data: the stock prices are much more volatile compared to what discounted future dividends would imply. Our paper closely replicates the results for the S&P 500 index. For an updated sample between 1963 and 2018, we find that the excess volatility puzzle is still strong, but it has diminished by a third relative to the sample period in Shiller, 1981.

\section*{Keywords:} excess volatility, present value model.

\section*{JEL Codes:} E5.

\textsuperscript{1}We thank Iwo Welch, an anonymous reviewer, as well as Adrian Cantemir Călin for their comments and observations.
1 Introduction

The contribution by Shiller, 1981 is one of the reference papers that concerns volatility tests. Similar papers that appeared at about the same time are due to Shiller, 1979, Singleton, 1980 or LeRoy and Porter, 1981. In the original interpretation, the paper was regarded as indicating an excess volatility of stock market prices compared to what efficient market models would predict, LeRoy, 1989. In essence, the paper constructed a set of bounds for the volatility of asset prices based on the volatility of discounted future dividends and then compared the real market volatility with the theoretical bounds.

The main result of the paper is quite clear: the volatility found in asset prices is clearly higher compared to the bounds estimated using expected real dividends, contradicting thus the "efficient market model", see Shiller, 1981. The results were verified for both the stock market, Shiller, 1981, LeRoy and Porter, 1981, the bond market, Shiller, 1979 or the term structure of interest rates, see Singleton, 1980.

However, the interpretation of the original findings has been disputed in the following years. Cochrane, 1991 and the even more influential review of Fama, 1991 suggested that the results of volatility bounds tests are not informative about stock market efficiency. As shown by Fama, 1991, the key assumptions in these papers are constant expected returns and the fact that the variation in stock prices is only driven by shocks to expected dividends. However, these assumptions were contradicted by the subsequent literature on the expected stock (as well as bond) returns. Thus, according to Fama, 1991, the early volatility tests are best understood as showing that the expected returns vary in time. We should also note further advances with respect to volatility tests that try to condition the tests on standard consumption-based asset pricing models, see Campbell and Shiller, 1988. However, Shiller, 2014 maintains its original interpretation of the results as indicating a lack of efficiency of financial markets.

The contribution by Shiller, 1981 remains a key paper in what concerns the excess volatility phenomenon. In this paper, we aim to both replicate the original results and to extend them to an updated sample. We also study whether the excess volatility effect has changed in time since it was originally observed in the literature. Although there is a recent discussion by Shiller, 2014 that uses more recent data, the results are not fully comparable with the original ones, because they are neither detrended,
nor are they based on a quantitative assessments volatility differences. A further contribution in our paper is to establish whether the excess volatility phenomena has increased or decreased in recent years and to quantify by how much.

2 Notes on the Methodology

We follow Shiller, 1981 and compute detrended prices and dividends by adjusting real prices and dividends with their long-run exponential growth trend, \( V_t = (1 + g)^{(T-t)}\tilde{V}_T \), where \( g \) is the exponential growth trend over the entire sample, \( V_t \) represents the real detrended value of dividends (d) or stock prices (p) in year t, and \( \tilde{V}_T \) is the unadjusted real value of dividends or stock prices in the terminal year T.

Similarly to Shiller, 1981, in our replication we use a single growth rate for both dividends and prices, this being the average long-run exponential growth trend in prices, \( g = \frac{T-1}{\sqrt{p_T p_0^{-1}}} - 1 \), where \( p_T \) is the real stock price in the terminal year and \( p_0 \) is the real stock price at the beginning of the sample. For assessing the robustness of results, we also consider using the long-run exponential growth trend in dividends, the average increase in log differences for both prices and dividends, or averages of the different growth rates, but the results do not significantly change when using the different possible growth rates.

The present value of real discounted dividends is computed using the backward recursive algorithm of Shiller, 1981, \( p_t^* = (1 + r)^{-1}(p_{t+1}^* + d_t) \), where the terminal value \( p_T^* \) is chosen as the average real detrended stock price over the entire sample.

In following the approach in the original paper, we assume\(^2\) that there is a (deterministic) trend in earnings and, consequently, in stock dividends and prices. Criticism of this assumption was swift in the aftermath of the publication of the paper, and there has not been too much discussion about using detrended data since then. However, many researchers still implicitly assume and employ the idea of a trend in the stock data that has to be eliminated, while movements of stock prices around that assumed trend are thought to contain information about future dividends.

\(^2\)We thank the reviewer for pointing this out.
3 Data Description

The data used in this study are a continuation of the data reported in Chapter 26 (Data Appendix) by Shiller, 1989. The nominal S&P500 index is the January stock price index taken from Standard and Poor’s Statistical Service Security Price Index Record. The nominal dividend series is based on the Standard and Poor’s Statistical Service Security Price Index Record after 1926, and on Cowles, 1939 data series Da-1 before 1926, being adjusted for base year differences. The real stock price and dividend series are computed by dividing the nominal series with the U.S. Bureau of Labor Statistics’ Producer Price Index (PPI), which replaced the Wholesale Price Index (WPI) in 1978. Price data prior to 1913 are obtained from Warren and Pearson, 1935 and are adjusted for base year differences.

When converting from nominal to real values in the replication analysis, we follow Shiller, 1981 (Appendix A, Data Set 1) and adjust the stock price series with the annual average price index before 1900 and with the January price index starting in 1900, both scaled to 1.00 in the base year 1979. The dividend series is adjusted with the annual average price index, scaled to 1.00 in the base year 1979. In the extension analysis, we adjust both the nominal price and dividend series with the Consumer Price Index-All Urban Consumers, taken from the U.S. Bureau of Labor Statistics starting in 1913, which is spliced to the price index of Warren and Pearson, 1935 for years before 1913 by multiplying it by the ratio of the indexes in January 1913. In this case, both are scaled to 1.00 in the base year 2018.

We decided to focus on the S&P500 index and not to analyze the Dow Jones Industrial Average index for brevity and two additional reasons. First, the Dow includes a maximum of 30 stocks, which makes it more limited and less relevant compared to the broader S&P500 index. Second, and more importantly, the Dow is a simple price-weighted index, which makes it less effective in tracking the real market value of the companies in the index. Shiller, 1981, pp.434 recognizes this and explains that “The published Dow Jones Industrial Average, however, is not ideal in that stocks are dropped and replaced and in that the weighting given an individual stock is affected by splits. Of the original 30 stocks, only 17 were still included in the Dow Jones Industrial Average at the end of our sample.” Thus, because of the significant measurement error risk associated with either using the raw version of the index, or with recomposing the index from raw data (which is the solution adopted by Shiller, 1981), we decide to drop the Dow from
4 Results

4.1 Replication of the Original Results

In this section we aim to replicate the key results in Shiller, 1981. Following the explanation about the data choice, our results are limited to the data based on S&P500. Namely, we replicate Figures 1 and 3 from the original paper, as well as Table 2. The original results can be seen in Figure 1 in the main paper and Figure S1 from the supplementary material, as well as Table 1 (left side). Our replication of the corresponding results are in Figure 2, Figure S2 in the supplementary material, as well as Table 1 (right side).

The effect of choosing a different terminal condition is shown in Figure S1 (the original results), while the replications are in Figure S2 in the supplementary material.

In terms of graphical depictions, our results are very similar to the original ones. While the replication is not perfect (because, in the end, the dataset are slightly different), Table 1 shows that the corresponding statistics are very close to the original ones.

4.2 An Extension of the Original Results

In this section, we extend the results from Figures 1 and 3 and Table 2 in the original paper to an updated sample between 1963 and 2018. The results can be seen in Figure 3 (for Figure 1 in the original paper), Figure S3 in the supplementary material (for Figure 3 in the original paper), as well as Table 2 (corresponding to Table 2 in the original paper). Additionally, we also consider the results for a full sample between 1871 and 2018. However, due to the limited space, the results are in the supplementary material, see Figures S4 and S5, although we do include the statistics in Table 2.

Figure 3 basically shows that the excess volatility phenomena is still present in recent data, although it does not show whether it has become stronger or weaker (we discuss this in section 5). On the other hand, Figure S3 shows that changing the assumptions regarding the terminal value has some impact on the results in later years. The results are comparable with those in Figure 3 in the original paper. In the figure based on the full sample
Figure 1: Real S&P500 Stock Price Index (p) and ex-post rational price (p*).

Description: This figure is Figure 1 from Shiller, 1981. It plots the real S&P 500 index (solid line) and the ex-post rational price (dotted line), computed as the present value of real dividends.

Interpretation: The main features of the data series estimated by Shiller, 1981 are replicated quite closely. The figure confirms the excess volatility phenomena of market prices versus ex-post rational prices, for the original sample from 1871 to 1979.
Source: Own Computations

Figure 2: Real S&P500 Stock Price Index (p) and ex-post rational price (p*).

Description: This figure replicates Figure 1 in Shiller, 1981. It plots the real S&P 500 index (solid line) and the ex-post rational price (dotted line), computed as the present value of real dividends. Both series are from 1871 to 1979 and are detrended by dividing a long-run exponential growth factor.

Interpretation: The series of real stock market prices is more volatile compared to the series of real discounted dividends, which proxies the unobserved fundamental value of stock prices. The visual analysis hints to the failure of the constant-growth dividend discount model-labeled as the "efficient market model" by Shiller, 1981-to reliably explain the movements of stock market prices.
Table 1: Sample Statistics for Price and Dividend Series - Original Results and Replication

<table>
<thead>
<tr>
<th>Sample Period</th>
<th>Original</th>
<th>Replication</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1871−1979</td>
<td>1871−1979</td>
</tr>
<tr>
<td>1) E(p)</td>
<td>145.5</td>
<td>134.4</td>
</tr>
<tr>
<td>E(d)</td>
<td>6.989</td>
<td>6.373</td>
</tr>
<tr>
<td>2) (\bar{r})</td>
<td>0.0480</td>
<td>0.0473</td>
</tr>
<tr>
<td>(\bar{r}_2)</td>
<td>0.0984</td>
<td>0.0970</td>
</tr>
<tr>
<td>3) (b = ln\lambda)</td>
<td>0.0148</td>
<td>0.0139</td>
</tr>
<tr>
<td>(\hat{\sigma}(b))</td>
<td>(.0011)</td>
<td>(.0011)</td>
</tr>
<tr>
<td>4) (cor(p,p^*))</td>
<td>.3918</td>
<td>.3922</td>
</tr>
<tr>
<td>(\sigma(d))</td>
<td>1.4811</td>
<td>1.324</td>
</tr>
</tbody>
</table>

Elements of Inequalities:

Inequality(1):

5) \(\sigma(p)\) | 50.12 | 46.68 |
6) \(\sigma(p^*)\) | 8.968  | 8.419 |

Inequality(11):

7) \(\sigma(\Delta p + d_{-1} - (\bar{r})p_{-1})\) | 25.57 | 23.75 |

\(\min(\sigma)\) | 23.01 | 21.37 |
8) \(\sigma(d)/\sqrt{\bar{r}_2}\) | 4.721  | 4.251 |

Inequality(13):

9) \(\sigma(\Delta p)\) | 25.24  | 23.47 |

\(\min(\sigma)\) | 22.71  | 21.12 |
10) \(\sigma(d)/\sqrt{2\bar{r}}\) | 4.777  | 4.301 |

**Description:** This table displays the original results and the replicated results for the sample statistics reported in Table 2 in Shiller, 1981. \(p\) represents the real detrended stock prices, \(d\) represents the real detrended stock dividends, \(\bar{r}\) and \(\bar{r}_2\) represent the 1- and 2-period real discount rates for detrended series, \(\lambda\) represents the trend factor for price and dividend series and \(p^*\) represents the ex-post rational stock price index. \(E\) denotes sample mean, \(\sigma\) denotes standard deviation, \(\hat{\sigma}\) denotes standard error and \(\Delta\) denotes the first difference operator. \(\min(\sigma)\) is the lower bound on \(\sigma\) computed as a one-sided \(\chi^2\) 95 percent confidence interval. The top part of the table reports sample statistics for the relevant variables in the analysis, while the bottom part reports volatility estimates and their corresponding bounds, which are obtained from equations (1), (11) and (13) in Shiller, 1981.

**Interpretation:** The volatility bounds are significantly violated: for all equations, the volatility estimates computed from real stock market prices are more than 5 times higher compared to their theoretical bounds computed from real discounted dividends. In addition, for equations (11) and (13), the lower volatility thresholds computed using a one-sided \(\chi^2\) 95 percent confidence interval is more than 4.75 times higher compared to the theoretical bounds. All differences are significant at the 1 percent confidence level in a standard F-test.
between 1871 and 2018, it appears clearer that the impact of choosing different terminal values is insignificant for most of the sample.

We further present the comparable statistics in Table 2 for both the shorter updated sample (1963 to 2018) and an longer updated sample (1871 to 2018). Although the magnitudes are higher (due to the changes in the corresponding terminal values), the key statistics indicate comparable results with those in the original paper, see inequalities (1), (11) and (13) or the correlation coefficient in row 4).

5 Discussion of Results

The volatility tests have an important role in the development of the finance field. Whether they can be really interpreted as indicating a failure of market efficiency, like Shiller, 1981 or Shiller, 2014 sustain, or just showing that expected returns vary in time, see Fama, 1991, they are nevertheless important in understanding what drives the changes in stock market prices.

Since the original paper by Shiller, 1981, the literature has highlighted and addressed several limiting assumptions of this analysis, such as choosing and adjusting for a growth trend in prices/dividends, fixing or varying the real discount rate. Also, alternative volatility bounds tests have been proposed and used in empirical setups. However, the subsequent research has not overturned the general conclusion that excess volatility is a real stock market phenomena. Real discounted dividends are much smoother compared to real stock prices, regardless of how the former are estimated.

The main results in this paper indicate that the excess volatility of stock prices compared to the original sample has remained significant. A further analysis of the correlations, see row 4) in Tables 1 and 2, indicates a stronger correlation between real detrended stock prices \( p \) and ex-post rational stock prices \( p^* \). This is a first indication of a weakening excess volatility phenomena, because it basically shows that the actual prices follow closer the ex-post rational stock prices implied by the present value model. Further analysis of the key inequalities (1), (11) and (13) seems to suggest the same thing: a significant decline in excess volatility when the sample is updated to recent years. This is however partially true for inequality (1) that indicates a higher excess volatility for the short extended sample, but still a lower excess volatility for the full extended sample.

This is more evident in Table S1 in the supplementary material where
Figure 3: Real S&P500 Stock Price Index (p) and ex-post rational price (p*).

Description: This figure extends Figure 1 in Shiller, 1981 for the sample between 1963 and 2018. It plots the real S&P 500 index (solid line) and the ex-post rational price (dotted line), computed as the present value of real dividends. Both series are from 1963 to 2018 and are detrended by dividing a long-run exponential growth factor.

Interpretation: The series of real stock market prices is more volatile compared to the series of real discounted dividends, which proxies the unobserved fundamental value of stock prices. The visual analysis hints to the failure of the constant-growth dividend discount model-labeled as the "efficient market model" by Shiller, 1981-to reliably explain the movements of stock market prices.
Table 2: Sample Statistics for Price and Dividend Series - Updated Sample

<table>
<thead>
<tr>
<th>Sample Period</th>
<th>Smaller Sample 1871−2018</th>
<th>Full Sample 1963-2018</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) E(p)</td>
<td>2054.5</td>
<td>2055.7</td>
</tr>
<tr>
<td>E(d)</td>
<td>86.416</td>
<td>55.182</td>
</tr>
<tr>
<td>2) $\bar{r}$</td>
<td>0.0421</td>
<td>0.0268</td>
</tr>
<tr>
<td>$\bar{r}_2$</td>
<td>0.0859</td>
<td>0.0544</td>
</tr>
<tr>
<td>3) $b = \ln \lambda$</td>
<td>.0152</td>
<td>.0187</td>
</tr>
<tr>
<td>$\hat{\sigma}(b)$</td>
<td>(.0009)</td>
<td>(.0045)</td>
</tr>
<tr>
<td>4) $\text{cor}(p, p^*)$</td>
<td>.7603</td>
<td>.6265</td>
</tr>
<tr>
<td>$\sigma(d)$</td>
<td>49.505</td>
<td>19.383</td>
</tr>
</tbody>
</table>

Elements of Inequalities:

Inequality(1):

5) $\sigma(p)$   | 896.10                      | 727.53                 |
6) $\sigma(p^*)$ | 682.43                      | 122.15                 |

Inequality(11):

7) $\sigma(\Delta p + d_{-1} - (\bar{r})p_{-1})$ | 350.59                      | 329.46                 |
$\min(\sigma)$ | 320.05                      | 285.01                 |
8) $\sigma(d)/\sqrt{\bar{r}_2}$ | 168.91                      | 83.098                 |
Inequality(13):

9) $\sigma(\Delta p)$ | 347.51                      | 327.04                 |
$\min(\sigma)$ | 317.23                      | 282.92                 |
10)$\sigma(d)/\sqrt{2\bar{r}}$ | 170.68                      | 83.654                 |

**Description:** This table reports the sample statistics for the price and dividend series in the updated sample. $p$ represents the real detrended stock prices, $d$ represents the real detrended stock dividends, $\bar{r}$ and $\bar{r}_2$ represent the 1- and 2-period real discount rates for detrended series, $\lambda$ represents the trend factor for price and dividend series and $p^*$ represents the ex-post rational stock price index. $E$ denotes sample mean, $\sigma$ denotes standard deviation, $\hat{\sigma}$ denotes standard error and $\Delta$ denotes the first difference operator. $\min(\sigma)$ is the lower bound on $\sigma$ computed as a one-sided $\chi^2$ 95 percent confidence interval.

The top part of the table reports sample statistics for the relevant variables in the analysis, while the bottom part reports volatility estimates and their corresponding bounds, which are obtained from equations (1), (11) and (13) in Shiller, 1981.

**Interpretation:** The volatility bounds continue to be violated in the updated sample between 1963 and 2018: for all inequalities, the volatility estimates computed from real stock market prices are more than 2 times higher compared to their theoretical bounds computed from real discounted dividends. However, the volatility ratio exceeded the value of 5 in the original sample, this implying that the excess volatility of real stock market prices has declined in the updated sample. The same conclusion can be extracted from analyzing the lower volatility bound computed using a one-sided $\chi^2$ 95 percent confidence interval in equations (11) and (13) in Shiller, 1981. This violates the theoretical bound by more than 1.85 times in the updated sample. However, this is smaller compared to the 4.85 value in the original sample.
we present a comparative quantitative analysis of the main inequalities. A few conclusions can be drawn. First, the F-test indicates significant differences between the volatility measures: the volatility ratios are thus significant. Second, except the first inequality indicating a slight increase (but only for the 1963 to 2018 sample), there is lower excess volatility for both the shorter sample (by about 30%) and the full sample (by about 60%).

Given the use of annual data, there might be some issues with the statistical significance related to the power of these statistical tests. While this might apply more for the case of the shorter sample, the full sample between 1879 and 2018 is clearly less affected by the lack of power.

An adequate line of research would be to address the sources of this reduced excess volatility: did the market became more efficient (is it absorbing better new information), when analyzing the model from the perspective of Shiller, 1981 or Shiller, 2014, or it just indicates a reduced volatility in expected returns, see Fama, 1991.

References


