# REAL OPTIONS, TAXES AND FINANCIAL LEVERAGE*

Stewart C. Myers [a]
MIT Sloan School of Management and NBER

James A. Read, Jr. [b]
The Brattle Group, Inc.

March 30, 2018

**Abstract**

We show how to calculate the after-tax values of real options, including the value of interest tax shields on debt supported or displaced by the options. The correct discount rate is after-tax. Our valuation method reveals the option's "debt capacity," which can be calculated from the adjusted present value (APV) and target debt ratio for the underlying asset, the option delta, and the amount of risk-free borrowing or lending that would be needed for replication. The debt capacity of a real call option is usually negative. The debt capacity of a real put option is always positive and greater than the value of the put. We review empirical implications for firms' capital structure choices when real options are important.

Keywords: real options, financing policy (JEL: G31, G32)

---

* We thank William Shore and Wei Dou for research assistance and several helpful comments. We also thank Richard Brealey, Christian Laux, and two anonymous referees for their comments and suggestions.

[a] MIT Sloan School of Management, 100 Main Street, E62-620, Cambridge, MA 02142, telephone: (617) 253-6696, email: scmyers@mit.edu.

[b] The Brattle Group, One Beacon Street, Suite 2600, Boston, MA 02108, telephone: (617) 864-7900, email: james.read@brattle.com.

This paper answers three questions. *Question 1*: How should the *after-tax* value of a real option be calculated? Should a before-tax or after-tax risk-free interest rate be used to discount option payoffs? *Question 2*: What is the debt capacity of a real option? How does it depend on the target debt ratio for the underlying asset? *Question 3*: How do real options affect firms' observed debt ratios? The answers to Questions 1 and 2 are closely related, and they in turn answer Question 3, with modest additional downhill reasoning.

Much of this paper is about valuation of real options. We assume rational valuation based on fundamentals. Although we understand that agency and information problems could distort investment decisions, we do not consider such distortions here.[1] We assume that the firm's capital structure is a mix of debt and equity. Rational valuation means that $1 of off-balance-sheet debt counts the same as $1 of explicit debt.

Here is the answer to Question 1: Discount after-tax payoffs to real options at the after-tax risk-free rate. Use the adjusted present value (APV) of the underlying real asset—including the present value of interest tax shields on debt that would be supported by the asset—and forecast the after-tax payoffs to the option as if the expected rate of return on the underlying were equal to the after-tax risk-free rate.[2] This procedure yields the APV of the real option, which includes the value of interest tax shields on debt the option supports or displaces.

The answer to Question 2: "Debt capacity" is a term of art in corporate finance. It is not the maximum amount that could be borrowed against an asset or real option, but

---

1 There is a large literature on the interaction of investment and financing decisions in the presence of agency problems. Mauer and Ott (2000) and Mauer and Sarkar (2005) explore how agency can affect the exercise of real options. Others explore how capital structure choices (debt maturity, debt priority, and the mix of bank and market debt) can mitigate distortions related to agency. See Childs, Mauer, and Ott (2005), Hackbarth, Hennessey, and Leland (2007), Hackbarth and Mauer (2012), and Sundaresan, Wang, and Yang (2015). There is also a literature on the impact of asymmetric information on investment and financing. Papers along these lines include Morellec and Schurhoff (2011), Grenadier and Malenko (2011), and Purnanandam and Rajan (2018).

2 APV is defined narrowly in this paper to be the present value of unlevered cash flows plus (or minus) the present value of interest tax shields on debt supported (or displaced) by those cash flows. The concept of adjusted present value was introduced by Myers (1974) as a general method for incorporating the financial side effects of investment decisions, not just interest tax shields, so in another context APV could refer to a present value that has been adjusted for, say, the value of a lease contract or the cost of issuing new securities.

the target amount of borrowing. If the target debt ratio for the underlying asset is $\lambda$, the debt capacity of a real call (growth) option is:

$$\text{Debt capacity (call)} = \lambda\delta\text{APV} - D_C$$

where APV is the adjusted present value of the underlying asset, $\delta$ is the option delta and $D_C$ is the amount of debt that would be needed to replicate the option. (APV, $\delta$, $D_C$ and perhaps $\lambda$ will of course vary over time, but we omit time subscripts for simplicity.)

We do not calculate the debt capacity of a real option as a stand-alone investment. Instead we assume that a financial manager has, by some combination of observation, analysis and judgment, settled on a target debt ratio $\lambda$ for the underlying asset that will be acquired if the real call option is exercised. We then calculate the debt capacity of the option, assuming consistent application of the same combination of observation, analysis and judgment. Thus the debt capacity of the option is derived from the target debt ratio for the underlying asset, much as the value of the real option is derived from the value of the underlying asset.

The debt capacity of a real call option is usually negative. Negative debt capacity means that the option leverage $D_C$ displaces explicit borrowing, in the same way that off-balance-sheet debt displaces on-balance-sheet debt in a world of full information and rational valuation. The debt capacity of a real put (abandonment) option is always positive and greater than the value of the put itself.

The option leverage $D_C$ equals the present value of the option exercise price times the risk-neutral probability that the option will end up in the money. One may ask whether $D_C$ is really debt; there is no legal obligation to pay the exercise price. But there is a cost of *not* exercising an in-the-money option: the loss of the option's NPV. Option leverage is equivalent to a collateralized non-recourse loan, which can be defaulted on without necessarily compromising the firm's overall solvency. The cost of defaulting is loss of the value of the collateral.

Question 3: Suppose the tradeoff theory holds in its basic form, which predicts that the firm will trade off the marginal values of interest tax shields versus possible future bankruptcy costs or other costs of debt. Suppose that the firm has considered this tradeoff and arrived at constant target debt ratio $\lambda$ for its real assets. If it has no real options, it will keep its debt ratio in a range around $\lambda$. Assume for now that the range is tight enough that the firm's observed debt ratio tracks its target ratio approximately.

Now suppose that the firm acquires valuable growth (call) options to expand assets in place. It will usually appear to operate at a too-conservative debt ratio, because the negative debt targets for the options reduce explicit borrowing. Option leverage displaces explicit leverage. Also the firm will *not* operate at a constant (explicit) debt ratio, and will usually move to a *lower* (explicit) debt ratio when the values of its assets and options increase. This may help to explain why profitable growth firms use so little debt.

These predictions are reversed for a mature firm with valuable abandonment (put) options. Again suppose that the target debt ratio for assets in place is $\lambda$. Then debt ratios for mature firms with valuable abandonment options will be greater than $\lambda$. This result may help to explain why mature firms tend to operate at high debt ratios, especially in LBOs.

There are other intriguing implications. Suppose that the assets underlying the real call or put options have the same business risks as assets in place. If the firm stays close to debt targets for real options as well as assets in place, then equity risk (standard deviation or beta) will not depend on the value or other characteristics of its real options. The real options are riskier than the assets in place, but debt policy compensates.[3] Conventional measures of the after-tax weighted average cost of capital (WACC) will nevertheless be overstated for growth firms—not because the cost of equity is too high, but because the observed (that is, explicit) debt ratio understates the firm's true debt ratio, which includes option leverage. Conventional measures of WACC will be understated for declining firms.

If option leverage displaces explicit leverage, then growth firms should *not* calculate WACC using market-value weights of outstanding debt and equity. Instead they should use the target debt and equity ratios for their *assets in place*. That is, they should

---

[3] Growth options are long the underlying asset. If the assets underlying the firm's growth options are of the same type (risk and debt capacity) as its assets in place, a combination of the firm's assets in place and its growth options is riskier than the assets in place alone. Abandonment options, on the other hand, are short the firms' underlying assets, so a combination of the firm's assets in place and its abandonment options is less risky than its assets in place.

use what their target financing ratios *would be* if the firm held no growth options. This result contradicts almost all corporate-finance authorities, but follows inescapably if firms adhere closely to target debt ratios and real options are important.

We will also show that exercise of a real call option increases the firm's overall debt capacity by the amount of the exercise price. Thus the immediate source of financing for new capital investment can be 100% debt, even though the target capital structure for the firm as a whole is a mix of debt and equity.

This paper fills two major gaps in the theory and practice of corporate finance. The first is a gap in capital budgeting. Although research in real options has been underway for over 30 years and textbooks have covered the topic for most of that time, the literature has been silent on whether and how to incorporate taxes and debt capacity in real-option valuation. As a result, the treatment of two closely related problems—the valuation of real options and the valuation of assets in place—has been disconnected. On the one hand, the basic textbook advice for calculating the value of real assets is to discount unlevered after-tax cash flows at an after-tax weighted average cost of capital. On the other hand, textbooks recommend discounting real-option payoffs at a before-tax risk-free rate. Our real-option valuation procedure can be viewed as the (heretofore missing) real-option corollary to the standard textbook procedure for calculating the value of real assets in place.

The second gap this paper fills is in research on capital structure. Some empirical results that seem inconsistent with the basic tradeoff theory may actually be explained by the dynamics of debt targets for growth or abandonment options. For example, the low debt ratios of profitable growth firms is a direct prediction of the tradeoff theory if growth opportunities are real call options and option leverage displaces explicit borrowing. On the other hand, the dynamics are complex. If real options are important, then the basic tradeoff theory does *not* predict constant observed debt ratios nor does it predict debt ratios that follow any simple target-adjustment model. Thus we can rationalize some financing patterns that seem inconsistent with the basic tradeoff theory, but it is difficult to confirm or reject the theory for firms holding valuable real options. Nevertheless it is essential to understand the debt capacity of real options in order to implement a valid test of the basic tradeoff theory or of more complex theories.

This problem arises because the econometrician observes only explicit borrowing, that is, the debt that appears on the company's balance sheet, not total debt, which includes the implicit, off-balance-sheet debt created by real options. (Similar problems could arise from other off-balance-sheet obligations, including derivatives.[4]) We examine this problem in the context of the basic tradeoff theory, but the same problem arises in testing any capital structure theory on samples of firms with valuable real options. A solution may be to focus such tests on samples of mature, profitable firms for which the value of real growth and abandonment options is small relative to the value of assets in place.

We discuss empirical implications in section 5 below. Prior research on real options and tests of capital structure theories is reviewed there. The remainder of this introduction summarizes research on taxes and discounting and previews our valuation results and our formula for a call option's debt capacity. We close the introduction by summarizing the simplifying assumptions that set the boundaries of the paper. Section 2 explains step by step how taxes affect real-option values and debt capacity. Section 3 presents debt- and tax-adjusted binomial and Black-Scholes-Merton option-valuation formulas. Section 4 investigates how real options can affect targets for explicit borrowing and how the targets evolve as the options mature. If the firm respects these targets, at least approximately, then it follows a precautionary debt policy that limits explicit borrowing. The policy builds up reserve debt capacity so that exercise prices of real calls can be 100% debt-financed. This section also digs deeper into the costs of financial distress that may be created by option leverage and considers how option leverage could affect the optimal amount of debt in a tradeoff model of capital structure. Section 5 covers empirical implications. Section 6 concludes.

---

[4] A further complication: the firm may have to post collateral to protect counterparties, but the collateral is not normally available to secure on-balance-sheet lenders.

## I. Prior Research on Taxes, Discount Rates and Real Options

The effects of capital structure and taxes on the value of the firm have been investigated, it seems, from every possible angle.[5] Also, the interplay of taxes, financing, and the cost of capital has been well-covered in the finance literature on the valuation of capital-investment projects. Key papers include Myers (1974), which introduces the APV method, Miles and Ezzell (1980), Ruback (1986), and Taggart (1991).[6] But as far as we know the effects of financial leverage and taxes on the valuation of real options have never been addressed.[7]

Real options are normally valued as derivatives by the risk-neutral method, that is, by calculating the payoffs as if the expected rate of return on the underlying asset is equal to the risk-free rate and then discounting at the risk-free rate. The before-tax risk-free rate is used in practice, with no adjustments for taxes or leverage. Dixit and Pindyck "largely ignore taxes" (1994, p. 55). Trigeorgis (1996), Copeland, and Antikarov (2001), and Smit and Trigeorgis (2004) use a before-tax risk-free rate. Brennan and Schwartz's (1985) real-options analysis of natural resource investments values cash flows after taxes but uses a before-tax rate. All of the examples in McDonald's (2006) practice-oriented survey discount at the before-tax rate. The case studies on the Real Option Group's website follow the same procedure.[8] Hull (2012, ch. 35) and Amram and Kulatilaka (1999) do not consider whether risk-free rates should be before- or after-tax. Brealey, Myers, and Allen (2017) cover taxes and leverage in detail when explaining WACC and APV, but then use the before-tax risk-free rate in their chapter on real options. Ross, Westerfield, and Jaffee (2011) and Berk and DeMarzo (2014) follow suit.

---

5 We will not attempt a comprehensive review of research on capital structure, which is enormous. Review articles include Harris and Raviv (1991), Myers (2003), Frank and Goyal (2008), and Graham and Leary (2011). None of these reviews covers the tax and debt-capacity implications of real options.

6 For a summary, see Brealey, Myers, and Allen (2017, Chapter 19). See also Ruback (2002) and Gamba, Sick, and Leon (2008).

7 Morellec and Schurhoff (2010) explore the implications for corporate financing and investment decisions, including decisions to exercise real options, when shareholders' have options to defer capital gains taxes under the personal tax code. We focus on corporate taxes, the value of interest tax shields, debt capacity, and leverage. Morellec and Schurhoff do discount at an after-tax rate but do not explain why and do not otherwise consider the valuation and financing issues addressed in this paper.

8 The case studies' DCF valuations discount at the after-tax weighted average cost of capital (WACC), but option valuations just use a risk-free rate with no mention of taxes. See http://rogroup.com.

Ruback (1986) explained why the after-tax risk-free rate $r(1-T)$ should be used to value risk-free cash flows.[9] Intuition suggests that the after-tax rate should also be used to discount certainty-equivalent option payoffs. We will show that the intuition is correct.

Modigliani and Miller (MM) (1963) derive the tax- and leverage-adjusted discount rate $r_i{*}=r_i(1-\lambda T)$, where $r_i$ is the opportunity cost of capital for assets in risk class $i$ and $\lambda$ is the target debt ratio for the firm.[10] Nowadays $r_i$ would be defined as an unlevered cost of capital, likely derived in practice from a capital asset pricing model. Myers, Dill, and Bautista (1976) derive the discount rate for financial leases as $r=r_D(1-\lambda T)$, where $r_D$ is the before-tax cost of debt and $\lambda$ is the fraction of debt supported or displaced by the lease. These discount rates are useful for calculating APVs in some circumstances, but are not designed to value real options. But we will derive a similar discount-rate formula for valuing certainty-equivalent cash flows from assets in place.

### A. Preview of results

Taxes are important, not just because corporations pay them, but also because in practice interest tax shields are assumed to be valuable. To value a real option, we must determine the amount of debt supported or displaced by the option.

Black-Scholes (1973) and Merton (1973) demonstrated that the instantaneous payoffs to a call option can be replicated by delta ($\delta$) units of the underlying risky asset less a specific amount of riskless borrowing. Therefore, we can express the value of a real option as the net value of the two components of a replicating portfolio. The first component for a call ($\delta$ units of the underlying APV) is perfectly correlated with the underlying asset and should therefore have the same proportional debt target over the next short time interval, that is, $\lambda\delta\mathrm{APV}$. The second component (borrowing of $D_C$) is off-balance-sheet debt, which in a world of rational valuation displaces explicit debt dollar for dollar. Therefore,

---

9 See also Brealey, Myers, and Allen (2017, ch. 19 Appendix).

10 Myers (1974) shows that the MM formula works only for level perpetuities and fixed borrowing. The correct formula for finite-lived projects with expected cash flows that vary over time is in Miles and Ezzell (1980).

$$\text{Debt capacity (call)} = \lambda\delta\text{APV} - D_C$$

Thus the target debt for a real option depends on the value and target debt ratio of the underlying asset and the amount of implicit borrowing in the replicating position. Interest on the implicit debt is not tax-deductible, however. There is a tax cost when implicit option debt displaces explicit borrowing.

Notice that our debt targets are not derived from a model of optimal debt. Instead we start with the firm's target debt ratio for the underlying asset and derive the debt capacity for the real option that is consistent with that target.

The debt-capacity equation holds only for an instant. The parameters change as time passes and the APV of the underlying asset fluctuates. We show in Section 4 how borrowing evolves if the firm stays close to the debt targets that we derive.

We show how to calculate real-option values after-tax, including the values of interest tax shields. Our method is most easily implemented by the following two steps.

1. Calculate the APV of the underlying real asset, including the present value of interest tax shields on debt supported by the asset. Use this APV as the value of the underlying asset.
2. Calculate the after-tax payoffs to the real option as if the expected rate of return on the underlying asset were equal to the after-tax risk-free rate. Then discount the option payoffs to present value at the after-tax rate.

The second step operates in an *after-tax* risk-neutral setting, where the risk-neutral rate is set so that the expected rate of return on the underlying asset equals the after-tax risk-free rate.

### B. Simplifying Assumptions

We make important simplifying assumptions in order to convey the paper's chief ideas. These assumptions set the boundaries of our analysis:

1. We assume a world of rational valuation, in which agency or information problems do not spoil a clear view of option leverage and do not distort option-exercise decisions.

2. We assume the firm is sure to pay corporate income tax and that interest tax shields are valuable.
3. We assume the firm has arrived at a target debt ratio $\lambda$ for the assets underlying the real calls and puts. We do not rely on any particular model of capital structure, although later in the paper we discuss option leverage in the context of the basic tradeoff model. We do not adjust for possible feedback effects of real options on the debt target. We discuss this possibility in Section 4, however.
4. We assume exercise prices are known and fixed, although this assumption could be relaxed.

## II. Calculating After-Tax Real Option Values

The logic of after-tax option valuation is best introduced in one-period binomial examples. After-tax binomial and Black-Scholes-Merton formulas are derived in the next section. We will concentrate on European calls, although the same logic applies to puts. We assume a before-tax risk-free rate of $r$ = 6%. For simplicity we also assume that the corporation can borrow or lend at 6% pre-tax. The opportunity cost of capital is $k$ = 10%. The marginal corporate tax rate is $T$ = 35%.

### A. APV of the Underlying Asset

Start with the value of the underlying real asset; the growth option will arrive in a moment. Assume that the asset generates a single expected cash payoff of $V_1$ = 110 after corporate taxes at t = 1. There are no intermediate cash flows between t = 0 and t = 1. Discounting at the opportunity cost of capital gives a present value $V_0$ = 110/1.10 = 100. The asset's payoff can also be expressed as a certainty-equivalent payoff of CEQ($V_1$) = 106. Discounting at the 6% risk-free rate gives the same $V_0$ = 100.

This valuation assumes all-equity financing and does not include the value of interest tax shields on the debt supported by the asset. Assume that the firm's target debt ratio for assets in place is $\lambda$ = 50%, expressed as a fraction of APV.

$$\begin{aligned}\mathrm{APV}(V_1) &= \frac{\mathrm{CEQ}(V_1)}{1+r} + \frac{\lambda r T\,\mathrm{APV}(V_1)}{1+r} = \frac{\mathrm{CEQ}(V_1)}{1+r(1-\lambda T)} \\ &= \frac{106}{1+0.06\,(1-0.5\times 0.35)} = \frac{106}{1.0495} = 101\end{aligned} \tag{1}$$

APV($V_1$) includes the value of one period's interest tax shields on debt of $\lambda$APV($V_1$) = 50.5. The tax shields are worth 1.0. Note from Eq. (1) that the certainty-equivalent discount rate is $r(1-\lambda T)$. The term $(1-\lambda T)$ adjusts for leverage and taxes. Note too that the relationship between PV and the tax- and leverage-adjusted APV is

$$\mathrm{APV}(V_1) = \mathrm{PV}(V_1)\left(\frac{1+r}{1+r(1-\lambda T)}\right)$$

We will work with certainty-equivalent cash flows to give a clearer comparison of discount rates for real assets and real options. But the APV of 101 could have been calculated by discounting the expected payoff of 110 at an after-tax weighted average cost of capital (WACC). WACC can be calculated using the Miles-Ezzell (1980) formula

$$\begin{aligned}\mathrm{WACC} &= k - \lambda r T\left(\frac{1+k}{1+r}\right) \\ &= 0.10 - 0.5\times 0.06\times 0.35\times\left(\frac{1.10}{1.06}\right) = 0.0891\end{aligned}$$

Discounting the expected cash flow of 110 at 8.91% gives the APV of 101.

The WACC can also be calculated from the costs of debt and equity. The cost of debt $r_D$ is the risk-free rate of $r$ = 6% and the levered cost of equity $r_E$ is 13.92%.[11]

$$\begin{aligned}\mathrm{WACC} &= r_D(1-T)\lambda + r_E(1-\lambda) \\ &= 0.06\times(1-0.35)\times 0.5 + 0.1392\times 0.5 = 0.0891\end{aligned}$$

## B. APV of a Forward Contract

The simplest derivative is a one-period forward contract to purchase the asset for a fixed price $X$. The long position in the forward contract receives the cash flow $V_1$ and pays the fixed price. The APV of the forward contract is

---

[11] An investor who buys a 50-50 portfolio of debt and equity has a claim on the unlevered asset, worth 100, and the safe interest tax shield, worth 1. The expected return on this portfolio is 9.96%. The cost of equity can be backed out as 13.92%.

$$\text{APV}(\text{forward}) = \text{APV}(V_1) - \text{APV}(X)$$

We have already calculated APV($V_1$), but now have to calculate APV($X$). Assume that the contract price is $X = 100$, so the forward is in the money. If we forget about debt and taxes, the PV (not APV) of the forward purchase price is $X/(1 + r) = 100/1.06 = 94.34$. But $X$ is a fixed, debt-equivalent obligation, which substitutes for explicit debt one for one if valuation is rational. Therefore we discount at the after-tax risk-free rate $0.06(1-0.35) = 0.039$, which gives APV($X$) = 100/1.039 = 96.25. This APV calculation follows from Eq. (1) when $\lambda = 1.0$.

Define $D_F$ = APV($X$) as the amount of debt needed to replicate the forward contract. The APV of the forward contract is APV($V_1$) − $D_F$ = 101 − 96.25 = +4.75. Notice that calculating the contract's APV requires two discount rates, $r(1-\lambda T)$ for the underlying asset and $r(1-T)$ for the forward purchase price. Notice also that using the after-tax rate to discount the payment $X$ grosses up its PV by the factor $\dfrac{1+r}{1+r(1-T)}$.

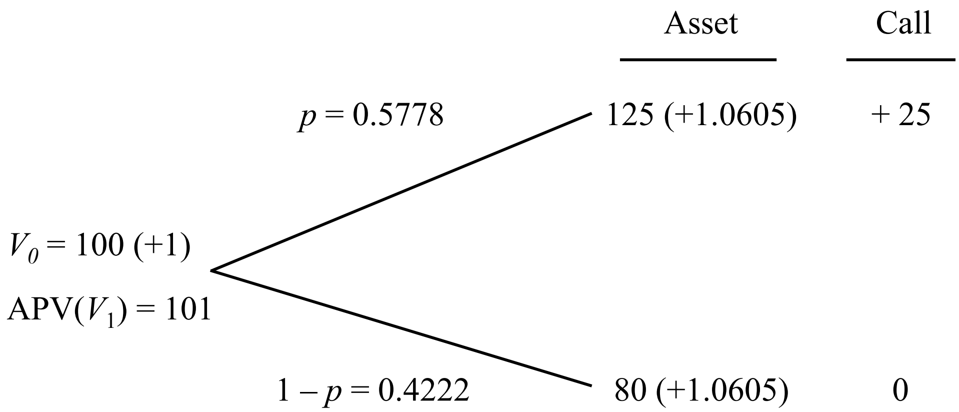
The debt capacity of the forward contract is negative in this example. APV($V_1$) supports debt of 50.50. APV($X$) displaces explicit borrowing of 96.25. The net debt capacity is −45.75. A firm holding the forward contract would borrow 45.75 less against its other assets and appear to operate at an excessively conservative debt ratio.[12]

### C. APV of a Real Call Option

Now consider a one-period call option for the same real asset with an exercise price of $X$ = 100. We start with standard valuation practice, using a before-tax risk-free rate (6%) and a one-period binomial event tree with rate of return outcomes of +25% (the "up" branch) and –20% (the "down" branch). The risk-neutral probabilities of the up and down branches are $p = 0.5778$ and $1-p = 0.4222$, so that the expected rate of return

[12] Marking the forward contract to its market value of 4.75 and putting this net value on the asset side of the balance sheet does not solve this problem. The firm would still appear to have unused debt capacity from its assets in place. If the accounting were fully revealing, it would split the forward contract in two, showing APV($V_1$) on the left side of the balance sheet and $D_F$ = APV($X$) as an explicit debt-equivalent liability on the right.

equals 6%. Here is a tree diagram showing the possible outcomes for the value of the underlying asset and the corresponding payoffs to the call option:




Interest tax shields on debt supported by the asset are split out and shown in parentheses. The tax shields are determined by borrowing at $t = 0$ and do not depend on the outcome at $t = 1$. The payoffs to the call do not yet take account of interest tax shields.

The PV (not yet APV) of the call is (0.5778×25)/1.06 = 13.63. We can break out this value as the difference between two values, just as for the forward contract. The two values are $\delta = 0.5556$ units of the underlying asset and implicit debt of $D_C = 41.93$.

$$\text{PV(call)} = \delta V_0 - D_C = 55.56 - 41.93 = 13.63$$

These two components comprise the replicating portfolio for the call.[13] The first term is $\delta$ units of the real asset. The second term $D_C$ is the PV of a future fixed payment of 41.93 plus interest of $0.06 \times 41.93 = 2.52$. We now have to calculate the APV of each term.

The first term $\delta V_0$ has exactly the same risk as the asset in place over the next short time interval and therefore the same target debt ratio of $\lambda = 50\%$. Thus we replace $\delta V_0$ with $\delta\text{APV}(V_1)$, which is worth 0.5556×101 = 56.11.[14] The second term $D_C$ has to be recomputed, because it displaces debt dollar for dollar and its APV must take account of the tax shields lost on the debt it displaces. We therefore discount the future payment of 41.93 + 2.52 = 44.45 at the after-tax risk-free rate of 3.9%. So $D_C = 44.45/1.039 = 42.78$. The APV of the call is

---

[13] Most real options cannot be explicitly replicated and hedged, but that does not change valuation principles. See Brealey, Myers, and Allen (2017, p. 589).

[14] The option delta does not depend on whether $V_1$ or APV($V_1$) is used for the underlying real asset.

$$\text{APV(call)} = \delta\text{APV}(V_1) - D_C = 56.11 - 42.78 = 13.33$$

Now $D_C$ is the amount of debt in a portfolio that replicates the option payoffs, including the associated interest tax shields.

The APV of the real call option is only 30 cents less (13.33 vs. 13.63) than its PV calculated by standard methods, which ignore debt and taxes. This is a small difference, but the example looks at only one time step in a binomial tree. We show later that the valuation errors from ignoring debt and taxes can be large in more realistic cases.

As for the forward contract, the calculation of APV(call) uses two tax-adjusted discount rates. APV($V_1$) can be valued by discounting the certainty equivalent of $V_1$ at $r(1-\lambda T)$. $D_C = \text{APV}(D(1+r))$ is valued by discounting at the after-tax risk-free rate $r(1-T)$. The calculation of $D_C$ as an APV grosses up our first calculation by a factor of

$$\frac{1+r}{1+r(1-T)}$$

The debt capacity of this call is negative. The debt capacity of the first term is $\lambda\delta\,\text{APV}(V_2) = 0.5 \times 0.5556 \times 101 = +28.06$. But the option's implicit leverage $D_C$ displaces debt of 42.78. The net debt capacity is the difference: 28.06 − 42.78 = − 14.72. Real call options usually have negative debt capacity, although debt capacity can be positive if the options are far enough in the money.[15]

Suppose the firm has one asset in place worth APV = 101 and a real call option to buy one more. If the firm's debt is on target, its market-value balance sheet (showing explicit debt only as $D$) is

[15] Change our example so that $X = 25$ rather than 100. Now $\delta = 1$ and $\delta\lambda\text{APV}(V_1) = 50.50$. The APV of the option debt is $D_C = 25.51$. The net contribution to debt capacity is 50.50 − 25.51 = +24.99.

| | | | |
|---|---|---|---|
| APV = | 101.00 | $D$ = | 35.78 |
| APV(call) = | 13.33 | $E$ = | 78.55 |
| | 114.33 | | 114.33 |

The asset in place has debt capacity of 50.50, but the firm only borrows 35.78, because the call's debt capacity is −14.72. The market debt ratio would be recorded as $35.78/114.33 = 0.313$, far below the target debt ratio of 50% against the APV of assets in place. The firm would appear to operate at a conservatively low debt ratio. In fact it is at its debt target. It is borrowing the same 50% of the APV of the asset in place and of $\delta\text{APV}(V_1)$, the option's position in an identical asset by way of the call. But it is also reducing borrowing dollar for dollar to compensate for the APV of the leverage in the call.

We can also write the balance sheet making the option leverage explicit:

| | | | |
|---|---|---|---|
| APV = | 101.00 | $D$ = | 35.78 |
| | | $D_C$ = | 42.78 |
| $\delta\text{APV}(V_1)$ = | 56.11 | $E$ = | 78.55 |
| | 157.11 | | 157.11 |

Notice that total debt (save for one penny added by rounding) is exactly 50% of firm value $\left((35.87 + 42.78)/157.11 = 0.50\right)$. Therefore the standard deviation or beta of the equity is not changed by the growth option.[16] The risk on the left of this balance sheet is the same with or without the growth option, because $\delta\text{APV}(V_1)$ is strictly proportional to

[16] William Shore was the first to notice this result.

the APV of the asset in place over the next interval of time. The financial risk on the right is the same, because the debt ratio remains at $\lambda = 0.5$ when $\delta\text{APV}(V_1)$ is shown as an asset and option leverage is recognized. Of course the growth option is riskier than the asset in place, but the firm offsets the additional risk by reducing explicit borrowing.

Suppose our firm undertakes to calculate its WACC for assets in place. If valuation is rational, the market risk of the firm's equity (its standard deviation and beta) will reflect the debt implicit in its real options as well as its explicit debt. In fact, its equity risk will be the same as it would be if the firm didn't hold a real option but did have explicit debt equal to 50% of the value of its underlying asset. Suppose the firm manages to measure its costs of debt and equity correctly at 6% and 13.92%. If it uses the 50% debt ratio in this last balance sheet, it gets the right answer: 8.91%, as calculated earlier. But it will probably use the 31.3% debt ratio from the previous balance sheet, which does not show option leverage. It will obtain a WACC estimate of 10.78%, almost 200 basis points too high.

$$\text{WACC (estimated)} = 0.06 \times (1 - 0.35) \times 0.313 + 0.1392 \times 0.687 = 0.1078$$

The example shows why conventional measures of WACC can be overstated for growth firms that reduce explicit debt in order to offset growth-option leverage. The overstatement does not occur because the cost of equity is too high, but because the observed debt ratio is too low. The observed ratio does not include option leverage.

The opposite error occurs for declining firms with valuable put (abandonment) options. In this case the market-value debt ratio is too high, because the firm can exploit the positive debt capacity created by the puts.

These measurement errors can be corrected by a simple change in how WACC is calculated. Disregard the authorities who recommend weighting the costs of debt and equity by the firm's market-value debt and equity ratios. Use the target debt and equity ratios for assets in place. This correction clearly works in our examples, and it works generally, provided, of course, that the firm stays at or near its target debt for assets in place and real options combined.

The possible effects of real options on measurement of WACC were first noted by Myers and Turnbull (1977, p. 332) and recently by Bernardo, Chowdhry and Goyal (2012). But these papers did not consider the displacement of explicit borrowing by option leverage. In effect they assumed that the debt capacity of real call options is small or zero. We show that assumption is not correct.

Use of WACC in practice almost always assumes a constant target debt ratio $\lambda$. Our valuation results do not require this assumption, however, because we calculate option value and debt capacity at a single point in time, and $\lambda$ may change over time. Option leverage $D_C$ will also change over time as the replicating portfolio for the option evolves.

### D. APV in an After-Tax Binomial Tree

We have calculated APVs in a conventional binomial event tree—a tree in which the risk-neutral drift rate equals the before-tax interest rate. Suppose we switch to an *after-tax* binomial tree. That tree is identical to the conventional tree except for the probabilities, which are reset so that the expected return is the after-tax rate $r(1-T)=0.06(1-0.35)=0.039$. The new probabilities are $p$ = 0.5311 and 1 − $p$ = 0.4689. The APV of the underlying asset is not changed. But option leverage increases, because the discount rate is lower. In this tree, option leverage of 42.78 is calculated in one step, but the result is exactly equal to $D_C$ calculated as an APV in the before-tax tree.

Switching from the before-tax to the after-tax tree increases option leverage by the factor $\frac{1+r}{1+r(1-T)}$, the same as the gross-up required in the before-tax tree. Therefore we can bypass the conversion of $D_C$ to APV and just value real-option leverage in an after-tax risk-free setup. Details are in the next section.

## III. After-Tax Binomial and Black-Scholes-Merton Formulas for Real Options

Now we generalize to a real call option on a longer-lived asset. We present valuation formulas for a European call on the underlying asset at each time interval in a binomial event tree. The formulas converge to Black-Scholes-Merton as calendar time is divided into smaller intervals and the number of intervals becomes very large.

Assume that the asset generates periodic cash flows at a known rate $y$ that is proportional to end-of-period APV. We will call this rate the cash flow yield.[17] The payoff at date t + 1 from purchasing the asset at date t is $\text{APV}_{t+1}(1 + y) + \lambda r T\text{APV}_t$. $\text{APV}_{t+1}$ captures the present value at t + 1 of all subsequent cash flows and associated interest tax shields.

The asset can be valued recursively, discounting expected payoffs at a tax- and leverage-adjusted discount rate. If the expected payoffs are certainty equivalents, as in Eq. (1), $\text{APV}_0$ is

$$\text{APV}_0 = \frac{\text{CEQ}\left[\text{APV}_1(1+y) + \lambda r T\,\text{APV}_0\right]}{1+r} = \frac{\text{CEQ}\left[\text{APV}_1(1+y)\right]}{1+r(1-\lambda T)}$$

Once $\text{APV}_0$ is calculated we can proceed to value a call on the asset. We start as before with a conventional, before-tax binomial event tree with return outcomes $u$ and $d = 1/u$. We use APV as the value of the underlying asset.

| | Asset | Call |
|---|---|---|
| | $p\,[u\text{APV}_0\,(1+y) + \lambda r T\text{APV}_0]$ | $C^u - rTD_C$ |
| $\text{APV}_0$ | | |
| | $(1-p)\,[d\text{APV}_0\,(1+y) + \lambda r T\text{APV}_0]$ | $C^d - rTD_C$ |

The option payoffs in this event tree include two terms. The first terms $C^u$ and $C^d$ are standard. For example, if the option matures in the next period,

$$C^u = \text{Max}\{u\text{APV}_0 - X, 0\}$$
$$C^d = \text{Max}\{d\text{APV}_0 - X, 0\}$$

(The option does not get the cash flow yield $y\text{APV}_0$ or the interest tax shield $\lambda r T\text{APV}_0$ on debt supported by the underlying real asset.) The second term $rTD_C$ captures the interest tax shields lost because of debt displaced by the option's implicit leverage.

17 A constant cash-flow yield implies that the asset is a growing, declining, or level perpetuity. The cash flow yield for a finite-lived asset must increase as the asset approaches the end of its life.

The replicating portfolio is described by two equations in two unknowns, the option delta ($\delta$) and option leverage ($D_C$)

$$\begin{aligned} C^u \; - \; rTD_C \; &= \; \delta\, u \mathrm{APV}_0 \; - \; D_C(1+r) \\ C^d \; - \; rTD_C \; &= \; \delta\, d \mathrm{APV}_0 \; - \; D_C(1+r) \end{aligned}$$

These equations simplify to

$$\begin{aligned} C^u \; &= \; \delta u \; \mathrm{APV}_0 \; - \; D_C\left(1+r(1-T)\right) \\ C^d \; &= \; \delta u \; \mathrm{APV}_0 \; - \; D_C\left(1+r(1-T)\right) \end{aligned} \tag{2}$$

Eq. (2) implies that the fractional debt capacity of a real call option is always less than for the underlying asset. The debt capacity would be the same ($\lambda$) but for option leverage $D_C$, which enters as a negative term in Eq. (2).

The lost tax shields $rTD_C$ are the same in the up and down states, so the formula for the option delta is $\delta = \dfrac{C^u - C^d}{(u-d)APV_0}$. Given $\delta$, the amount of debt displaced is

$$D_C = \frac{\delta u\, APV_0 - C^u}{1 + r(1-T)} = \frac{\delta d\, APV_0 - C^d}{1 + r(1-T)} \tag{3}$$

Eq. (3) for $D_C$ discounts at the after-tax rate $r(1 - T)$. The standard formula, which ignores interest tax shields on displaced debt, would discount at the before-tax rate $r$. Thus Eq. (3) grosses up option leverage by the now-familiar factor $(1+r)/(1+r(1-T))$.

Of course the gross-up is not needed in an after-tax binomial tree, where the expected risk-neutral rate of return on the asset is $r(1-T)$. The APV of option leverage is generated automatically in the after-tax tree, with no required explicit adjustment for interest tax shields lost because of displaced debt. The after-tax tree is identical to the conventional tree, except for the risk-neutral probabilities, which depend on the after-tax risk-free rate. The probability of the "up" branch is

$$p \; = \; \frac{\dfrac{1+r(1-T)}{1+y} - d}{u-d}$$

Moving from the before-tax to after-tax tree does not change the state variable $\mathrm{APV}_0$. The amount of debt in the replicating portfolio changes, because of the gross-up for lost interest tax shields on explicit debt displaced by option leverage.

## A. Volatility of APV in discrete time

We used $APV_0$ as the state variable in the binomial event tree. $APV_0$ includes the interest tax shield $\lambda rTAPV_0$, which is fixed by borrowing at $t = 0$ and does not depend on the outcome at $t = 1$. This tax shield does not affect the payoffs to the call or the dollar spread between up and down payoffs to the real asset, but it does reduce the risk of the next payoff to the real asset. Therefore the tax shield could affect volatility and the choice of $u$ and $d$, the rate of return outcomes in the tree. Thus we pause to consider what determines volatility when APV is used as the state variable in a discrete-time binomial setup.

The call does not receive next period's interest tax shield or the cash flow $yAPV_1$. The call's value depends only on $APV_1$. Suppose that the firm rebalances its capital structure in each period to stay close to a target debt ratio. (This assumption underlies use of WACC for discounting.) Then $APV_1$ has the same volatility as the unlevered asset value $V_1$. Thus the up and down moves $u$ and $d = 1/u$ should be based on the volatility of $APV_1$.[18]

The only safe interest tax shield is the first one, $\lambda rTAPV_0$. Later tax shields share the risks of future cash flows and APVs, provided that debt is rebalanced as APV fluctuates. For example, firms that discount at WACC must assume, at least implicitly, that future debt levels will be adjusted to keep debt at a constant fraction of APV. In that case, the value at date $t = 1$ of the tax shield at date $t = 2$ is proportional to the realized $V_1$ and $APV_1$. It is proportional because the firm is assumed to rebalance at date $t = 1$ and borrow the fraction $\lambda$ of the realized outcome $APV_1$. Viewed from date $t = 0$, the interest tax shield for $t = 2$ has exactly the same risk as $APV_1$ or the unlevered payoff $V_1$. So do the interest tax shields for $t = 3, 4, \ldots$ . When the firm rebalances its capital structure every period, the tax shields for $t = 2$ and beyond have the same risk as future cash flows and unlevered asset values.

The assumption that firms rebalance their capital structures in each future period is of course a stretch. But the assumption is common in practice when firms discount at WACC, and it provides a useful reference point.

---

[18] The cash flow yield is proportional to $APV_1$ and does not affect percentage volatility.

The general APV method does not require rebalancing every period. It can incorporate fixed debt repayment schedules or debt policies that adjust with lags to changes in target debt levels. The assumption of regular rebalancing for the underlying could also be relaxed in valuing real options. Valuation principles would not change, although the volatility parameter would not be constant. We do not pursue such extensions here. Our examples will assume that the volatility of the APV of the underlying asset is stable over time.[19] This is consistent with the implicit assumptions underlying the use of WACC to discount unlevered after-tax cash flows.

### B. Valuing puts

Corporations also hold real put options, that is, options to abandon existing assets. If the put matures in the next period, the payoffs are

$$\begin{aligned} P^u &= \text{Max}\{X - u\text{APV}_0, 0\} \\ P^d &= \text{Max}\{X - d\text{APV}_0, 0\} \end{aligned}$$

Puts are valued in the same way as calls, except that puts support debt, not displace it. Thus we distinguish $D_P$, the debt supported by a put's negative leverage, from $D_C$, the debt displaced by the option leverage in a call. The replication equations for a put in the before-tax binomial tree are:

$$\begin{aligned} P^u + rTD_P &= (\delta - 1)u\text{APV}_0 + D_P(1+r) \\ P^d + rTD_P &= (\delta - 1)d\text{APV}_0 + D_P(1+r) \end{aligned}$$

Option leverage $D_P$ now has a positive sign, because a put is replicated by a short position in the underlying asset plus lending. The put delta is $\delta - 1$, where $\delta$ is the delta for a call with the same maturity and exercise price.[20] These equations simplify to:

$$\begin{aligned} P^u &= (\delta - 1)u\text{APV}_0 + D_P(1 + r(1-T)) \\ P^d &= (\delta - 1)d\text{APV}_0 + D_P(1 + r(1-T)) \end{aligned} \tag{4}$$

The amount of lending required to replicate the put is:

---

[19] The volatility could also fluctuate due to operating leverage, that is, fixed costs embedded in the cash flows to assets in place. Our option valuation procedure could be modified to incorporate operating leverage, but we do not address that modification here.

[20] The delta of a plain vanilla European put is equal to one minus the delta of a vanilla European call with the same maturity and exercise price. This relationship is not true of American options, however.

$$D_P = \frac{P^u - (\delta - 1)uAPV_0}{1 + r(1-T)} = \frac{P^d - (\delta - 1)dAPV_0}{1 + r(1-T)} \qquad (5)$$

Notice that $D_P$ is calculated by discounting at the after-tax risk-free rate. The discounting is automatic in the after-tax binomial tree.

The fractional debt capacity of a real put option is always positive, always greater than for the underlying asset, and always greater than 100% of the value of the put. Compare the formula for put-option replication with the put's debt capacity:

Replication: $\text{APV}(put) = (\delta - 1)\text{APV}(V_1) + D_P$

Debt capacity: $\lambda(\delta - 1)\text{APV}(V_1) + D_P$

APV(*put*) is non-negative, so $D_P \geq -(\delta - 1)\text{APV}(V_1)$. The put's debt capacity must therefore be positive. Also, debt capacity must be greater than APV(*put*), because $\lambda(\delta - 1)\text{APV}(V_1) > (\delta - 1)\text{APV}(V_1)$. Put value and put debt capacity can approach zero when the put is far out of the money, however.

### C. A Modified Black-Scholes-Merton Formula

Cox, Ross and Rubinstein (1979) derived a multi-period binomial option pricing model by recursive application of this single-period valuation logic, assuming that the replicating portfolio is revised each period in each possible state conditional on the price of the underlying. They also showed that in the continuous-time limit, as the length of the time interval approaches zero and the return parameters *r*, *u* and *d* are rescaled accordingly, their binomial option pricing formula converges to the Black-Scholes-Merton formula. The convergence proof does not depend on the nature of the underlying asset (here an APV) or on the definition of the risk-free rate (here the after-tax rate $r(1-T)$).[21] Therefore, we can modify the Black-Scholes-Merton formula to take account of taxes and leverage by (1) substituting the APV of the underlying asset for the PV and (2) substituting the after-tax risk-free rate for the before-tax rate.

[21] The Black-Scholes-Merton formula was derived by imagining that the replicating portfolio is revised continuously as time elapses and the price of the underlying stock evolves. Our modified formula reflects an additional, tacit assumption that corporate borrowing is revised continuously to maintain the target debt ratio in market value terms.

Suppose the underlying asset generates all-equity cash flows at a known rate $y$ in proportion to its market value. If $\tau$ denotes the time remaining to expiration, the APV of a call is

$$\begin{aligned}\text{APV}(\text{call}) &= N(d_1)\text{APV}(V_\tau) - N(d_2)\text{APV}(X) \\ &= N(d_1)\text{APV}_0(1+y)^{-\tau} - N(d_2)X(1+r(1-T))^{-\tau}\end{aligned}$$

where

$$d_1 = \frac{\ln\left\{\dfrac{\text{APV}_0}{X}\right\} + \left(\ln\{1+r(1-T)\} - \ln\{1+y\} + \tfrac{1}{2}\sigma^2\right)\tau}{\sigma\sqrt{\tau}}$$

$$d_2 = d_1 - \sigma\sqrt{\tau}$$

$\text{APV}_0$ is the spot value of the stream of cash flows and associated interest tax shields generated by the underlying real asset. If the asset generates income before date $\tau$ ($y > 0$), the APV of the underlying delivered immediately will be greater than APV($V_\tau$), which is a deferred claim. This is accounted for by "discounting" at the cash-flow yield rate $y$.[22]

The Black-Scholes-Merton formula for the value of a real put option is:

$$\begin{aligned}\text{APV}(\text{put}) &= N(-d_2)\text{APV}(X) - N(-d_1)\text{APV}(V_\tau) \\ &= N(-d_2)X(1+r(1-T))^{-\tau} - N(-d_1)APV_0(1+y)^{-\tau}\end{aligned}$$

The drift term in the modified valuation formulas is the after-tax interest rate adjusted for payouts. Thus, real options are valued *as if* the APV of the underlying real asset is expected to appreciate at the after-tax risk-free rate. One might therefore call the modified formula "risk-and-debt-neutral valuation". Thus whether one applies an analytical formula (such as the modified Black-Scholes-Merton formula), simulation methods, or some other numerical approximation procedure, one can work with the APV of the underlying asset and use an after-tax risk-neutral drift rate (equal to the after-tax risk-free rate adjusted for payouts).

Notice that the target debt ratio $\lambda$ does not appear in the valuation formulas. Thus, the value of a real option does not depend directly on the debt capacity of the underlying asset, just as the value of an option does not depend directly on the expected rate of return

[22] Merton (1973) derives the formula for the value of an option on the common stock of a firm that follows a proportional dividend policy. The cash flow yield $y$ plays the same role in our analysis as the dividend yield does in Merton's analysis.

on the underlying asset. Debt capacity and expected rate of return influence real option values only insofar as they affect the APV of the underlying.

The modified Black-Scholes-Merton formula is useful chiefly as a means to explore the effects of taxes and financial leverage on the value and debt capacity of real options. In practice, real options almost always have timing flexibility and/or other features that require the application of numerical approximation techniques, such as Monte Carlo simulation. But the essential elements of our real option valuation procedure can be applied when using any numerical approximation technique: (1) the APV of the underlying asset is used as the state variable and (2) expected option payoffs (using risk-neutral probabilities) are discounted at the after-tax risk-free rate.

A natural extension of the modified model—again, with a view to exploring the effects of taxes and financial leverage on the value and debt capacity of real options—would be to allow for uncertain interest rates or an uncertain exercise price, as in Fischer (1978) or Margrabe (1978). That extension is straightforward in principle, but it would require an analysis of the debt displaced by the uncertain exercise price.

### D. Examples

The modified Black-Scholes-Merton formula will tell us whether the valuation errors from ignoring taxes and the debt capacity of real options are practically important. Table 1 reports the values of real call options with exercise prices of 100. The underlying asset values (APVs) range from 40 to 200. The options are European, with maturities of 1, 3, or 5 years. The volatility of the underlying is 20%. The tax rate is 35%. Panel A assumes a before-tax risk-free interest rate of 6% and a cash flow yield of 10%. Panels B, C, and D show results for other combinations of the interest rate and cash flow yield.

[Table 1 here]

Each entry in Table 1 gives the correct option value, taking account of the option's debt capacity and the interest tax shields lost because of debt displaced by the option, and below it the absolute (dollar) and percentage errors from *not* adjusting for taxes and leverage. Take for example the 5-year at-the-money option in panel A. The option is worth 5.01 using the modified Black-Scholes-Merton formula. Valuing the option by the conventional method—that is, Black-Scholes-Merton with a pre-tax interest rate—would value the option at 6.79, an overvaluation of 1.78 or 36%.

Table 1 shows that real call options are always *overvalued* by the conventional method. Dollar and percentage errors are material. Percentage errors are larger for longer maturities and larger for out-of-the-money calls than for in-the-money calls. Absolute errors are large for in-the-money calls. Dollar and percentage errors are also greater when the before-tax interest rate is high (Panel C). This is as expected, since the chief difference between the standard and modified Black-Scholes-Merton model is use of an after-tax interest rate. The tax adjustment to the interest rate matters more when interest rates are high.

Table 2 shows values and valuation errors for real put options. Puts are systematically *undervalued* by conventional methods. Dollar and percentage errors are again material. For example, the 5-year at-the-money put option in panel A is worth 25.50 using the modified Black-Scholes-Merton formula. The conventional method would value the option at 19.42, a valuation error of −6.08 or −24%.

[Table 2 here]

These examples indicate that ignoring taxes and debt capacity can introduce serious errors in valuing real options. Of course there will also be cases where the errors are not material and where conventional option-valuation practice is an acceptable approximation.[23] We do not see the point of the approximation, however, since the correct tax- and leverage-adjusted valuation method is no more complicated or burdensome than the conventional method.

## IV. Option Leverage

This section investigates debt targets for options and how the targets evolve as options mature. We review the similarities and differences between explicit borrowing and option leverage.

---

[23] There may be cases where mistakes in valuing the underlying real asset create errors that offset the errors in Tables 1 and 2. We have investigated how the errors change if real option payoffs are discounted at the before-tax risk-free rate but the values of interest tax shields are excluded from the values of the underlying assets. We find that the two wrongs do not make a right answer. In some cases the errors in Tables 1 and 2 are little changed. In other cases the signs of the errors are reversed. The net effect depends on the extent to which the option is in or out of the money, the debt capacity of the underlying asset, and on other inputs. Overall accuracy is not improved by ignoring the value of interest tax shields.

It is helpful to begin with a real forward contract. Recall that when an option is deep in the money, it is nearly a foregone conclusion that the option will be exercised. In that case a call option is effectively a long position and a put option is effectively a short position in a forward contract.

The debt capacity $B$ of a long position in a real forward contract depends on the APV of the contract price, the target debt ratio for the underlying asset, and the APV of the underlying at the forward delivery date:

$$B(\text{forward}) = \lambda\,\text{APV}(V_0)(1+y)^{-\tau} - X\left(1+r(1-T)\right)^{-\tau}$$

A forward contract may displace or support borrowing. A long position will displace (support) borrowing if the APV of the exercise price is greater (smaller) than $\lambda\,APV(V_0)(1+y)^{-\tau}$, which is the debt capacity of the underlying. These conclusions are of course reversed for a short position. The debt capacity of a real forward will fluctuate with the value of the underlying asset if the target debt ratio is greater than zero.

Plain-vanilla European calls and puts are equivalent to simple combinations of digital options. A plain-vanilla call (put) option is equivalent to the combination of a short (long) cash-or-nothing call (put) and a long (short) asset-or-nothing call (put), both with the same strike price and time to expiration. The debt capacity of a real option is equivalent to a weighted combination of the same digital components, where the weight on the asset-or-nothing option is equal to $\lambda$ and the weight on the cash-or-nothing option is equal to one. Therefore, the debt capacities of real European calls and puts can be calculated using the modified Black-Scholes-Merton formulas with one further change: the APV of the underlying asset is weighted by its debt capacity $\lambda$. The debt capacities of calls and puts are:

$$B(\text{call}) = \lambda\,N(d_1)\text{APV}_0\,(1+y)^{-\tau} - N(d_2)X\left(1+r(1-T)\right)^{-\tau}$$

$$B(\text{put}) = N(-d_2)X\left(1+r(1-T)\right)^{-\tau} - \lambda\,N(-d_1)\text{APV}_0\,(1+y)^{-\tau}$$

The debt capacity of a call depends on the APV of the underlying, offset by the debt that would be needed for replication, that is, $D_C = N(d_2)X\left(1+r(1-T)\right)^{-\tau}$. Option leverage $D_C$ equals the APV of the exercise price multiplied by the probability that the option will mature in the money and the exercise price paid. The replicating loan for a put is $D_P$ =

$N(-d_2)X(1+r(1-T))^{-\tau}$ , which is the APV of the exercise price multiplied by the probability that the put will mature in the money and the exercise price received.

Figures 1 and 2 are graphs of option debt capacity versus the APV of the underlying asset. The options have a strike price of 100 and a time to expiration of one year. The debt capacity of the underlying asset is 0%, 25%, or 50%. Figure 1 shows debt capacity for real call options and Figure 2 shows debt capacity for real puts. Option debt capacity for calls is always less than or equal to zero *if* the underlying asset has no debt capacity. Call option debt can be positive if the underlying asset has positive debt capacity and the option is deep in the money, however. Option debt capacity for puts, in contrast, is always greater than or equal to zero. The debt capacity of the underlying is small compared to the debt capacity of the exercise price when puts are near or in the money.

[Figure 1 and Figure 2 here]

### A. Is Call-Option Leverage Really Debt?

One may ask whether option leverage $D_C$ is really off-balance-sheet debt; there is no claim held by an outside lender. We argue that option leverage acts like debt and would be regarded as debt-equivalent by a rational CFO.

Debt creates financial risk, which shows up as a higher equity standard deviation or beta. Option leverage likewise adds financial risk. Suppose the firm decides not to "pay off" the leverage in a call option by exercising the option. That is not an event of default. The firm is not exercising a default put.

A default put does not necessarily require putting all the firm's assets to lenders. A collateralized non-recourse loan can be defaulted on without compromising the firm's overall solvency. The cost of defaulting is loss of the value of the collateral.

A real call option is equivalent to a collateralized non-recourse loan. The firm can default on payment of the exercise price if it gives away the value of the collateral, which is the PV of the underlying asset. The cost is loss of the NPV of the option.

A real put option is equivalent to holding the underlying asset, but retaining an insurance policy giving the right to swap the asset for the exercise price. The insurance policy puts a floor under asset value and allows the firm to borrow more than it could otherwise. The holder of a put is a lender, who retains the right to receive cash if he or she does not want the underlying asset.

### B. Option Leverage and Costs of Financial Distress

Consider a tradeoff model, in which the PV of interest tax shields is balanced at the optimal margin by the PV of costs of bankruptcy and financial distress. Can the leverage in real call options create costs of financial distress? How does option leverage affect a tradeoff optimization?

Tradeoff models must assume material adjustment costs if the firm encounters a severe downside shock to profitability and asset value. This assumption is too often left implicit. If the adjustment costs were trivial, it would be easy to escape financial distress by issuing a large block of fresh equity and paying down debt. The firm could operate at high debt ratios but remain solvent by frequent debt rebalancing. The firm could also ignore option leverage and wait until the last minute to raise a package of debt and equity to finance option exercise.

Adjustment costs can include agency or incentive problems that prevent the firm from rebalancing. In addition, the time required to decide on and implement rebalancing transactions can be long enough that firms need to allow an extra equity cushion to keep the probability of financial distress remote, especially if asset values can jump (change discontinuously) and volatility is stochastic.

Assume that adjustment costs are important. Suppose a firm starts near its target but considers as a thought experiment acquisition of real call options without cutting back explicit borrowing. The thought experiment would take the firm to a higher overall debt ratio when option leverage is included.

Option leverage amplifies financial risk in the same way as explicit debt. A new call option is equivalent to acquiring a new asset encumbered by a non-recourse collateralized loan, as in project finance. Project-finance debt adds financial risk even when failure of the project cannot force failure of the firm.

Suppose a shock to profitability and asset value puts the firm in serious financial distress. Cash is constrained because operating profitability has vanished, at least temporarily, and adjustment costs limit access to fresh equity. Explicit debt faces default risk. Then part of the value added from paying the call exercise price (thus paying off the implicit option debt) goes to holders of explicit debt. The standard debt-overhang problem described in Myers (1977) arrives and the firm cuts back investment. What is the resulting cost of financial distress? Loss of the NPV of the option. This cost could be large, especially for growth companies with extensive investment opportunities, and especially if the investment would help the firm recover from the downside shock.

If the firm does not recognize option leverage and does not reduce explicit debt to compensate, then option leverage can lead to costs of financial distress: (1) Call-option leverage increases financial risk and the probability that the firm will land in financial trouble and face the usual costs of financial distress. (2) Trouble means that exercise of real call options can be hobbled by debt-overhang problems. Cutting back investment can in turn increase the odds of default on explicit debt.

Now consider how call-option leverage affects a tradeoff optimization. Once the call is on the balance sheet, the firm can change its total debt ratio only by adding or subtracting explicit debt. Suppose it considers borrowing $1 more or less. The PV of interest tax shields should be the same at this margin with as without the call. If the marginal PV of distress costs is likewise the same, the firm has no incentive to depart from its optimal debt ratio for assets in place only.

The call is of course worth less because the implicit interest on option leverage is not tax-deductible. Our valuations reduce call values to compensate, which may in turn reduce the size or number of real calls that the firm decides to hold. But here we are assuming that the call is already on the firm's market-value balance sheet. The inability to get tax shields on the call's implicit interest is a sunk cost, which does not affect the marginal conditions for a tradeoff optimum.

The effect of a real call on a tradeoff optimization should therefore depend on whether the marginal PV of distress costs is increased or reduced by call-option leverage. It could go either way. This marginal PV could be reduced, because failure to "pay off" option leverage by exercising the call will not trigger covenants and is not an event of default. In that case the firm can expand explicit debt and move to a higher total debt ratio. But the PV could be higher at the margin if more explicit debt increases debt-overhang problems and the likelihood that valuable calls will not be exercised. In that case the firm can reduce explicit debt and move to a more conservative total debt ratio.

Our valuation methods and examples have not considered how option leverage might affect the marginal PV of distress costs and therefore shift the firm's overall target debt ratio. Ignoring this possible feedback effect could be an important simplification that may reward further investigation.

### C. How Debt Capacity Evolves as a Call Option Matures

If real-option leverage acts like debt, and the CFO regards it as a debt-equivalent obligation, then it should displace explicit borrowing. Let us see how the displacement works as time passes and the option approaches maturity. We assume that the target debt ratio $\lambda$ for the underlying assets is constant over time.

Table 3 gives examples of the debt capacity of a call with exercise price = 100 on an asset worth APV = 125 in Panel A or 80 in Panel B. For simplicity we hold the underlying APVs constant as the time remaining to maturity declines in steps from five periods to zero. Line (1) gives the value of the call and Line (2) gives the call's debt capacity. Debt capacity is the sum of Line (3), the option leverage $D_C$ (the product of the APV of the exercise price and the risk-neutral probability that the call will end up in the money) and Line (4), the call's positive contribution to debt capacity (the product of the APV of the underlying asset and the option delta).

[Table 3 here]

In Panel A, where the option stays in the money until maturity, the option's positive contribution to debt capacity (Line 4) starts at 32.81 with five periods to maturity and increases to 37.50 the instant before exercise, when $\delta = 1$. Option leverage (Line 3) increases from 62.69 with five periods to maturity to 100 the instant before exercise, when the probability of exercise is 1.0. Overall debt capacity gets more and more negative as time passes and the probability of exercise increases. It is −62.50 the instant before exercise.

In Panel B, where the option begins and remains out of the money, debt capacity starts negative, but smaller in absolute amount than in Panel A. Option leverage $D_C$ goes away as time passes and the probability of exercise goes to zero.

The examples illustrate how a CFO would adjust explicit borrowing targets in a company with valuable real call options. Assume the calls have negative debt capacity, as in Table 2. (Calls can have positive debt capacity if they are far enough in the money.)

The CFO would recognize each call's positive contribution $\lambda\delta$APV, but also its option leverage $D_C$. The probability-weighted obligation to pay the exercise price (or lose the PV of the underlying) steadily increases for options that stay in the money, and steadily decreases for options that stay out of the money. A CFO with a large in-the-money call would set more and more conservative targets for explicit borrowing as time passes and the probability of exercise increases. A CFO with a large out-of-the-money call would start with a conservative target for explicit borrowing, but that target would recover towards the debt ratio for assets in place as time passes and the probability of exercise falls.

Suppose the CFO sets targets in this fashion. Consider the instant just before a call option is exercised. At that point $\delta = 1$ and $D_C = X$, the exercise price. So the option's debt capacity the instant before exercise is $\lambda \text{APV} - X$. The instant after exercise, when the new asset is paid for, the debt capacity is $\lambda$APV. The option leverage is extinguished, so the firm's overall debt capacity has increased by the amount of the exercise price $X$. The firm can now borrow that amount explicitly. Thus the immediate source of financing for exercising a real call option can be 100% debt, even though the target capital structure for the firm as a whole is a mix of debt and equity.

Thus a CFO who cuts back targets for explicit debt to offset the negative debt capacity of (most) real call options preserves financial flexibility, that is, reserve borrowing power to finance future investment. Graham and Harvey (2001, Table 5) asked managers to identify the most important considerations affecting their decisions about debt policy. Financial flexibility was at the top of the list.

## V. Empirical Implications

We have assumed a target debt ratio $\lambda$ for the underlying asset, as is customary when valuing capital-investment projects. We have recognized option leverage as a debt-equivalent obligation and valued it as such. If the CFO likewise recognizes option leverage as debt-equivalent, we can calculate how his or her targets for explicit borrowing change and evolve when the firm holds real options.

We now move to Question 3: What does our analysis predict for the debt policy of corporations holding valuable real options? We will focus on the most basic tradeoff theory, which predicts that firms settle on target debt ratios and can adjust their mix of debt and equity to stay in a range around the target. We assume that ranges are tight enough so that observed debt ratios can reveal useful information about targets. If that assumption holds, then our predictions about targets for explicit borrowing are in principle testable.

In focusing on the basic tradeoff theory, we are aware that there are competing theories of capital structure and that empirical tests of the basic tradeoff theory have yielded results that can at best be described as mixed.[24] We are also aware that the basic tradeoff theory has been generalized to incorporate financial frictions, including adjustment (especially recapitalization) costs.[25] Thus we do not endorse the basic tradeoff theory as complete and correct. It is nevertheless important to work out its empirical implications when the firm holds real options that account for a significant fraction of firm value. First, it is not possible to carry out a valid test of a theory unless its implications are understood. We argue that the implications of the tradeoff theory for firms that hold real options have *not* heretofore been understood. Our results may help explain some empirical findings cited as evidence against the theory. Second, the problems that real options create for tests of the basic tradeoff theory also lurk in tests of more complex theories. The problems are caused by the off-balance-sheet leverage created by real call options (or the implicit lending position in real put options). Third, it is useful to know how real options could influence observed debt ratios and distort measurements of WACC.

---

[24] Graham and Leary (2011, p. 310), in a literature review that focuses on the tradeoff and pecking order theories, conclude that "Each view succeeds in explaining several broad patterns in observed capital structures…. However, neither view has succeeded in explaining much of the observed heterogeneity in capital structures, leverage changes, or security issuance decisions."

[25] Fischer, Heinkel, and Zechner (1989) conclude that small adjustment costs can lead to large variations in debt ratios over time. They express target debt ratios in terms of ranges rather than point values. Leary and Roberts (2005) find that when capital structure adjustments are costly, equity price shocks have persistent effects even if firms actively rebalance their capital structures. Flannery and Rangan (2006), after analyzing the impact of adjustment costs, conclude that firms have target capital structures, but that they move towards long-run targets at a rate of only about 30% per year. Strebuleav (2007) observes that due to financial frictions firms will most of the time be at capital structures that differ from targets. Using a model of dynamic capital structure that incorporates debt and equity issuance costs and costly asset sales, he simulates cross sectional capital structure data and finds that the results can lead to rejection of the very financial model on which they are based.

### A. Prior Research

We are not the first to consider how real options could affect capital structure choices. Some empirical results have been attributed in a qualitative way to the presence of growth options. For example, growth firms with high market-to-book ratios tend to use less debt, which makes sense if growth options increase market values but have little, no or negative debt capacity. See Barclay, Smith, and Watts (1995) for an example of this view of the evidence.

Mauer and Triantis (1994) develop a model of a firm with operating flexibility and financial flexibility. The firm can start and stop production in response to uncertain product prices, subject to specified start and stop costs. It can also issue and retire debt and equity subject to specified recapitalization costs. They conclude that operating and financial flexibility increase debt capacity, that operating and financial flexibility are substitutes, and that the impact of debt on operating and investment decisions is "economically insignificant." Note that the embedded options in their model are operating options, not investment options.

Barclay, Morellec, and Smith (2006) present an agency model in which growth options have negative debt capacity and present empirical results indicating that firms with growth options tend to operate at low book debt ratios. We agree with these authors that growth options usually have negative debt capacity, but our modeling approach is different. They present an agency model in which debt constrains managers' impulse to overinvest. Their empirical predictions are qualitative. Our paper uses standard option valuation methods—no agency issues—and derives debt targets for real options from the targets for assets in place. We can calculate the implications of growth options for the basic tradeoff theory.

Tserlukevich (2008) develops empirical implications if firms do not borrow against growth options because there is no tax advantage from doing so. He in effect assumes that real options have zero debt capacity. We show that this assumption is incorrect.

Some papers consider how growth options affect the risk of levered common equity. Bernardo, Chowdhry, and Goyal (2012), Gomez and Smidt (2010), Hackbarth and Johnson (2015) and Jacquier, Titman, and Yalcin (2010) are recent examples. The latter article shows that equity betas are more closely linked to the implicit leverage in growth options than to explicit borrowing.[26] We agree that the implicit debt in options creates financial risk, but we demonstrate that corporations following the basic tradeoff theory should compensate by reducing explicit borrowing. Thus the Jacquier et al. article's results can be read as saying that the tradeoff theory is *not* working.

Hackbarth and Johnson (2015) model a firm with both growth and abandonment options. They demonstrate how the risk (standard deviation or beta) responds to changes in the profitability and value of the firm's assets in place. They do not consider whether debt policy could mitigate or offset the changes in risk. They assume that the firm is all-equity financed, or that real options have the same fractional debt capacity as assets in place.[27] We show that real options do not have the same fractional debt capacity as assets in place.

### B. Predictions of the Basic Tradeoff Theory for Firms with Valuable Real Options

The basic tradeoff theory makes two broad predictions. The first is cross-sectional: Target debt ratios should vary depending on the firm's tax status, the risk or other attributes of its assets (tangible versus intangible assets, for example) and on how much value would be lost in financial distress. The second, time-series prediction is that observed debt ratios will fluctuate, but revert towards target debt ratios over time.

The implicit leverage in real options has immediate implications for cross-sectional tests. The debt capacity of real call options is usually negative and always less than the debt capacity of assets in place. Therefore a firm with valuable growth options will appear to operate at a too-conservative debt ratio, compared to the debt capacity of its assets in place. A mature firm with valuable abandonment (put) options will appear to operate at an aggressively high debt ratio, because puts contribute debt capacity rather than displacing it.

---

[26] Jacquier, Titman, and Yalcin (2010) refer to option leverage as "operating leverage."

[27] The main model in Hackbarth and Johnson (2015) assumes all-equity financing, but their Appendix B adds a simplified tradeoff model in which optimal debt is a constant fraction of the firm's market value, which includes the value of real options.

The example in Table 3 shows that a firm that cuts back explicit borrowing to offset option leverage maintains reserve borrowing power and can finance 100% of capital investment with new debt. Therefore tests that examine the immediate sources of financing for capital investment cannot distinguish the basic tradeoff theory, if followed faithfully by financial managers, from the pecking order theory, which also predicts that debt is the financing source at the margin.

### C. Cross-sectional tests of the basic tradeoff theory

Most cross-sectional tests start with a standard list of variables to "explain" differences in market or book debt ratios. See Rajan and Zingales (1995) or Graham and Leary (2011). The most common variables include the following (plus or minus signs in parentheses show typical effects on debt ratios in the cross section[28]): profitability (−); market-to-book ratio (−); tangibility of assets, for example the ratio of property, plant, and equipment to total assets (+), and size, usually the log of total assets (+). Many other variables have also been tested, but it is useful to start here with these old standards.

The positive signs for size and tangibility seem reasonable. Large firms ought to borrow more; they are presumably safer and more likely to pay taxes. Firms with more tangible assets are less likely to be damaged in financial distress and should therefore have higher target debt ratios.

The negative signs for profitability and market-to-book are harder to rationalize. The market-to-book ratio might measure intangible assets in place, which are more liable to damage in financial distress. But intangibles ought to contribute some positive debt capacity and at least increase *book* debt ratios. In fact the market-to-book ratio usually correlates with lower book as well as market debt ratios. The negative relationship between profitability and book debt ratios is even harder to explain. Higher profitability increases debt servicing capacity and also increases taxable income and the potential value of interest tax shields. Therefore profitable firms should operate at higher book debt ratios, but by and large they don't.

---

[28] There are of course exceptions to these typical effects. See Appendix C in Antoniou et al. (2008), which tabulates results from dozens of prior research papers that have used these and other variables.

These typical results are easier to square with the tradeoff theory when we recognize real options. Firms with higher market-to-book ratios are more likely to be firms with more valuable growth options. We say that such firms will operate at *lower* observed debt ratios, because implicit option leverage, though off balance sheet, nevertheless displaces explicit debt. The negative relationship between profitability and financial leverage also makes sense if more profitable firms have more valuable growth options.

Large firms are usually mature firms, which should borrow more if assets in place account for a greater fraction of market value than for younger growth firms. Mature firms are also likely to have valuable abandonment options, which add debt capacity off balance sheet. Recognizing real put options therefore reinforces the tradeoff theory's prediction that large firms should borrow more.

### D. Examples[29]

Table 4 reports debt capacities and debt ratios for firms that have assets in place and real call options to invest 100 in the same assets in year 3. The APV of each underlying asset is equal to the APV of each asset in place. That is, the firms have options to double in size in year 3. Option values are from Panel A of Table 1: the tax rate is 35%, the before-tax risk-free interest rate is 6%, volatility is 20% and the cash-flow yield is 10%. (Versions of Table 4 based on 1- or 5-year maturities or on other panels of Table 1 tell similar stories but are not included here.) The top panel of Table 4 assumes a target debt ratio of 25% for both assets in place and underlying assets. The bottom panel assumes the target debt ratio is 50%. The firm is assumed on-target with respect to the debt capacity of its option as well as its asset in place. The value of the asset in place ranges from 40 to 200.

[Table 4 here]

Table 4 reports negative debt capacity for the call option in all cases. The negative debt capacity can be double or triple the option's APV. For example, the at-the-money option in Panel A is worth 5.62. Its debt capacity is −15.47.

---

[29] William Shore (2012) provides further examples of the impact of real options on debt financing.

Also, with one exception, option debt capacity *decreases* as the option moves farther in the money. The exception occurs in Panel B when the underlying asset value increases from 150 to 200. In this case option debt capacity increases (becomes less negative) from −16.54 to −9.39. Debt capacity would continue to increase for still-higher asset values and eventually turn positive. In the limit, where the ratio of asset value to exercise price becomes extremely high, the fractional debt capacity of a real call option approaches $\lambda$, the target ratio for the underlying asset.

Table 4 also reports the debt ratio for the option—the ratio of the option's debt capacity to its value—which always increases (becomes less negative) as the option moves more in the money.

Negative debt capacity means that the option's implicit leverage displaces reported leverage. Thus the firm's market debt ratio (explicit debt only) in Table 4 is always less than the 25% and 50% target debt ratios for assets in place. In Panel A, the firm operates at a *negative* debt ratio when underlying asset value is 150. The firm appears to be a net lender. The "book" debt ratio—the ratio of debt to assets in place—is also less than the target ratios in Table 4. The book ratio would eventually move above the targets, however, because option debt capacity turns positive when the option is far enough in the money.

The last line of each panel of Table 4 reports the amounts by which a conventional estimate of WACC overstates the WACC for investment in assets in place. Because growth options usually have negative debt capacity, firms with growth options will appear to borrow less than their target debt ratios. That is, growth firms will use less explicit debt in order to compensate for the debt implicit in their growth options. In the examples, where the firm has an option to double its scale in three years' time, the errors that result from using observed debt ratios to calculate the WACC are modest when the growth options are out of the money, but become substantial when the options are near or in the money. Panel A shows that when the option is at the money and the target debt ratio is 25%, the estimated WACC is 10.64%, which is 118 basis points above the 9.46% WACC for the assets in place.

Table 5 reports debt ratios for an aging firm that has assets in place and options to abandon (put) its assets for 100 in year 3. Put option values come from Panel A of Table 2. Format and inputs are the same as in Table 4.

[Table 5 here]

The firm's market and book debt ratios in Table 5 are always greater than the 25% and 50% target debt ratios for assets in place. The debt capacity of a real put option can be far greater than the value of the put itself. For example, the at-the-money put option in Panel A of Table 5 is worth 19.64. Its debt capacity is 54.91.

The last line of each panel of Table 5 reports the amounts by which a conventional estimate of WACC *understates* the WACC for assets in place. The understatements occur because the debt capacity of a put option is always positive and always greater than the value of the put itself. Thus a firm with valuable put (abandonment) options will operate at a reported debt ratio *higher* than the target ratio for assets in place. The errors attributable to using observed borrowing ratios to calculate the WACC are even greater when the firm has an abandonment option than when it has a growth option of comparable scale. In Table 5 Panel A, the firm with a target debt ratio of 25% operates at a 67% reported debt ratio when it has an at-the-money option to abandon all of its assets in three years' time. The estimated WACC is 6.36%, 310 basis points *below* the true WACC of 9.46%.

Table 5 probably overstates the debt capacity of real put options, because it assumes that the exercise price is fixed and risk-free. In practice the proceeds from abandonment will be uncertain, and will not support debt dollar for dollar. Nevertheless, debt capacity from puts can help explain why takeovers of mature, cash-cow firms are often highly levered. For example, LBOs are often diet deals motivated by options to sell assets and shrink operations. The options increase debt capacity relative to target debt ratios for assets in place.

The results summarized in Tables 4 and 5 assume that the firm holds a single call or put. We have confirmed the results in more realistic examples, in which the firm holds a portfolio of calls and/or puts with a range of expiration dates and exercise prices.

The numerical experiments confirm our cross-sectional predictions for market and book debt ratios when firms follow the basic tradeoff theory and do not drift too far away from their debt targets.

1. The target debt ratio for the APV of a real call option is always less than the target debt ratio for the underlying asset. Therefore growth firms with valuable real call options will have lower market debt ratios. Table 4 indicates that growth firms should operate at (explicit) debt ratios that are much lower than their targets for assets in place. In some cases optimal explicit debt ratios are negative. This result gives a possible explanation why some large growth firms have accumulated "cash mountains" invested in debt securities.
2. The debt capacity of real call options is in most cases negative. Therefore growth firms will in most cases reduce borrowing relative to assets in place. If book values are adequate proxies for assets in place, then firms with more valuable growth options will usually operate at lower book debt ratios. The differences between observed debt ratios and targets for assets in place can be dramatic.
3. The predictions for mature firms with valuable abandonment (put) options and few growth options are reversed. These firms will operate at higher market and book debt ratios. Table 5 suggests that the differences between observed debt ratios and targets for assets in place can be dramatic.

One lesson of this paper is that it is useful to consider the tradeoff theory's predictions for book as well as market debt ratios. The impact of real options on observed book and market debt ratios can be different. Of course book ratios are less informative when book values are poor measures of the values of assets in place.

### E. Changes in debt ratios over time

Our analysis does not make life easier for time-series tests of the basic tradeoff theory. Targets for observed debt ratios will clearly evolve in complex ways, even when target debt ratios for assets in place are constant. Thus it should be no surprise that tests of target-adjustment behavior for debt ratios have generated mixed results.[30] If the tradeoff theory is correct and real options are important, any time-series test that assumes a constant target book or market debt ratio (for explicit debt only) is misspecified.[31]

Our numerical experiments only hint at the complexity likely to be encountered in real firms. It is not just a matter of changes in underlying asset values and option values and debt capacities. Firms will invest to create new options, but also discard options that are too far out of the money. Changes in technology and competition will extinguish options but also create new ones.

But we can say something more definite about how target debt ratios should change in response to random shocks to the firm's profitability and the value of its assets in place. We will assume for simplicity that the firm's portfolio of real options is held constant.

A positive shock to the value of a firm's assets in place moves the firm from left to right in Tables 4 and 5. Notice that the *target market debt ratio falls when market value increases*. In other words, the derivative of the target debt ratio for explicit debt with respect to value is *negative*, both for growth firms (Table 4) and for mature firms with abandonment options (Table 5).[32] For growth firms, the debt ratio falls even though the debt capacity of real options increases (becomes less negative). The reason is that a positive shock to profitability and asset values increases the proportion of market value accounted for by the growth options, which always have less debt capacity than assets in place.

Our numerical experiments are simplified, but it is nevertheless reasonable to predict that firms with valuable real options—abandonment options as well as growth

---

30 See Graham and Leary (2011, pp. 30-33) for a review of these tests.

31 Of course, real options are not the only challenge facing the econometrician who wishes to test capital structure theory. Operating leverage and financial leverage are substitutes, so empirical tests of the tradeoff theory may need to incorporate measures of operating leverage. See, for example, Chen, Harford, and Kamara (2014). Measuring off-balance-sheet debt is another potential challenge.

32 Our theory does not require this negative effect for real call options—the derivative turns positive when growth options are deep in the money. The negative effect is required for puts, because puts have higher fractional debt capacity than the underlying assets in place. This additional debt capacity dissipates when asset value increases and the put moves out of the money.

options—will target a *lower* market debt ratio when market value goes up. They may also target a lower book debt ratio—notice how the ratio of debt to assets in place declines from left to right in Tables 4 and 5.

This negative relationship could explain otherwise difficult puzzles in the empirical literature on capital structure. Take Welch (2004) as an example. This paper finds that year-to-year changes in market debt ratios are mostly explained by changes in stock price, and that net issues of debt and equity do not counteract the effects of stock-price changes, at least not in the short run. In addition, Welch finds that net issuing activity appears to amplify the effect of stock price changes on market debt ratios, at least in the short run. "Over one year, firms respond to poor [stock price] performance with more debt issuing activity and to good performance with more equity issuing activity." (Welch 2004, p. 111.) This behavior is a deep puzzle if one ignores real options and assumes that the firms are following the basic tradeoff theory with a stable target debt ratio. The behavior makes sense if real options are important and target ratios (for explicit borrowing only) incorporate the options' debt capacity off balance sheet. If poor stock price performance means less valuable growth options and more valuable abandonment options, then observed debt ratios should go up, leading to more debt issuing activity. Good stock price performance should have the opposite effect.

### F. Predictions about equity risk

Assume the firm has a target debt ratio $\lambda$ for its assets in place, based on the risks of those assets. If the firm recognizes the debt capacities of its real options, then it should rebalance its capital structure to keep its *total* leverage—the sum of explicit borrowing and the implicit debt in its real options—equal, or at least close to, $\lambda$ times the sum of (1) the APVs of assets in place plus (2) the sum of the APVs for its real options, each multiplied by its current option delta. (The first part of a replicating portfolio for a real call is $\delta \lambda APV$.) Assume that the assets underlying the real options have the same business risks as assets in place. Then equity risk (standard deviation or beta) will not depend on the value of its real options. The real options are riskier than the assets in place, but debt policy compensates.

Equity risk will be strictly constant if asset risk is constant and the firm can fully adjust capital structure every period, with no lags or frictions. If capital structure is rebalanced with lags, as in a tradeoff model with adjustment costs, then a positive (negative) shock to profitability should at first increase (decrease) a growth firm's equity risk. But equity risk will tend to revert to a target level. This is a testable prediction of the basic tradeoff theory.

If the firm does not adjust capital structure to take account of the debt capacity of its real options, then equity risk will evolve in complex ways as the profitability and value of its assets in place and the composition of its portfolio of real calls and puts changes. See Hackbarth and Johnson (2015).

We have noted how real options can distort estimates of WACC for firms with valuable growth options. There are two polar cases. If the tradeoff theory holds and firms take account of option leverage, then the cost of equity is right but the observed, explicit debt ratio too low. If the firm ignores option leverage and adheres to a target debt ratio for assets in place, then the debt ratio may be right but the cost of equity is too high. In both cases the value of assets in place is understated when the resulting WACC is used as a discount rate.

## VI. Summary and Conclusions

This paper analyzes the after-tax value of real growth (call) and abandonment (put) options, including option debt capacity and the value of the associated interest tax shields. The existing theoretical and practical literatures on real options, in contrast, ignore debt capacity and interest tax shields. Our results include the following:

1. Valuing assets in place: The correct discount rate for unlevered certainty-equivalent cash flows is $r(1-\lambda T)$, where $r$ is the pre-tax interest rate, $T$ is the marginal corporate tax rate, and $\lambda$ is the target debt ratio expressed as a fraction of the asset's tax- and leverage-adjusted present value (APV).
2. Valuing real options: Calculate after-tax payoffs to real options using a tax- and leverage-adjusted APV for the value of the underlying real asset. Discount the after-tax real-option payoffs (in a risk-neutral setting) at the *after-tax* interest rate

$r(1-T)$. For example, the modified Black-Scholes-Merton formula uses APV as the underlying asset value and $r(1-T)$ as the risk-free interest rate.

3. A real call option is equivalent to a collateralized non-recourse loan. Its fractional debt capacity is usually negative, and always less than the fractional debt capacity of the underlying asset. Call options that are in the money can have positive debt capacity.
4. A real put option gives the firm an insurance policy that puts a floor under asset value and allows the firm to borrow more than it could otherwise. The fractional debt capacity of a real put option is always positive and greater than the fractional debt capacity of the underlying asset. A put's absolute debt capacity (in dollars) is always greater than the value of the put itself.

If the basic tradeoff theory holds and the firm recognizes option leverage as debt-equivalent, then:

5. The implicit debt in real call options displaces explicit borrowing. Observed debt ratios for firms with valuable growth options will be less than target debt ratios for assets in place. These results may help explain why profitability and market-to-book ratios are negatively correlated with debt ratios. Our examples also suggest that observed debt ratios can be negative for firms with abundant, valuable real call options. This may help explain why some successful growth firms carry little debt but hold large portfolios of cash and marketable securities.
6. A firm with in-the-money real call options will follow a precautionary debt policy, reducing explicit borrowing as the option's maturity approaches. The reduction in explicit debt equals the call's exercise price at the instant before exercise. Paying the exercise price increases the firm's capacity for explicit borrowing dollar for dollar. Thus the immediate source of financing for capital investment can be 100% debt, even though the firm's overall capital structure is a mix of debt and equity. New explicit debt replaces the option leverage that is cancelled by exercise.

7. The implicit lending in real put options supports additional explicit borrowing. Debt ratios for firms with valuable abandonment options will be greater than target debt ratios for assets in place. This result may help explain why large, mature firms tend to operate at high debt ratios, especially in LBOs.
8. When a growth firm's profitability and value improve, its target market debt ratio (for explicit debt only) falls. When profitability and value decline, the target ratio increases. (This result assumes that growth options are valuable, but not too far in the money.)
9. Capital structures adjust to offset the additional risks of real options, leaving the standard deviation or beta of equity at a constant level determined by the risk of assets in place and real assets underlying the firm's options.
10. Therefore, conventional calculations of WACC, which use debt ratios based on observed borrowing only, mismeasure the WACC required for evaluating investment in assets in place. Conventional WACCs are too high for growth firms and too low for declining firms with valuable abandonment options. WACC should be calculated using the target market debt ratio for assets in place.

Of course results 5 through10 assume the simplest, basic specification of the tradeoff theory, in which the firm rebalances its capital structure to stay close to a target debt ratio. This case is a useful reference point, although reality is probably more complicated.

Our analysis predicts financing patterns that can be much more complex than conventional empirical specifications of the tradeoff theory. The complexity has at least three sources. First, the implicit debt in real options does not appear on balance sheets. Second, the relative market values of the firm's assets in place and its real options must fluctuate as time elapses and the values of the underlying assets evolve. For growth firms, positive profitability shocks increase the relative value of its real options. Third, the fractional debt capacity of real options is not constant, even if the target debt ratio for the underlying assets is constant. Additional complications arise when business risk changes (due to operating leverage, for example) and when new real options are added to the firm's portfolio. There is also the possibility that the firm invests in two or more classes of assets, each with its own target debt ratio.

Our analysis of real options and financing can rationalize some financing patterns that seem inconsistent with the tradeoff theory. On the other hand, it is probably too easy to create examples that are consistent with known patterns in the data. (The two most dangerous words in empirical corporate finance are "consistent with.") The complexity of financing under the basic tradeoff theory when real options are important may make it impossible to confirm or reject the theory using conventional empirical specifications when real options account for a substantial fraction of firm value.

The problems that real options create for tests of the basic tradeoff theory all arise because the econometrician sees the explicit debt on firms' balance sheets, but not the off-balance-sheet leverage implicit in real options. The same problems arise in tests of more complex capital structure theories, for the same reason. There is perhaps a way around these problems: test capital-structure theories on samples of mature, profitable firms only—firms that hold relatively few growth and abandonment options.

**References**

Antoniou, A., Y. Guney, and K. Paudyal (2008), “The determinants of capital structure: capital market-oriented versus bank-oriented institutions,” *Journal of Financial and Quantitative Analysis* 43, 59–92.

Amram, M., and N. Kulatilaka (1999), *Real Options* (Harvard Business School Press, Boston).

Barclay, M., E. Morellec, and C. Smith (2006), “On the debt capacity of growth options,” *Journal of Business* 79, 39–59.

Barclay, M., C. Smith, and R. Watts (1995), “Determinants of corporate leverage and dividend policies,” *Journal of Applied Corporate Finance* 7, 4–19.

Berk, J., and P. DeMarzo (2014), *Corporate Finance*, 3rd ed. (Pearson Education, Boston).

Bernardo, A. E., B. Chowdhry, and A. Goyal (2012), “Assessing project risk,” *Journal of Applied Corporate Finance* 24, 94–100.

Black, F. and M. S. Scholes (1973), “The pricing of options and corporate liabilities,” *Journal of Political Economy* 81, 637–654.

Brealey, R. A., S. C. Myers, and F. Allen (2017), *Principles of Corporate Finance*, 12th ed. (McGraw-Hill Education, New York).

Brennan, M. J. and E. S. Schwartz (1985), “Evaluating natural resource investments,” *Journal of Business* 58, 135–157.

Chen, Z., J. Harford and A. Kamara (2014), “Operating Leverage, Profitability and Capital Structure,” SSRN 2209070.

Childs, P. D., D. C. Mauer, and S. H. Ott (2005), “Interactions of corporate financing and investment decisions: The effects of agency conflicts”, *Journal of Financial Economics* 76, 667–690.

Copeland, T. and V. Antikarov (2001), *Real Options: A Practitioner’s Guide* (Texere, New York).

Cox, J. C., S. A. Ross, and M. Rubenstein (1979), “Option pricing: a simplified approach,” *Journal of Financial Economics* 7, 229–263.

Dixit, A.K. and R.S. Pindyck (1994), *Investment under Uncertainty* (Princeton University Press, Princeton, NJ).

Fischer, E., R. Heinkel, and J. Zechner (1989), “Dynamic capital structure choice: theory and tests,” *Journal of Finance* 44, 19–40.

Fischer, S. (1978), “Call option pricing when the exercise price is uncertain, and the valuation of index bonds,” *Journal of Finance* 33, 169–176.

Flannery, M. J. and K. P. Rangan (2006), “Partial adjustment toward target capital structures,” *Journal of Financial Economics* 79, 469–506.

Frank, M. and V. Goyal (2008), “Tradeoff and pecking order theories of debt,” in E. Eckbo, ed., *Handbook of Corporate Finance: Empirical Finance*, v. 2 (North Holland, Amsterdam).

Gamba, A., G. A. Sick, and C. A. Leon (2008), “Investment under uncertainty, debt and taxes,” *Economic Notes* 37, 31–58.

Gomez, J. F. and L. Schmid (2010), “Levered returns,” *Journal of Finance* 65, 467–494.

Graham, J. R. and C. R. Harvey (2001), “The theory and practice of corporate finance: Evidence from the field,” *Journal of Financial Economics* 60, 187–243.

Graham, J. R. and M. T. Leary (2011), “A review of capital structure research and directions for the future,” *Annual Review of Financial Economics*, v. 3, 1–59.

Grenadier, S. and A. Malenko (2011), “Real options signaling games with applications to corporate finance,” *Review of Financial Studies* 24, 3993-4036.

Hackbarth, D., C. A. Hennessey, and H. E. Leland (2007), “Can the trade-off theory explain capital structure?” *Review of Financial Studies* 20, 1389–1428.

Hackbarth, D. and D. C. Mauer (2012), “Optimal priority structure, capital structure, and investment,” *Review of Financial Studies* 25, 747–796.

Hackbarth, D. and T. C. Johnson (2015), “Real options and risk dynamics,” *Review of Economic Studies* 82, 1449–1482.

Harris, M. and A. Raviv (1991), “The theory of capital structure,” *Journal of Finance* 46, 297–355.

Hull, J. C. (2015), *Options, Futures, and Other Derivatives* (Prentice-Hall, Boston).

Jacquier, E., S. Titman, and A. Yalcan (2010), “Predicting systematic risk: implications from growth options,” *Journal of Empirical Finance* 17, 991–1005.

Leary, M. T. and M. R. Roberts (2005), “Do firms rebalance their capital structures?” *Journal of Finance* 60, 2575–2619.

Margrabe, W. (1978), “The value of an option to exchange one asset for another,” *Journal of Finance* 33, 177–186.

Mauer, D. C. and S. H. Ott (2000), “Agency costs, underinvestment, and optimal capital structure: the effect of growth options to expand”, chapter 9 in M. J. Brennan and L.

Trigeorgis (eds.), *Project Flexibility, Agency, and Competition* (Oxford University Press, New York).

Mauer, D. C. and S. Sarkar (2005), “Real options, agency conflicts, and optimal capital structure,” *Journal of Banking & Finance* 29, 1405–1428.

McDonald, R. L. (2006), “The role of real options in capital budgeting: theory and practice,” *Journal of Applied Corporate Finance* 18, 28–39.

Merton, R. C. (1973), “Theory of rational option pricing,” *The Bell Journal of Economics and Management Science* 4, 141–183.

Miles, J. and R. Ezzell (1980), “The weighted average cost of capital, perfect capital markets, and project life: a clarification,” *Journal of Financial and Quantitative Analysis* 15, 719–730.

Miller, M. H. and F. Modigliani (1966), “Some estimates of the cost of capital to the electric utility industry, 1954–57,” *American Economic Review* 56, 333–391.

Modigliani, F. and M. H. Miller (1963), “Corporate income taxes and the cost of capital: a correction,” *American Economic Review* 53, 433–443.

Morellec, E. and N. Schurhoff (2010), “Dynamic investment and financing under personal taxation,” *Review of Financial Studies* 23, 101–146.

Morellec, E. and N. Schurhoff (2011), “Corporate investment and financing under asymmetric information,” *Journal of Financial Economics* 99, 262-288.

Myers, S. C. (1974), “Interactions of corporate financing and investment decisions: implications for capital budgeting,” *Journal of Finance* 29, 11–25.

Myers, S. C. (1977), “Determinants of Corporate Borrowing,” *Journal of Financial Economics* 5, 147–175.

Myers, S. C. (2003), “Financing of corporations,” in G. Constantinedes, M. Harris and R. Stulz, eds., *Handbook of the Economics of Finance*, Elsevier.

Myers, S. C., D. A. Dill, and A. J. Bautista (1976), “Valuation of financial lease contracts,” *Journal of Finance* 31, 799–819.

Myers, S. C. and J. A. Read, Jr. (2012), “Real options, taxes and financial leverage,” National Bureau of Economic Research, Working Paper 18148.

Myers, S. C. and S. M. Turnbull (1977), “Capital budgeting and the capital asset pricing model: good news and bad news,” *Journal of Finance* 32, 321–333.

Purnanandam, A. and U. Rajan (2018), “Growth option exercise and capital structure,” *Review of Finance* 22, 177–206.

Rajan, R. and L. Zingales (1995), "What do we know about capital structure? Some evidence from international data," *Journal of Finance* 56, 1421–1460.

Ross, S. A., R. W. Westerfield, and J. Jaffee (2011), *Corporate Finance*, 9th ed. (McGraw-Hill Irwin, New York).

Ruback, R.S. (1986), "Calculating the market value of risk-free cash flows," *Journal of Financial Economics* 15, 323–339.

Ruback, R. S. (2002), "Capital cash flows: a simple approach to valuing risky cash flows," *Financial Management* 31, 85–103.

Sundaresan, S., N. Wang, and J. Yang (2015), "Dynamic investment, capital structure, and debt overhang," *Review of Corporate Finance Studies*.

Shore, W. T. (2012), "On Testing the Tradeoff Theory of Capital Structure with Real Options," Unpublished M.S. thesis, Massachusetts Institute of Technology.

Smit, H. T. J. and L. Trigeorgis (2004), *Strategic Investment: Real Options and Games* (Princeton University Press, Princeton, NJ).

Strebulaev, I. A. (2007), "What do tests of capital structure theory show?" *Journal of Finance* 62, 1747–1787.

Taggart, R. S. Jr. (1991), "Consistent cost of capital and valuation expressions with corporate and personal taxes," *Financial Management* 20, 8–20.

Trigeorgis, L. (1996), *Real Options* (MIT Press, Cambridge, MA).

Tserlukevich, Y. (2008), "Can real options explain financing behavior?" *Journal of Financial Economics* 89, 232–252.

Welch, I. (2004), "Capital structure and stock returns," *Journal of Political Economy* 112, 106–131.

**Figure 1: Call option debt as a function of the APV of the underlying asset**

This figure shows the debt capacity of a real call option with an exercise price of 100 and a time to expiration of one year. There are three cases: the underlying assets have a debt capacity of 0%, 25%, or 50%. (The APV of the call option is also shown for reference.) In all cases the before-tax risk-free rate is 6% and the marginal tax rate is 35%; the cash-flow yield and volatility of the underlying are 10% and 20%, respectively. Call option debt is usually negative and increases in absolute value with the APV of the underlying asset. However, call option debt can be positive and it can decrease with the APV of the underlying if the underlying has some debt capacity ($\lambda > 0$) and the option is very deep in the money (to the right-hand side of the graph, beyond the range of underlying APVs shown).

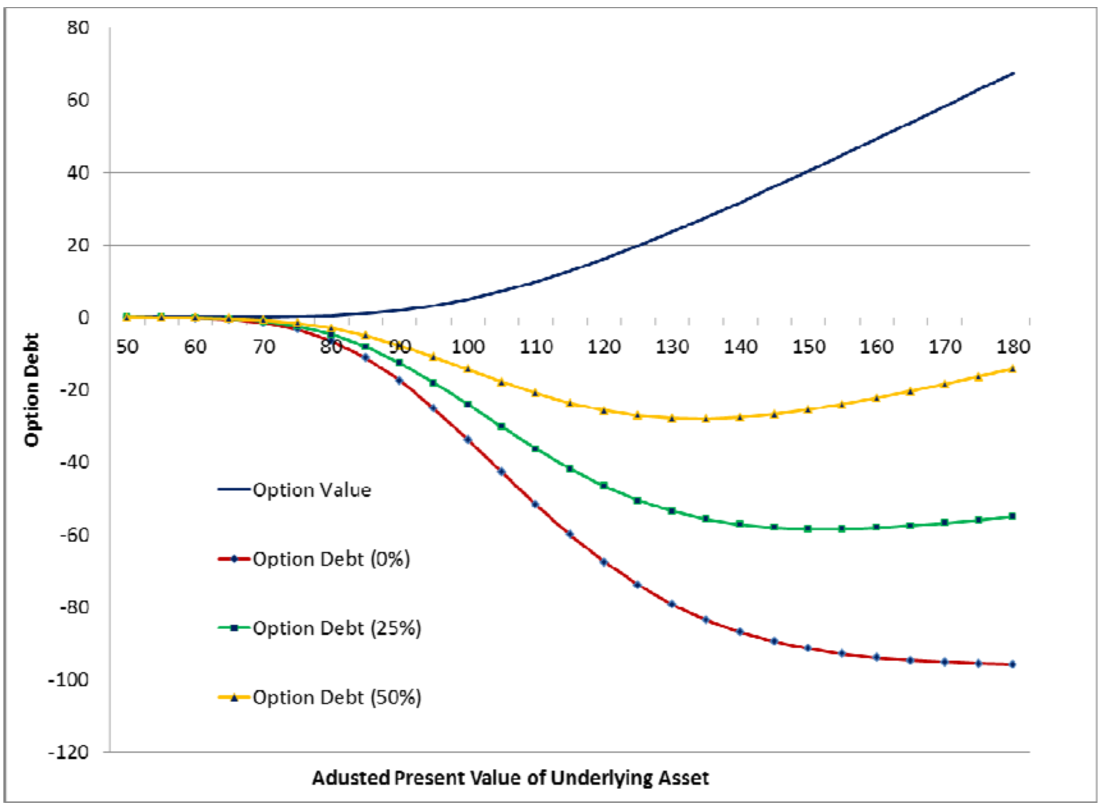
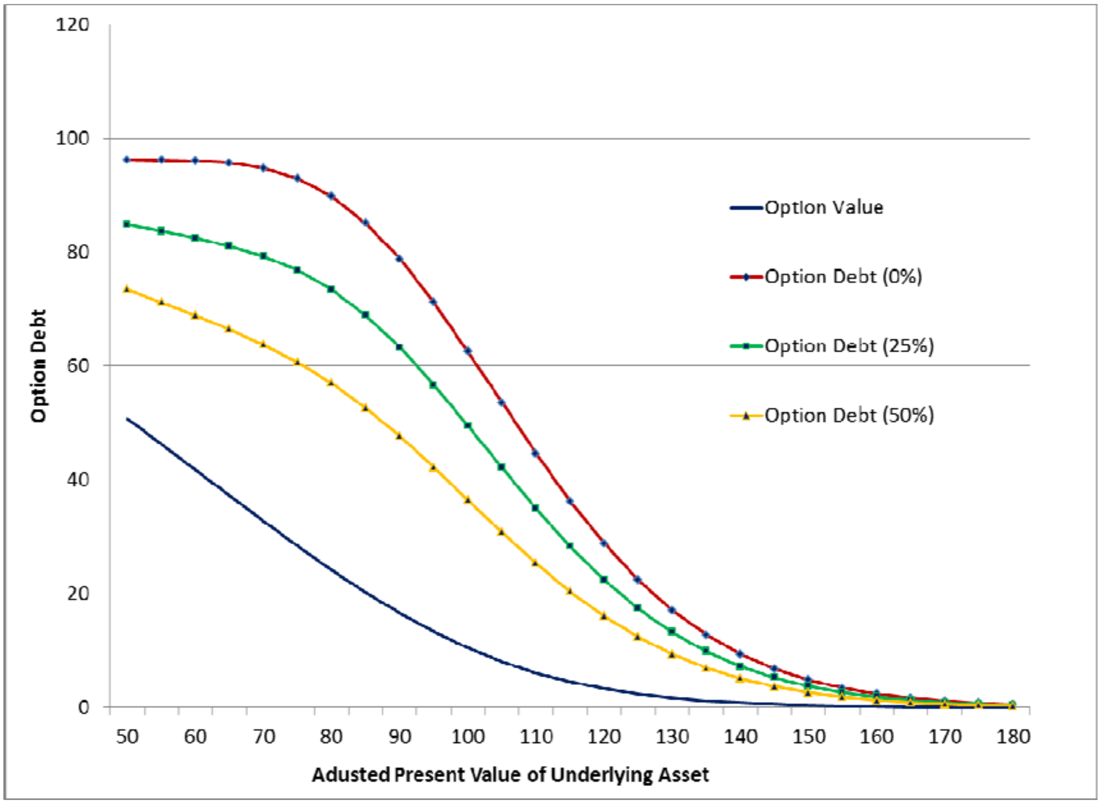


**Figure 2: Put option debt as a function of the APV of the underlying asset**

This figure shows the debt capacity of a real put option with an exercise price of 100 and a time to expiration of one year. There are three cases: the underlying assets have a debt capacity of 0%, 25%, or 50%. (The APV of the put option is also shown for reference.) In all cases the before-tax risk-free rate is 6% and the marginal tax rate is 35%; the cash-flow yield and volatility of the underlying are 10% and 20%, respectively. Put option debt decreases with the APV of the underlying asset. It is always positive and always greater than the APV of the underlying.



### Table 1: Tax and leverage-adjusted value of real call (growth) options

Table 1 reports the value of a real call option with an exercise price of 100 as a function of the APV of the underlying asset. Present values are calculated using the modified Black-Scholes-Merton (BSM) formula, which accounts for option leverage and taxes. Absolute and percentage errors from using the standard BSM model are reported below the correct present values. Present values and errors are reported for various combinations of the time to expiration ($\tau$), the pre-tax risk-free rate ($r$), and the cash-flow yield on the underlying asset ($y$). The volatility of the underlying ($\sigma$) is 20% and the marginal tax rate ($T$) is 35%. Standard BSM overstates real call values. Absolute errors are largest for in-the-money options, but percentage errors are largest for out-of-the-money options.

| | | Maturity | APV of Underlying Asset | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | (years) | 40 | 60 | 80 | 100 | 125 | 150 | 200 |
| Panel A | Interest rate = 6%, Yield = 10% | | 0.00 | 0.01 | 0.61 | 5.09 | 19.76 | 40.49 | 85.58 |
| | | 1 | 0.00 | 0.00 | 0.14 | 0.70 | 1.49 | 1.82 | 1.91 |
| | | | 66% | 38% | 23% | 14% | 8% | 4% | 2% |
| | | | 0.00 | 0.20 | 1.60 | 5.62 | 15.11 | 28.66 | 62.25 |
| | | 3 | 0.00 | 0.10 | 0.57 | 1.46 | 2.73 | 3.75 | 4.79 |
| | | | 80% | 52% | 36% | 26% | 18% | 13% | 8% |
| | | | 0.02 | 0.37 | 1.80 | 5.01 | 11.82 | 21.37 | 45.98 |
| | | 5 | 0.02 | 0.23 | 0.83 | 1.78 | 3.15 | 4.42 | 6.19 |
| | | | 93% | 63% | 46% | 36% | 27% | 21% | 13% |
| Panel B | Interest rate = 3%, Yield = 10% | | 0.00 | 0.01 | 0.50 | 4.47 | 18.38 | 38.75 | 83.74 |
| | | 1 | 0.00 | 0.00 | 0.06 | 0.32 | 0.75 | 0.94 | 1.00 |
| | | | 31% | 19% | 12% | 7% | 4% | 2% | 1% |
| | | | 0.00 | 0.13 | 1.17 | 4.43 | 12.72 | 25.20 | 57.59 |
| | | 3 | 0.00 | 0.03 | 0.22 | 0.62 | 1.27 | 1.85 | 2.54 |
| | | | 38% | 26% | 19% | 14% | 10% | 7% | 4% |
| | | | 0.01 | 0.22 | 1.21 | 3.64 | 9.20 | 17.47 | 40.08 |
| | | 5 | 0.00 | 0.07 | 0.29 | 0.70 | 1.37 | 2.07 | 3.19 |
| | | | 44% | 32% | 24% | 19% | 15% | 12% | 8% |
| Panel C | Interest rate = 9%, Yield = 10% | | 0.00 | 0.01 | 0.74 | 5.74 | 21.15 | 42.19 | 87.35 |
| | | 1 | 0.00 | 0.01 | 0.25 | 1.13 | 2.22 | 2.63 | 2.73 |
| | | | 108% | 59% | 34% | 20% | 11% | 6% | 3% |
| | | | 0.01 | 0.30 | 2.13 | 6.96 | 17.63 | 32.14 | 66.71 |
| | | 3 | 0.01 | 0.23 | 1.09 | 2.50 | 4.29 | 5.57 | 6.72 |
| | | | 127% | 77% | 51% | 36% | 24% | 17% | 10% |
| | | | 0.04 | 0.58 | 2.56 | 6.65 | 14.74 | 25.46 | 51.73 |
| | | 5 | 0.06 | 0.53 | 1.66 | 3.20 | 5.15 | 6.77 | 8.76 |
| | | | 144% | 92% | 65% | 48% | 35% | 27% | 17% |

**Table 1 (continued)**

| | | Maturity (years) | APV of Underlying Asset | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | 40 | 60 | 80 | 100 | 125 | 150 | 200 |
| Panel D | Interest rate = 6% Yield = 0% | 1 | 0.00 | 0.05 | 1.68 | 9.83 | 29.73 | 53.86 | 103.75 |
| | | | 0.00 | 0.02 | 0.31 | 1.06 | 1.71 | 1.88 | 1.91 |
| | | | 59% | 33% | 19% | 11% | 6% | 3% | 2% |
| | | 3 | 0.07 | 1.58 | 7.61 | 19.13 | 39.01 | 62.00 | 111.00 |
| | | | 0.04 | 0.57 | 1.79 | 3.10 | 4.22 | 4.78 | 5.13 |
| | | | 60% | 36% | 24% | 16% | 11% | 8% | 5% |
| | | 5 | 0.56 | 4.33 | 13.13 | 26.28 | 46.69 | 69.48 | 117.92 |
| | | | 0.33 | 1.61 | 3.36 | 4.90 | 6.22 | 6.99 | 7.62 |
| | | | 59% | 37% | 26% | 19% | 13% | 10% | 6% |

**Table 2: Tax and leverage-adjusted value of real put (abandonment) options**

Table 2 reports the value of a real put option with an exercise price of 100 as a function of the APV of the underlying asset. Present values are calculated using the modified Black-Scholes-Merton (BSM) formula, which accounts for option leverage and taxes. Absolute and percentage errors from using the standard BSM model are reported below the correct present values. Present values and errors are reported for various combinations of the time to expiration ($\tau$), the pre-tax risk-free rate ($r$), and the cash-flow yield on the underlying asset ($y$). The volatility of the underlying ($\sigma$) is 20% and the marginal tax rate ($T$) is 35%. Standard BSM understates real put values. Absolute errors are largest for in-the-money options, but percentage errors are largest for out-of-the-money options.

| | | Maturity | APV of Underlying Asset | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | (years) | 40 | 60 | 80 | 100 | 125 | 150 | 200 |
| Panel A | Interest rate = 6%, Yield = 10% | | 59.88 | 41.71 | 24.13 | 10.42 | 2.37 | 0.38 | 0.01 |
| | | 1 | -1.91 | -1.90 | -1.77 | -1.20 | -0.42 | -0.09 | 0.00 |
| | | | -3% | -5% | -7% | -12% | -17% | -23% | -32% |
| | | | 59.11 | 44.28 | 30.65 | 19.64 | 10.35 | 5.12 | 1.14 |
| | | 3 | -5.19 | -5.09 | -4.62 | -3.74 | -2.47 | -1.45 | -0.41 |
| | | | -9% | -11% | -15% | -19% | -24% | -28% | -35% |
| | | | 57.77 | 45.70 | 34.71 | 25.50 | 16.79 | 10.82 | 4.38 |
| | | 5 | -7.84 | -7.63 | -7.03 | -6.08 | -4.71 | -3.45 | -1.67 |
| | | | -14% | -17% | -20% | -24% | -28% | -32% | -38% |
| Panel B | Interest rate = 3%, Yield = 10% | | 61.72 | 43.55 | 25.86 | 11.65 | 2.83 | 0.48 | 0.01 |
| | | 1 | -1.00 | -1.00 | -0.94 | -0.68 | -0.25 | -0.06 | 0.00 |
| | | | -2% | -2% | -4% | -6% | -9% | -12% | -17% |
| | | | 64.32 | 49.42 | 35.44 | 23.67 | 13.18 | 6.88 | 1.70 |
| | | 3 | -2.86 | -2.82 | -2.64 | -2.24 | -1.59 | -1.00 | -0.32 |
| | | | -4% | -6% | -7% | -9% | -12% | -15% | -19% |
| | | | 65.97 | 53.76 | 42.33 | 32.34 | 22.38 | 15.13 | 6.69 |
| | | 5 | -4.53 | -4.46 | -4.24 | -3.83 | -3.16 | -2.47 | -1.35 |
| | | | -7% | -8% | -10% | -12% | -14% | -16% | -20% |
| Panel C | Interest rate = 9%, Yield = 10% | | 58.11 | 39.94 | 22.49 | 9.30 | 1.99 | 0.30 | 0.00 |
| | | 1 | -2.73 | -2.72 | -2.48 | -1.60 | -0.51 | -0.10 | 0.00 |
| | | | -5% | -7% | -11% | -17% | -25% | -33% | -44% |
| | | | 54.27 | 39.54 | 26.34 | 16.15 | 8.04 | 3.76 | 0.76 |
| | | 3 | -7.09 | -6.87 | -6.01 | -4.60 | -2.82 | -1.53 | -0.38 |
| | | | -13% | -17% | -23% | -28% | -35% | -41% | -49% |
| | | | 50.46 | 38.58 | 28.15 | 19.82 | 12.38 | 7.58 | 2.80 |
| | | 5 | -10.21 | -9.73 | -8.61 | -7.07 | -5.11 | -3.49 | -1.50 |
| | | | -20% | -25% | -31% | -36% | -41% | -46% | -54% |

**Table 2 (continued)**

| | | Maturity (years) | APV of Underlying Asset | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | 40 | 60 | 80 | 100 | 125 | 150 | 200 |
| Panel D | Interest rate = 6% Yield = 0% | 1 | 56.25 | 36.29 | 17.93 | 6.08 | 0.98 | 0.11 | 0.00 |
| | | | -1.91 | -1.89 | -1.59 | -0.85 | -0.20 | -0.03 | 0.00 |
| | | | -3% | -5% | -9% | -14% | -20% | -26% | -35% |
| | | 3 | 49.23 | 30.74 | 16.77 | 8.28 | 3.16 | 1.16 | 0.15 |
| | | | -5.15 | -4.63 | -3.40 | -2.10 | -0.97 | -0.41 | -0.06 |
| | | | -10% | -15% | -20% | -25% | -31% | -35% | -43% |
| | | 5 | 43.15 | 26.92 | 15.71 | 8.87 | 4.28 | 2.07 | 0.51 |
| | | | -7.54 | -6.26 | -4.51 | -2.96 | -1.64 | -0.88 | -0.25 |
| | | | -17% | -23% | -29% | -33% | -38% | -42% | -49% |

**Table 3: Debt capacity of real call options—an example**

This is an example to illustrate how corporate borrowing can accommodate the debt capacity of a real call option as time elapses. The exercise price of the option is 100. Panel A describes a scenario in which the value of the underlying begins and remains at 125. Panel B describes a scenario in which the value of the underlying begins and remains at 80. The volatility, cash flow yield, and target debt ratio of the underlying are 20%, 0%, and 30%, respectively. The interest rate is 6% and the marginal tax rate is 35%. Debt capacity is the sum of the option leverage and the debt capacity contributed by the underlying. The debt capacity contributed by the underlying is the product of the target debt ratio for the underlying $(\lambda)$, the option delta$(\delta)$, and the APV of the underlying. Option leverage is the product of the risk-neutral probability that the option will be exercised $(N(d_2))$ and the APV of the exercise price. In Panel A, when the call option is exercised, the corporation can finance the acquisition of the underlying asset entirely by borrowing, implicitly using 37.50 of debt capacity in the underlying asset and 62.50 of reserve borrowing capacity from assets in place. In Panel B the option expires out of the money.

| | | Time Remaining to Expiration | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | 5 | 3 | 1 | 0.5 | 0.25 | 0.001 | 0 |
| Panel A | APV of underlying | 125 | 125 | 125 | 125 | 125 | 125 | 125 |
| | (1) Option value | 46.69 | 39.01 | 29.73 | 27.17 | 25.99 | 25.00 | 25.00 |
| | (2) Option debt capacity | -29.87 | -37.53 | -50.79 | -57.07 | -60.75 | -62.50 | 37.50 |
| | (3) Option leverage | -62.69 | -70.33 | -85.30 | -93.18 | -97.92 | -100.00 | |
| | (4) Underlying | 32.81 | 32.80 | 34.51 | 36.10 | 37.17 | 37.50 | 37.50 |
| Panel B | APV of underlying | 80 | 80 | 80 | 80 | 80 | 80 | 80 |
| | (1) Option value | 13.13 | 7.61 | 1.68 | 0.42 | 0.05 | 0.00 | 0.00 |
| | (2) Option debt capacity | -18.27 | -17.28 | -9.79 | -4.35 | -0.98 | 0.00 | 0.00 |
| | (3) Option leverage | -31.72 | -27.95 | -14.71 | -6.39 | -1.43 | 0.00 | |
| | (4) Underlying | 13.45 | 10.67 | 4.92 | 2.04 | 0.44 | 0.00 | 0.00 |

**Table 4: Debt capacity and target debt ratios for firms with real call options**

Table 4 shows the debt capacity and debt ratios of a firm with assets in place plus a real call option on the same type of assets. In particular, it shows how debt capacity and debt ratios vary with the value of the underlying assets. Option values are from Table 1, Panel A, for 3-year maturity. The firm has the option to double its scale at year 3. Target debt ratios for assets in place are 25% (Panel A) and 50% (Panel B). The option debt ratio is the ratio of the option's debt capacity to its market value. The value of the firm is the sum of the value of assets in place and the value of the real option. The market debt ratio is the ratio of the firm's debt capacity to the value of the firm. The "book" debt ratio, in contrast, is the debt capacity of the firm as a ratio to assets in place. The "estimated" WACC is calculated based on the apparent capital structure of the firm (explicit debt only), and the estimation error is the difference between the estimated WACC and the WACC of assets in place.

| | | | APV of Underlying Asset | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | 40 | 60 | 80 | 100 | 125 | 150 | 200 |
| Panel A | Debt Capacity = 25% | Value of assets in place | 40 | 60 | 80 | 100 | 125 | 150 | 200 |
| | | Debt capacity of assets in place | 10 | 15 | 20 | 25 | 31 | 38 | 50 |
| | | Debt ratio for assets in place | 25% | 25% | 25% | 25% | 25% | 25% | 25% |
| | | Option value | 0.00 | 0.20 | 1.60 | 5.62 | 15.11 | 28.66 | 62.25 |
| | | Option debt capacity | -0.03 | -1.03 | -5.94 | -15.47 | -29.04 | -39.15 | -45.21 |
| | | Option debt ratio | -729% | -512% | -372% | -275% | -192% | -137% | -73% |
| | | Value of firm | 40 | 60 | 82 | 106 | 140 | 179 | 262 |
| | | Debt capacity of firm | 10 | 14 | 14 | 10 | 2 | -2 | 5 |
| | | Market debt ratio | 25% | 23% | 17% | 9% | 2% | -1% | 2% |
| | | Book debt ratio | 25% | 23% | 18% | 10% | 2% | -1% | 2% |
| | | WACC for assets in place | 9.46% | 9.46% | 9.46% | 9.46% | 9.46% | 9.46% | 9.46% |
| | | Estimated WACC | 9.46% | 9.59% | 10.03% | 10.64% | 11.19% | 11.38% | 11.17% |
| | | Estimation error | 0.01% | 0.13% | 0.58% | 1.18% | 1.73% | 1.92% | 1.72% |
| Panel B | Debt Capacity = 50% | Value of assets in place | 40 | 60 | 80 | 100 | 125 | 150 | 200 |
| | | Debt capacity of assets in place | 20 | 30 | 40 | 50 | 63 | 75 | 100 |
| | | Debt ratio for assets in place | 50% | 50% | 50% | 50% | 50% | 50% | 50% |
| | | Option value | 0.00 | 0.20 | 1.60 | 5.62 | 15.11 | 28.66 | 62.25 |
| | | Option debt capacity | -0.02 | -0.62 | -3.43 | -8.44 | -14.32 | -16.54 | -9.39 |
| | | Option debt ratio | -453% | -308% | -215% | -150% | -95% | -58% | -15% |
| | | Value of firm | 40 | 60 | 82 | 106 | 140 | 179 | 262 |
| | | Debt capacity of firm | 20 | 29 | 37 | 42 | 48 | 58 | 91 |
| | | Market debt ratio | 50% | 49% | 45% | 39% | 34% | 33% | 35% |
| | | Book debt ratio | 50% | 49% | 46% | 42% | 39% | 39% | 45% |
| | | WACC for assets in place | 8.91% | 8.91% | 8.91% | 8.91% | 8.91% | 8.91% | 8.91% |
| | | Estimated WACC | 8.92% | 9.03% | 9.43% | 9.98% | 10.48% | 10.64% | 10.46% |
| | | Estimation error | 0.01% | 0.12% | 0.52% | 1.07% | 1.56% | 1.73% | 1.55% |

**Table 5: Debt capacity and target debt ratios for firms with real put options**

Table 5 shows debt capacity and target debt ratios for a firm with assets in place plus a real put option on those assets. Option values are from Table 2, Panel A, for 3-year maturity. The firm has the option to liquidate at year 3. Target debt ratios for assets in place are 25% (Panel A) and 50% (Panel B). The option debt ratio is the ratio of the option's debt capacity to its market value. The value of the firm is the sum of the value of assets in place and the value of the real option. The market debt ratio is the ratio of the firm's debt capacity to the value of the firm. The "book" debt ratio, in contrast, is the debt capacity of the firm as a ratio to assets in place. The "estimated" WACC is calculated based on the apparent capital structure of the firm (explicit debt only), and the estimation error is the difference between the estimated WACC and the WACC of assets in place.

| | | | APV of Underlying Asset | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | 40 | 60 | 80 | 100 | 125 | 150 | 200 |
| Panel A | Debt Capacity = 25% | Value of assets in place | 40 | 60 | 80 | 100 | 125 | 150 | 200 |
| | | Debt capacity of assets in place | 10 | 15 | 20 | 25 | 31 | 38 | 50 |
| | | Debt ratio for assets in place | 25% | 25% | 25% | 25% | 25% | 25% | 25% |
| | | Option value | 59.11 | 44.28 | 30.65 | 19.64 | 10.35 | 5.12 | 1.14 |
| | | Option debt capacity | 81.61 | 76.86 | 68.19 | 54.91 | 36.64 | 21.84 | 6.38 |
| | | Option debt ratio | 138% | 174% | 222% | 280% | 354% | 427% | 558% |
| | | Value of firm | 99 | 104 | 111 | 120 | 135 | 155 | 201 |
| | | Debt capacity of firm | 92 | 92 | 88 | 80 | 68 | 59 | 56 |
| | | Market debt ratio | 92% | 88% | 80% | 67% | 50% | 38% | 28% |
| | | Book debt ratio | 229% | 153% | 110% | 80% | 54% | 40% | 28% |
| | | WACC for assets in place | 9.46% | 9.46% | 9.46% | 9.46% | 9.46% | 9.46% | 9.46% |
| | | Estimated WACC | 4.46% | 4.78% | 5.40% | 6.36% | 7.59% | 8.47% | 9.23% |
| | | Estimation error | -5.00% | -4.67% | -4.05% | -3.10% | -1.86% | -0.98% | -0.22% |
| Panel B | Debt Capacity = 50% | Value of assets in place | 40 | 60 | 80 | 100 | 125 | 150 | 200 |
| | | Debt capacity of assets in place | 20 | 30 | 40 | 50 | 63 | 75 | 100 |
| | | Debt ratio for assets in place | 50% | 50% | 50% | 50% | 50% | 50% | 50% |
| | | Option value | 59.11 | 44.28 | 30.65 | 19.64 | 10.35 | 5.12 | 1.14 |
| | | Option debt capacity | 74.11 | 66.00 | 55.67 | 43.15 | 27.87 | 16.26 | 4.64 |
| | | Option debt ratio | 125% | 149% | 182% | 220% | 269% | 318% | 405% |
| | | Value of firm | 99 | 104 | 111 | 120 | 135 | 155 | 201 |
| | | Debt capacity of firm | 94 | 96 | 96 | 93 | 90 | 91 | 105 |
| | | Market debt ratio | 95% | 92% | 86% | 78% | 67% | 59% | 52% |
| | | Book debt ratio | 235% | 160% | 120% | 93% | 72% | 61% | 52% |
| | | WACC for assets in place | 8.91% | 8.91% | 8.91% | 8.91% | 8.91% | 8.91% | 8.91% |
| | | Estimated WACC | 4.41% | 4.70% | 5.26% | 6.12% | 7.23% | 8.03% | 8.71% |
| | | Estimation error | -4.51% | -4.21% | -3.65% | -2.79% | -1.68% | -0.89% | -0.20% |