Mispricing of Index Options with Respect to Stochastic Dominance Bounds? A Reply

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Abstract

Constantinides and Perrakis (2002, 2007) derive a lower bound on the price of an option such that an investor increases her utility by buying the option at the ask price if the ask price is lower than the lower bound; and by writing the option at the bid price if the bid price is higher than the upper bound. Contrary to the evidence in Constantinides, Jackwerth, and Perrakis (2009) and Constantinides, Czerwonko, Jackwerth and Perrakis (2011) who demonstrate several violations of mainly the upper bound on call prices and document a tradable anomaly by exploiting this mispricing, Wallmeier (2015) claims that practically all options on the S&P 500, Eurostoxx 50, and DAX indices lie within the bounds. The main reason for the discrepancy is that Wallmeier erroneously inflates the volatility input to the bounds by about 2% by using the at-the-money implied volatility which is approximately the risk-neutral volatility instead of the physical volatility, as required by the model.

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Introduction

Constantinides and Perrakis (CP 2002, 2007) consider an investor who trades in a risk free asset and the market index subject to proportional transaction costs. The investor maximizes the expectation of her increasing and concave utility function of cash wealth at the end of the trading interval. Now a third asset, a call or put option, becomes available for trade. CP (2002, 2007) apply standard results of stochastic dominance and derive a lower bound on the price of this option such that the investor increases her utility by borrowing and buying the option at the ask price if the ask price is lower than the lower bound; and they derive an upper bound on the price of this option such that the investor increases her utility by writing the option at the bid price if the bid price is higher than the upper bound, investing the proceeds in the risk free asset. The bounds are independent of the investor's initial wealth and particular utility function.

The intuitive interpretation of stochastic dominance is that the investor increases her expected utility by shifting payoffs from states where the index level is high to states where the index level is low, at zero net cost, while maintaining the same or higher expected portfolio return. Constantinides, Jackwerth, and Perrakis (CJP 2009) test the CP (2002) bounds for S&P 500 index options and demonstrate violations of mainly the upper bound on call option prices.

Several aspects of the CP (2002, 2007) proofs are critical in empirically applying the bounds. First the bounds involve an expectation under the physical probability measure, not the risk-neutral measure. Second it is well known since Constantinides (1979) that the investor's asset allocation decisions stay within the bounds of a no-trade zone and alter the asset proportions to the nearest boundary only when they move outside the zone. Consequently assuming a single trading period from the initial trade time till the maturity of an option is an acceptable approximation for short maturities thus contradicting Wallmeier's (2015, p.4) criticism of discrete trading. Last but not least there is no single equilibrium price for either calls or puts in an economy with frictions, let alone a single price pair implied by the put-call parity: an option trades within its bid and ask prices. In his empirical demonstration Wallmeier seems to be unaware of these three key aspects of the derivation of the CP bounds and the earlier literature on the investor asset allocation under proportional transaction costs.

Wallmeier recalculates the CP (2002) bounds by making a critical change in the calculation of the volatility under the real probability measure relative to CJP (2009) and presents evidence that practically all options on the S&P 500, Eurostoxx 50, and DAX indices lie within the bounds. He concludes that "[o]ur results indicate that index option markets might be much more efficient than previous literature suggests".

Our response consists of two parts. In Section 1 we present empirical evidence that there are mispriced index options in most months of the sample period by showing that a portfolio that includes the mispriced options stochastically dominates a portfolio that excludes them. This empirical evidence is *out-of-sample* and, therefore, is independent of whether Wallmeier's or our method is the correct method for finding mispriced options. In Section 2 we explain why Wallmeier's method of calculating the volatility under the physical probability measure is upward biased.

1 The Out-of-Sample Empirical Evidence

The significance of the out-of-sample empirical evidence is that it is independent of whether Wallmeier's or our method is the correct method of searching for mispriced options. One may treat the search method as a “black box” and focus on whether mispricing exists that gives rise to stochastically dominating portfolios. The out-of-sample test consists of a stochastic dominance comparison of two time series for which the only requirement is that the observations are serially uncorrelated.

Constantinides, Czerwonko, Jackwerth, and Perrakis (CCJP 2011) test the corresponding CP (2007) bounds for S&P 500 index futures options and demonstrate violations of mainly the upper bound on call option prices. They implement the trading strategy based on selling violating call options and show out of sample that such strategy increases utility relative to a strategy which does not include the violating call options.

Constantinides, Czerwonko, and Perrakis (CCP 2017) expand the search from single mispriced options to mispriced index option *portfolios* on the S&P, CAC, and DAX indices and show out of sample that the inclusion of the mispriced option portfolios increases utility relative to a strategy which does not trade in options in almost every month of the sample period. The mispriced portfolios include both short and long positions in call and put options and both out- and in-the-money (OTM and ITM) options.

2 Why Wallmeier’s Bounds Differ from Ours

We argue that Wallmeier’s procedure of estimating the volatility input which is supposed to be under the physical probability measure artificially inflates the volatility, thereby raising the upper call bound and reducing the number of violations which are mainly bid call price quotes above the upper bound. The expectation in the CP (2002, 2007) bounds is based on the physical probability measure, not the risk-neutral measure. Wallmeier initially follows CJP (2009) and CCJP (2011) in estimating the historical distribution of index returns. Then he crucially adjusts the volatility of the historical distribution so that the "average at-the-money (ATM) implied volatility lies in the middle of the bounds range," (Wallmeier, p. 8). Basically Wallmeier replaces the volatility under the physical measure with the ATM implied volatility (*IV*), that is, the risk-neutral volatility. Yet this ATM *IV* is a strongly upward biased predictor of the physical volatility of realized returns. The bias is major, about 2% in our sample from January 1990 to February 2013. Since by construction all the CP (2002 and 2007) bounds are based on expectations under the physical measure of the return distribution, their expectation with a more volatile distribution yields a higher value by Jensen's inequality, given the convexity of the payoff. This incorrect interpretation of the CP (2002, 2007) bounds accounts for the results in Wallmeier. We illustrate this point by replicating Wallmeier's results.

Wallmeier argues that the estimation of the ex-dividend index price is improved by using intra-daily data (transactions data) instead of end-of-day data and by using dividends implied by the put-call parity instead of the observed dividend yields. Both effects turn out to be trivially small as we show in our empirical results below where we implement both changes.

We use the same S&P 500 options data set as in (CCP, 2017) for 28-day options from January 1990 to February 2013, filtering the data for arbitrage violations. For the index price adjustment for dividends we follow Wallmeier and use five intraday cross-sections from 11:00 AM to 3:00 PM EST in hourly intervals. In each cross-section we find the dividend adjustment that best aligns put-call parity for the option pair closest to ATM. We use the median of the five adjustments as our final adjustment. Alternatively we use actual dividends to adjust the index level and the results hardly change. Both papers agree on using a risk premium for the index between 4% and 6% instead of the poorly estimated sample risk premium.

For our reference model we first set the volatility of the physical distribution equal to the realized volatility observed over the previous four weeks (based on daily index returns). Further we use two parametric models. We forecast the volatility of the physical distribution either with the Glosten, Jagannathan, and Runkle (GJR 1993) method or the exponential EGARCH method of Nelson (1991). For either GARCH application we estimate the model coefficients over a rolling window of 3800 daily observations, or approximately 15 years of data. Having estimated coefficients and one-day ahead conditional volatility we sum the forecasted conditional variances till the option expiration day, for which forecast there is a closed form formula for the GJR model while we simulate the EGARCH model with 100,000 repetitions. Our three estimates for the volatility of the physical distribution based on historical data are sensible as they give unbiased forecasts of the volatility over the near future.

Next we follow Wallmeier and inflate the three volatility estimates by 1% or 2%. We also set the volatility equal to the 11:00 AM ATM implied volatility and the 3:00 PM ATM implied volatility which are on average 2.13% and 2.03% higher than the realized volatility, respectively.

We derive the bounds using the trinomial tree assuming a 6% risk premium—decreasing the premium to 4% has minimal impact on the results. We assume 0.5% one-way transaction cost rate; this rate is conservative as a high transaction cost leads to fewer violations. We verify the violations by using option bid and ask quotes rather than trading prices (which may be bid or ask prices or prices in between) as in Wallmeier and do not impose the put-call parity on the bounds.[1] As noted above put-call parity is a meaningless concept in an economy with frictions and does not

[1] Wallmeier (p. 6) imposes the put-call parity on the CP upper bound by setting the transaction cost parameter k equal to zero on the grounds that "one-way transaction costs are typically below 10 basis points which means that they do not strongly affect the option price bounds". This reasoning is incorrect, since even with $k = 0.1\%$ the CP put upper bound is reduced by several dollars when we set $k = 0$. This weak bound was never used in CJP for screening overpriced options.

hold in Wallmeier's own trading price data; see the left panel in his figure 1. Besides option quotes yield fewer violations than trading prices of the key CP call upper bound and only quoted prices can be used for the out-of-sample tests.

Table 1 presents the percentage of violations of the call upper bound by the call bid prices (panel A) and the percentage of violations of the put lower bound by the put ask prices (panel B). The volatility forecast is: the realized volatility; the volatility estimated by the Glosten, Jagannathan, and Runkle (GJR, 1993) and Nelson (EGARCH, 1991) methods; the ATM *IV* estimated at 11:00 AM; and the ATM *IV* estimated at 3:00 PM. The columns with the headings "unadjusted" present the percentage of violations when no adjustment is made to the volatility forecast. The columns with the headings "1%", "2%", "-1%", and "-2%" present the percentage of violations when the volatility forecast is adjusted by the stated percentage.

[Table 1 here]

We find few violations of the put lower bounds in panel B of Table 1 and thus concentrate on the upper bound violations in panel A. We again establish our typical violation pattern of almost 30% of call options violating the upper bound for the realized volatility and the GJR-estimated volatility. This rises to 40% for the EGARCH-estimated volatility. We also see that an ad-hoc addition of 1% or 2% to the realized volatility leads to a reduction in violations from 30% to 14% and 8%, respectively.

We replicate Wallmeier's results by replacing the realized volatility with the 11:00 AM *IV* which corresponds to an ad-hoc addition of 2.13%, on average. This leads to an almost complete elimination of violations (0.59%). Using the 3:00 PM *IV* which corresponds to an ad-hoc addition of on average 2.03% leads to a similar result (0.55% violations). When we correct for the upward bias in the *IV* as a measure of the volatility under the physical probability measure by subtracting 2% from the *IV* we find 19.72% violations of the call upper bound.

We note that the imposition of the put-call parity has little influence on the results. The results are qualitatively similar when we use the put-call-parity-implied volatilities as in Wallmeier or the average of the put and the call implied volatilities. Furthermore the choice of 11:00 AM *IV* versus 3 PM *IV* has little influence on the results.

We conclude that Wallmeier finds practically no violations of the bounds because he uses an upward-biased volatility under the physical probability measure in the calculation of the bounds.

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Table 1: Violations of Options Bounds, January 1990 to February 2013

The table presents the percentage of violations of the call upper bound by the call bid prices (panel A) and the percentage of violations of the put lower bound by the put ask prices (panel B). The volatility forecast is: the realized volatility; the volatility estimated by the Glosten, Jagannathan, and Runkle (GJR, 1993) and Nelson (EGARCH, 1991) methods; the ATM *IV* estimated at 11:00 AM; and the ATM *IV* estimated at 3:00 PM. The columns with the headings "unadjusted" present the percentage of violations when no adjustment is made to the volatility forecast. The columns with the headings "1%", "2%", "-1%", and "-2%" present the percentage of violations when the volatility forecast is adjusted by the stated percentage.

A: Percent Violations of Call Upper Bound			
	unadjusted	+1%	+2%
realized	28.68	14.03	7.84
GJR	29.70	15.97	8.31
EGARCH	40.87	28.91	18.74
	unadjusted	-1%	-2%
11:00 AM *IV*	0.59	2.40	19.33
3:00 PM *IV*	0.55	3.47	19.72

B: Percent Violations of Put Lower Bound			
	unadjusted	+1%	+2%
realized	3.91	5.31	8.40
GJR	3.34	5.49	8.74
EGARCH	4.40	5.98	8.08
	unadjusted	-1%	-2%
11:00 AM *IV*	0.04	0	0
3:00 PM *IV*	0.11	0.02	0