Seasoned Offerings, Imitation Costs, and the Underpricing of Initial Public Offerings

Ivo Welch

Seasoned Offerings, Imitation Costs, and the Underpricing of Initial Public Offerings

IVO WELCH*

ABSTRACT

This paper presents a signalling model in which high-quality firms underprice at the initial public offering (IPO) in order to obtain a higher price at a seasoned offering. The main assumptions are that low-quality firms must invest in imitation expenses to appear to be high-quality firms, and that with some probability this imitation is discovered between offerings. Underpricing by high-quality firms at the IPO can then add sufficient signalling costs to these imitation expenses to induce low-quality firms to reveal their quality voluntarily. The model is consistent with several documented empirical regularities and offers new testable implications. In addition, the paper provides empirical evidence that many firms raise substantial amounts of additional equity capital in the years after their IPO.

EVER SINCE IBBOTSON (1975) FIRST rigorously documented the large underpricing of initial public offerings (IPOs), it has puzzled researchers. Rock (1986) has offered an equilibrium model for this phenomenon in which uninformed investors face a winner's curse when they submit an order for IPO shares. Since informed investors withdraw from the market when the issue is priced above its value, uninformed investors are more likely to receive a full allocation of shares if the offering is overpriced and a rationed allocation if it is not. Firms are forced to underprice their IPOs in order to compensate uninformed investors for this adverse selection. Beatty and Ritter (1986) extend the model to show that the value of information and, thus, both the bias against uninformed investors and the necessary underpricing are higher for issues for which there is greater uncertainty about their value.

One problem with the winner's curse explanation of underpricing is that it could be easily avoided. For example, underwriters could reduce the adverse selection problem by offering IPOs only in pools or by agreeing to withdraw an issue or compensate uninformed investors if demand from informed investors is not forthcoming (see also Ritter (1987)). Alternatively, venture capitalists could provide the expertise and capital funding to reduce or avoid IPO underpricing. Yet Barry, Muscarella, Peavy, and Vetsuypens (1988) find that venture-capital-backed IPOs are even more underpriced than non-venture-capital-backed issues.

Ibbotson, in listing possible reasons for the underpricing of IPOs, stated that the prevalent explanation on Wall Street is that issuers may want to " 'leave a good taste in investors' mouths' so that future underwritings from the same issuer could be sold at attractive prices." [1] This paper formalizes this argument in a two-period signalling model in which firms are rational actors with superior information in a perfectly competitive capital market. The main assumptions are that low-quality firm owners must incur imitation costs to appear to be high-quality firms, and that nature may nevertheless reveal the firm's true quality after the IPO but before a seasoned offering (SO). Consequently, low-quality issuers face a tradeoff. They can invest in imitation activity but face the possible loss of some of this investment if they are discovered, or they can reveal their quality and forego the higher price they could have received at the IPO and the SO had their imitation not been discovered. This paper shows that the additional costs of underpricing can induce low-quality firms to voluntarily reveal their quality when real imitation costs alone are not sufficient.

There are three distinctive features of this model. First, the information asymmetry is due to the firm owner knowing more about the firms' value than investors. In Rock's model, the asymmetry is between informed investors on one hand and uninformed firm owners and investors on the other. Second, in my model it is high-quality firms whose quality is not otherwise known by the market that underprice. Third, this model implies that high-quality firms *value* underpricing as a signalling device. Therefore, firms have no incentive to avoid underpricing. In Rock's model, firms reluctantly underprice only to keep uninformed investors in the market.

Three recent papers also model IPO underpricing as a signal from better-informed firm owners to less informed investors. In Nanda (1989), projects with higher expected return are also less risky. Since (unidentified) riskier firms would prefer to issue debt over equity because of limited liability, high-quality firms can use (underpriced) equity contracts as a signalling mechanism to communicate their quality. In Grinblatt and Hwang (1989) a generalization of Leland and Pyle (1977) is presented in which a continuum of risk-averse firm types signal two characteristics, the mean and the risk of their projects. To do so, firms employ a second signal, underpricing, in addition to Leland and Pyle's original signal, the fraction of the firm retained by the firm owner. In Allen and Faulhaber (1987), high-quality IPO firms offer to trade off a lower IPO price against a more favorable interpretation of future high dividends. Low-quality firms are more reluctant to offer this exchange, since they are less likely to experience high future cash flows and therefore are less likely to pay high future dividends. Consequently, investors can rationally interpret future dividends more favorably for firms that underprice at their IPO.

The latter two models are particularly closely related to this model insofar as, in all three, IPO underpricing results in higher proceeds for high-quality firms in future selling activity. In contrast, however, this model focuses on financial markets' ability to observe firms' past and current real activities at the IPO. I assume that there are direct costs that low-quality firms must incur in order to

---

[1] See Ibbotson (1975, p. 264).

imitate observable operations by high-quality firms. Therefore, this model offers comparative statics quite different from the above models.

The remainder of this paper is organized as follows: Section I provides the details of the model, including a description of the assumptions, the sequence of actions and events, and the definition of equilibrium. Section II analyzes the pricing and operating choices of issuers in different equilibria. Section III presents the empirical implications of the model. Section IV provides preliminary empirical evidence consistent with a central implication of the model: some firms should issue only a portion of their equity in the IPO and the remainder in SOs. I find that nearly a third of the approximately one thousand IPO firms during 1977–1982 had issued SOs by 1987. Section V contains concluding remarks.

I. The Model

A. Setting

Consider an economy in which risk-neutral individuals own one of two types of firms, high-quality firms, H, and low-quality firms, L. The utility of each firm owner depends only on the sum of the issuing proceeds from an initial public offering (IPO) and a single seasoned offering (SO). The risk-neutral, perfectly competitive market would pay V^H for a high-quality firm and V^L for a low-quality firm ($V^L < V^H$). However, investors cannot directly verify the quality of an individual firm; they know only the aggregate proportion of high-quality and low-quality firms, h and $1- h$, respectively. h can therefore be interpreted as the market's prior that a firm is high-quality. Both high-quality and low-quality owners know their firm's true value, but high-quality owners cannot credibly communicate their knowledge through simply announcing their quality. To receive V^H (at least at the SO), high-quality firms may be forced to adopt a mechanism that speaks louder than words.

Fortunately for the high-quality owners in this model, nature reveals the true firm value with probability r between the IPO and SO. This represents credible outside information that becomes available randomly. For example, an oil fountain may emerge from the drilling site in time or a disgruntled employee may reveal his employer's type. I shall sometimes refer to this type of information disclosure as *detection* or *revelation*.

Further, assume that it is costly for a low-quality firm to imitate a high-quality firm. There are publicly observable costs that a high-quality owner always finds optimal to incur but that are not optimal for low-quality owners. For example, a high-quality oil company may order a pipeline, while a low-quality oil company would rather not. I assume that these activities (sometimes referred to as *operations*)[2] are efficient for high-quality firms in the sense that, even in a world of perfect information, high-quality firms must perform them to be worth V^H; and that the imitation of these operations, with cost C, is of no value to low-quality firms other than to imitate the observable actions or attributes of high-

[2] A firm that does not *operate* may, of course, conduct business other than that necessary to appear to be of high quality.

Table I
Market Value of Firms Conditional on
Investors' Information Set and the Firm's
Operations (o)

A known high-quality firm (H) is worth V^H if it operates ($o = $ O), V^L if it does not operate ($o = $ NO). A known low-quality firm (L) is worth V^L if it does not operate. If it does operate, it expends costs (C) and is thus worth only $V^L - C$. If the market cannot distinguish between these firms, it is willing to pay the expected value of an unidentified firm. This is just the value of high-quality firms and low-quality firms weighted by their proportion in the market, h and $1 - h$, respectively.

	Known H Firm	Known L Firm	Unknown Firm
Firm operates ($o = $ O)	V^H	$V^L - C$	$hV^H + (1 - h)(V^L - C)$
Firm does not operate ($o = $ NO)	V^L	V^L	V^L

quality firms. For simplicity, I further assume that high-quality firms that do not operate are worth V^L, and that there are no default considerations ($V^L - C > 0$). Table I summarizes the market values of firms.[3]

Next, assume that firms are so wealth constrained that they must raise the capital necessary to fund their operations, and that C covers the full outlays necessary for a firm to operate.[4] (Recall that only high-quality firms eventually recoup this outlay.) I further assume that firms can raise their minimum certain value after beginning operations ($V^L - C$) from alternative sources, but that these cannot completely finance the firm's operations, that is $V^L - C < C$.[5] Without loss of generality, the minimum funding and maximum borrowing constraints can be combined into a minimum proceeds constraint

$$\left. \begin{array}{r} \alpha_1 P_1 + B \geq C \\ B \leq V^L - C \end{array} \right\} \ \alpha_1 P_1 \geq 2C - V^L \quad (>0), \tag{1}$$

where $\alpha_1 P_1$ denotes the proceeds of the IPO, and B the amount a firm borrows from alternative sources.

[3] Because V^H and V^L are defined to be the perfect information values of high-quality firms and low-quality firms, C is subtracted from a low-quality firm's value only if the low-quality firm attempts to imitate a high-quality firm. To illustrate, consider land that can be used for either farming or oil exploration. If there is exploration and oil is found, the value of the land is V^H. If there is no exploration, the value of the land is that to the farmer, V^L. These would be the full-information values. However, when a low-quality imitator explores oil-empty land, he or she (a) incurs exploration expenses and (b) foregoes farming rent. Therefore, such land would not be worth V^L, but only $V^L - C$. Note also that with limited stockholder liability, it is unclear who would bear possible default costs if I allowed $V^L < C$. In the strictest sense, one may therefore interpret this model as applying only to firms with $V^L > C$.

[4] This assumption was introduced by Leland and Pyle (1977) and has often been adopted in the IPO literature (e.g., see Rock (1986) and Allen and Faulhaber (1987)).

[5] Firms that can finance their operations by borrowing may not have to approach the IPO market for a while and thus may be able to—and, as shall be discussed in section IIE, may prefer to—advertise sufficiently early to signal quality (for a subsequent IPO) without Securities Exchange Commission (SEC) restrictions. It is also common for IPO firms to engage in several rounds of venture capital financing prior to going public. Firms communicate some proprietary information to venture capitalists, whose investments partially certify that the issuer is a high-quality firm.

B. Sequence of Actions and Events

The game played by managers and investors is now described in more detail, and the sequence of actions and events is specified. The players are the two types of firms and the market. The market is both passive and efficient. Investors purchase an entire offering if and only if, given their current information, its expected value weakly exceeds its price. Firm owners are active players. They maximize the expected sum of proceeds from two offerings, one in each of two stages.

In stage 1, the firm concurrently issues its IPO and begins operations, denoted by o ($o \in \{O, NO\}$, where O [NO] stands for operation [no operation]). The market observes o and can then purchase the offering, involving the proportion α_1 of the firm for price P_1. Firm owners in turn can use the proceeds of this offer to fund operations. Thus, the market uses information from the firm's action triple $(\alpha_1, P_1, o) \in ([0, 1] \times R^+ \times \{O, NO\})$ in its decision to purchase the IPO.

After the IPO, nature reveals to all players the firm's quality with probability r. Let RH (RL) be the state (denoted by R) in which the firm is revealed to be high quality (low quality), and NR the state where quality is not revealed. I assume that this information is not noisy—$\rho(L \mid RH) = \rho(H \mid RL) = 0$, where ρ stands for probability—and that the revelation probability is nonzero and independent of any other variables, in particular, of firm quality: $\rho(RH \mid H) = \rho(RL \mid L) = r$, and $\rho(NR \mid H) = \rho(NR \mid L) = 1 - r$.

In stage 2, the market observes another action by the firm, the seasoned offering (SO), analogously denoted $(\alpha_2, P_2) \in ([0, 1] \times R^+)$. Hence the market can use the information set $\{(\alpha_1, P_1, o), R, (\alpha_2, P_2)\}$ to judge firm quality in stage 2. The game ends with stage 2. Therefore each firm owner chooses α_2 to be $1 - \alpha_1$, and P_2 to be the highest price the market is willing to pay. Market beliefs about firm quality are in turn determined by known past events, first periods' actions by the firm (α_1, P_1, o) and the state of detection R. Consequently, only strategic signalling behavior in stage 1 needs to be considered, (α_2, P_2) contains no additional information about firm quality to the market, and $\{(\alpha_1, P_1, o), R\}$ is a sufficient statistic for market beliefs.

C. Definition of Equilibrium

I first introduce the notation used to define and describe equilibria. Let $\sigma_1[(\alpha_1, P_1, o) \mid Q]$ denote the probability of a firm of type $Q \in \{H, L\}$ to choose action (α_1, P_1, o) in stage 1, and let $\sigma_2[(\alpha_2, P_2) \mid Q, (\alpha_1, P_1, o), R]$ denote the probability of a firm of type Q to choose action (α_2, P_2) in stage 2 (after [a] having engaged in o operations, [b] offered α_1 of the firm for P_1 at the IPO, and [c] experienced detection R).

The manager's optimal strategy depends on the market's reaction to the specified strategy. Let $\rho_1[Q \mid (\alpha_1, P_1, o)]$ denote the market's belief in stage 1 that the firm is type Q (after having observed [a] the firm's o operations and [b] the firm's IPO of α_1 of the firm for P_1), and let $\rho_2[Q \mid (\alpha_1, P_1, o), R, (\alpha_2, P_2)]$ denote the market's belief in stage 2 that the firm is type Q (after having observed [a] the firm's o operations, [b] the firm's IPO of fraction α_1 of the firm for P_1, [c] the state of detection R, and [d] the firm's SO of α_2 of the firm for P_2). Since

[d] contains no information about firm quality, $\rho_2[Q \mid (\alpha_1, P_1, o), R, (\alpha_2, P_2)]$ is henceforth abbreviated to $\rho_2[Q \mid (\alpha_1, P_1, o), R]$ to reduce clutter.

The link between market beliefs and firm strategies is the equilibrium definition that specifies the way in which each is rational given the other. I employ the concept of Bayesian equilibrium. An equilibrium is a set of strategies σ_1, σ_2, and of (posterior) market beliefs ρ_1, ρ_2, such that (a) at each stage both high-quality and low-quality owners' choice of stragegy—given their own information, future (wealth-maximizing) strategies, and market beliefs—is wealth maximizing, and (b) at each stage the market's posterior beliefs about firm quality are consistent with Bayes' rule and the specified and observed strategies of firms.

Thus, for an equilibrium to exist, there must be a set of market beliefs that are self-fulfilling and against which firm's actions are optimal. In this model, this implies that the set of equilibria is defined against market beliefs that consider a firm that acts in an out-of-equilibrium fashion to be of low quality (with value $V^L - C$ if $o = O$, V^L if $o = NO$)—unless, of course, nature reveals this firm to be of high quality. However, as I proceed, I shall apply some of Cho and Kreps' (1987) equilibrium refinements. Since these constrain rational market beliefs such that investors do not consider out-of-equilibrium firms to be of low quality in certain situations, high-quality firms can have an opportunity to profitably deviate. This allows some economically implausible Bayesian equilibria to be eliminated.

For example, consider an equilibrium in which high-quality firms are strictly worse off and low-quality firms are strictly better off than in a second equilibrium. Cho and Kreps (1987) argue that the first equilibrium is untenable, because a high-quality firm could adopt the action of the second equilibrium, and make a speech to the market that its (out-of-equilibrium) action should not be interpreted to imply low quality. This is because only a high-quality firm could have an incentive to be compensated according to the second equilibrium. Therefore, all high-quality firms would pursue this out-of-equilibrium strategy, and the original equilibrium is no longer tenable.

II. Equilibria of the Issuing Game

In this section, three pure equilibria of the issuing game are presented.[6] The first equilibrium is a pooling equilibrium in which investors cannot distinguish at the IPO between low- and high-quality firms since both raise enough capital at their IPO to operate and do operate. The second is a first-best separating equilibrium in which all high-quality firms operate but do not underprice, and low-quality firms voluntarily reveal their identify. The third is a signalling equilibrium in

[6] The choice of presented equilibria is not exhaustive, and I do not even discuss the full set of these equilibria. Instead I present and contrast only some economically plausible equilibria. In particular, I omit an equilibrium where neither firm operates and the market believes every firm to be of value V^L, since it is pareto dominated by some other equilibrium over all possible parameter values. Further, I omit a mixed equilibrium in which some high-quality firms pool and others underprice. This equilibrium is not included in the text because an argument that only some high-quality firms underprice while others pool adds little to an understanding of why high-quality firms underprice (above and beyond the pure equilibria presented in the text). Notice, though, that the discontinuity of high-quality firms' expected total proceeds across equilibria (see Figure 2) is due to the omission of this mixed equilibrium.

which all high-quality firms underprice at their IPO and operate, while low-quality firms again disclose their identity by not operating. In both the pooling and underpricing equilibria, high-quality firms receive less than their value at the IPO. Yet there are bargains to investors only in the latter equilibrium.

A. A Pooling Equilibrium

When both high-quality and low-quality firms choose to raise sufficient funds to operate, and do operate, a pooling equilibrium results. Low-quality firms mimic the operations and pricing of high-quality firms in order to receive the proceeds of high-quality firms both at the IPO and at the SO if nature does not reveal their type. By imitating, low-quality firms impose an externality on high-quality firms because they lower the price unrevealed high-quality firms can receive.

The proof of the existence of the pooling—and subsequent—equilibria is constructive. A set of market beliefs is specified, and a program that assures that firms behave accordingly is constructed. For the pooling equilibrium, assume that the market believes that all firms offer a fraction α_1^P of the firm for P_1^P and operate. Against these beliefs, the best alternative for a pooling low-quality firm is to reveal its quality. This avoids the loss of C at the SO if the market were to discover its true quality (state RL) but foregoes the benefits of imitation. These benefits are a higher offering price—according to the average firm quality—both at the IPO and, if the market does not discover the firm's quality (state NR), at the SO. A high-quality firm has two alternatives. It can choose not to operate and accept V^L—which it prefers to pooling only if a low-quality firm prefers to accept V^L. Or it can sell a smaller proportion of the firm, $\hat{\alpha}_1 < \alpha_1$, for $V^L - C$ at the IPO and operate (if it can satisfy the minimum proceeds constraint), hoping for subsequent revelation. In summary, a pooling equilibrium exists if a pair (α_1^P, P_1^P) exists such that[7]

$$\alpha_1^P P_1^P + (1 - \alpha_1^P)[r(V^L - C) + (1 - r)U] \geq V^L, \tag{2}$$

$$\alpha_1^P P_1^P + (1 - \alpha_1^P)[rV^H + (1 - r)U] \geq V^L, \tag{3}$$

$$\alpha_1^P P_1^P + (1 - \alpha_1^P)[rV^H + (1 - r)U] \geq \max_{\hat{\alpha}_1, \hat{P}_1} \quad \hat{\alpha}_1 \hat{P}_1 + (1 - \hat{\alpha}_1) \tag{4}$$

$$[rV^H + (1 - r)\hat{P}_2]\},$$

$$\text{s.t.} \quad \hat{\alpha}_1 \hat{P}_1 \geq 2C - V^L$$

$$\hat{P}_1, \hat{P}_2 \leq V^L - C, \tag{5}$$

$$\alpha_1^P P_1^P \geq 2C - V^L, \tag{6}$$

$$P_1^P \leq U,$$

---

[7] To reduce clutter, this and subsequent programs have already been substantially simplified. In particular, since in stage 2 firms maximize their proceeds, I substitute $\alpha_2 = 1 - \alpha_1$ and

$$P_2 = \begin{cases} \rho_2[H \mid (\alpha_1, P_1, \text{NO}), R]V^H + \rho_2[L \mid (\alpha_1, P_1, \text{NO}), R]V^L & \text{if } o = \text{NO}, \\ \rho_2[L \mid (\alpha_1, P_1, O), R]V^H + \rho_2[L \mid (\alpha_1, P_1, O), R](V^L - C) & \text{if } o = O. \end{cases}$$

Furthermore, I substitute out the ρ notation for market beliefs. (The formal description follows in the proofs of equilibrium existence.) Similarly, I use r as equivalent for $\rho(\text{RH}) = \rho(\text{RL})$ and $1 - r$ for $\rho(\text{NR})$.

where U is the market value of an unidentified, operating firm:

$$U \equiv hV^H + (1 - h)(V^L - C). \tag{7}$$

Constraint (2) states that a low-quality firm does not prefer revealing its quality and accepting V^L to adhering to the pooling equilibrium. By adhering to the pooling equilibrium, for the fraction $1 - \alpha_1^P$ it offers at the SO, a low-quality firm receives $V^L - C$ in state RL but U in state NR. In stage 1, since the market does not know its quality, a low-quality firm can receive P_1^P for the fraction α_1^P of the firm that is sold in the pooling equilibrium. Constraints (3) and (4) state that high-quality firms prefer the pooling equilibrium's proceeds ($\alpha_1^P P_1^P$ at the IPO; $(1 - \alpha_1^P)V^H$ in state RH or $(1 - \alpha_1^P)U$ in state NR at the SO) to either accepting V^L, or selling a different proportion $\hat{\alpha}_1$ of the firm for $\hat{P}_1$ in order to raise the minimum proceeds and operate. Constraint (5) is the minimum proceeds constraint. And constraint (6) states that investors do not pay more for unidentified firms than their expected value. Market beliefs are rational because both low-quality and high-quality firms optimally choose to offer for (α_1^P, P_1^P) and operate.

The program in (2–6) defines the set of pooling equilibria that meet the Bayesian equilibrium definition. Yet, the following Cho and Kreps (1987)-style argument can further reduce this set. A high-quality firm is always better off in a pooling equilibrium in which only the fraction of the firm that is necessary to fund operations is sold. The opposite is the case for low-quality firms. Thus, when a firm approaches the market with an alternative issue, proposing to sell a smaller fraction of the firm just sufficient to permit to begin operations, the market should rationally believe that this firm is no worse than the average firm. Consequently, since the market recognizes the intent of such out-of-equilibrium behavior, every high-quality firm would pursue it, and any low-quality firm that would refuse to do the same would reveal itself as low-quality. The only equilibrium that survives this argument, henceforth referred to as the *best* pooling equilibrium, is one in which firms sell only the necessary proportion of the firm to begin operations in their IPO. Similarly, both high-quality and low-quality firms are strictly better off if the equilibrium IPO price is the maximum price that investors are willing to pay for firms of unknown quality. Consequently, investors can be convinced that if any firm offered for $P_1^P = U$ in a pooling equilibrium, this higher price should not imply low quality.

This restricted set of pooling equilibria can be described in a modified program in which constraints (5) and (6) are binding. Moreover, constraints (3) and (4) become redundant given (2) because high-quality firms no longer have an incentive to deviate if low-quality firms have no incentive. The restrictions on the parameters that permit the existence of this best pooling equilibrium are summarized in Lemma 1. Let $P_1^P \equiv U$ and $\alpha_1^P \equiv \dfrac{2C - V^L}{P_1^P}$.

LEMMA 1 (POOLING EQUILIBRIUM): *A best pooling equilibrium exists if*

$$U \geq V^L \tag{8}$$

and

$$r \le \left(\frac{U - V^L}{U - (V^L - C)}\right)\left(\frac{U}{U - (2C - V^L)}\right), \tag{9}$$

where

1. *investors believe that in stage 1 both high- and low-quality firms raise funds by selling a proportion α_1^P of the firm for P_1^P and operate:*

$$\rho_1[H \mid (\alpha_1^P, P_1^P, O)] = h, \qquad \rho_1[L \mid (\alpha_1^P, P_1^P O)] = 1 - h,$$

$$\rho_1[L \mid (\hat{\alpha}_1, \hat{P}_1, \hat{o})] = 1 \quad for all (\hat{\alpha}_1, \hat{P}_1, \hat{o}) \ne (\alpha_1^P, P_1^P, O),$$

$$\rho_2[H \mid (\alpha_1^P, P_1^P, O), NR] = h, \qquad \rho_2[L \mid (\alpha_1^P, P_1^P, O), NR] = 1 - h,$$

$$\rho_2[L \mid (\hat{\alpha}_1, \hat{P}_1, \hat{o}), NR] = 1 \quad for all (\hat{\alpha}_1, \hat{P}_1, \hat{o}) \ne (a_1^P, P_1^P, O),$$

$$\rho_2[H \mid (\alpha_1, P_1, o), RH] = \rho_2[L \mid (\alpha_1, P_1, o), RL] = 1 \quad for all (\alpha_1, P_1, o);$$

2. *and both high- and low-quality firms offer a proportion α_1^P of their firm for P_1^P and operate in stage 1:*

$$\sigma_1[(\alpha_1^P, P_1^P, O) \mid H] = 1, \qquad \sigma_1[(\alpha_1^P, P_1^P, O) \mid L] = 1,$$

$$\sigma_2[(1 - \alpha_1^P, U) \mid H, (\alpha_1^P P_1^P, O), NR] = 1,$$

$$\sigma_2[(1 - \alpha_1^P, U) \mid L, (\alpha_1^P P_1^P, O), NR] = 1,$$

$$\sigma_2[(1 - \alpha_1^P, V^H) \mid H, (\alpha_1^P P_1^P, O), RH] = 1,$$

$$\sigma_2[(1 - \alpha_1^P, V^L - C) \mid L, (\alpha_1^P P_1^P, O), RL] = 1,$$

Proof: Inequality (8) is a necessary restriction on (α_1^P, P_1^P) in (2) and a sufficient condition for (α_1^P, P_1^P) in a binding (5). Inequality (9) follows from substitution of binding constraints (6) and (5) into (2).   Q.E.D.

Inequality (8) assures both that low-quality firms are better off imitating high-quality firms at least when their imitation is not discovered than if they had revealed themselves, and that high-quality firms can raise the necessary funding to begin operating in stage 1. Inequality (9) assures that low-quality firms do not deviate from this equilibrium by revealing their quality.

I now derive the comparative statics of the pooling equilibrium. Since unidentified firms receive the average value of an operating firm, $hV^H + (1 - h)(V^L - C)$, both high- and low-quality firms are better off in a pooling equilibrium the smaller the proportion of low-quality firms in the pool $(1 - h)$ and the higher the value of high-quality firms (V^H). In contrast, an increase in the value of low-quality firms or a decrease in the cost of imitation improves the welfare of high-quality firms but not necessarily of low-quality firms. Both high- and low-quality firms can obtain a higher price for their IPO and their SO when their identities are not revealed. However, the simultaneous increase in borrowing opportunities permits a high-quality firm to reduce the fraction of the firm that must be sold at stage 1 to cover operations. Selling a smaller fraction of the firm at the IPO strictly benefits high-quality firms and strictly hurts low-quality firms (since

their type can be detected between issues). In addition, when V^L increases, the value of the best alternative of low-quality firms (revealing quality) increases. Similarly, the impact of changes in the probability of detection (r) on equilibrium compensation differs for high- and low-quality firms. High-quality firms benefit and low-quality firms suffer when the probability that they receive their true value at their SO increases.

LEMMA 2: *The set of parameter values that permit a pooling equilibrium increases with h and V^H and decreases with r.*

Proof: U is a strictly increasing function of h and V^H, neither of which appears elsewhere in (8) and (9). The partial derivative of the right-hand side of (9) with respect to U is

$$\frac{\partial}{\partial U} \cdot \frac{(V^L - C)[(V^L - U)^2 + 2C(2U - V^L)]}{(V^L - U - C)^2 (V^L + U - 2C)^2} \geq 0. \tag{10}$$

This completes the first part of the lemma. The comparative statics with respect to r are more easily obtained by inspection of (9).   Q.E.D.

B. A First-Best Announcement Equilibrium

The pooling equilibrium is feasible when the probability of detection is low. At the opposite extreme, when the probability of detection is very high, imitation by low-quality firms is so likely to be discovered that high-quality firms can sell their IPO for the maximum price, V^H. This occurs when the expected losses due to low-quality firms' operations—reduced proceeds at the SO if the imitation is uncovered—are so high that low-quality firms will not claim to be of high-quality (and receive V^H) at their IPO. This equilibrium is first-best (1) because no low-quality firm incurs the socially wasteful investment in imitation and (2) because owners receive their firm's true (full-information) value.

Again, this equilibrium is constructed by a program that ensures that our specification of market beliefs is rational. Assume that the market believes that operating firms that offer fraction α_1^A for $P_1^A = V^H$ in their IPO and operate are high quality and that any other action implies low quality. An announcement equilibrium exists if some fixed (α_1^A, P_1^A) exists such that

$$\alpha_1^A P_1^A + (1 - \alpha_1^A)[r(V^L - C) + (1 - r)V^H] \leq V^L, \tag{11}$$

$$\alpha_1^A P_1^A \geq 2C - V^L, \tag{12}$$

$$P_1^A = V^H. \tag{13}$$

Constraint (11) states that low-quality firms prefer revealing themselves to operating and selling fraction α_1^A for P_1^A at their IPO. Constraint (12) is the minimum proceeds constraint. Constraint (13) states that operating but unidentified firms receive as much as identified high-quality firms. High-quality firms, receiving V^H, have, of course, no incentive to deviate. Market beliefs are rational in that only high-quality firms operate and offer for (α_1^A, P_1^A). (Note also that whenever such an equilibrium exists, another equilibrium exists in which constraint (12) is binding. This equilibrium can be constructed by decreasing α_1^A.)

Lemma 3 presents the restrictions on the exogenous variables that permit the existence of this first-best equilibrium. Let $P_1^A \equiv V^H$ and $\alpha_1^A \equiv \dfrac{2C - V^L}{P_1^A}$.

LEMMA 3 (ANNOUNCEMENT EQUILIBRIUM): *A (first-best) announcement equilibrium exists if*

$$r \geq \left(\frac{V^H}{V^H - V^L + C}\right)\left(\frac{V^H - V^L}{V^H + V^L - 2C}\right), \tag{14}$$

where

1. *investors believe that only high-quality issuers operate and offer proportion α_1^A of the firm for price P_1^A:*

$$\rho_1[H \,|\, (\alpha_1^A, P_1^A, O)] = 1,$$

$$\rho_1[L \,|\, (\hat{\alpha}_1, \hat{P}_1, \hat{o})] = 1 \quad \text{for all } (\hat{\alpha}_1, \hat{P}_1, \hat{o}) \neq (\alpha_1^A, P_1^A, O),$$

$$\rho_2[H \,|\, (\alpha_1^A, P_1^A, O), \text{NR}] = 1,$$

$$\rho_2[L \,|\, (\hat{\alpha}_1, \hat{P}_1, \hat{o}), \text{NR}] = 1 \quad \text{for all } (\hat{\alpha}_1, \hat{P}_1, \hat{o}) \neq (\alpha_1^A, P_1^A, O),$$

$$\rho_2[H \,|\, (\alpha_1, P_1, o), \text{RH}] = \rho_2[L \,|\, (\alpha_1, P_1, o), \text{RL}] = 1 \quad \text{for all } (\alpha_1, P_1, o);$$

2. *and only high-quality firms offer proportion α_1^A of the firm for P_1^A and operate:*[8]

$$\sigma_1[(\alpha_1^A, P_1^A, O) \,|\, H] = 1, \qquad \sigma_1[(1, V^L, \text{NO}) \,|\, L] = 1,$$

$$\sigma_2[(1 - \alpha_1^A, V^H) \,|\, H, (\alpha_1^A, P_1^A, O), R] = 1 \quad \text{for } R \in \{\text{RH}, \text{NR}\},$$

$$\sigma_2[(0, V^L) \,|\, L, (1, V^L, \text{NO}), R] = 1 \quad \text{for } R \in \{\text{RL}, \text{NR}\}.$$

Proof: Substitution of (12) and (13) into (11) yields (14). Substituting (13) into (12), and using $(V^H - C) + (V^L - C) \geq 0$, the minimum proceeds constraint can be shown to be positive. Q.E.D.

I now derive the comparative statics of the announcement equilibrium. Low-quality firms do not invest in imitation if they cannot impose a high enough externality on high-quality firms to pay for their imitation expenses. Therefore, an announcement equilibrium is feasible when the potential gain to imitation (V^H) is low and the probability of revelation (r) and the value of low-quality firms (V^L) are high (since high-quality firms need to raise less funds in stage 1 to operate, and since low-quality firms receive more when they reveal themselves). Further, Lemma 4 proves that an increase in C does not favor the announcement equilibrium. The higher penalty that a discovered low-quality firm bears is outweighed by the increased fraction of the firm that high-quality firms must sell at the IPO to cover their own cost of operations (also C).

LEMMA 4: *The set of parameter values that permit an announcement equilibrium increases with r and V^L and decreases with C and V^H.*

---

[8] To reduce clutter, in both this and the underpricing equilibrium that follows, I assume, without loss of generality, that low-quality firms that are indifferent between selling in stage 1 and stage 2 choose to sell in stage 1.

Proof: Differentiating the right-hand side of (14) with respect to C and V^L yields

$$\frac{\partial}{\partial C} : \frac{V^H(V^H - V^L)(4C + V^H - 3V^L)}{(V^L - V^H - C)^2(V^L + V^H - 2C)^2} \geq 0, \tag{15}$$

$$\frac{\partial}{\partial V^L} : -\frac{V^H[(V^H - V^L)^2 + 2C(V^H - C)]}{(V^L - V^H - C)^2(V^L + V^H - 2C)^2} \leq 0. \tag{16}$$

The comparative statics for V^H and r are more easily obtained by inspection of the program and constraint, respectively.   Q.E.D.

C. An Underpricing Equilibrium

Neither the pooling nor the announcement equilibria can explain why IPOs are on average underpriced. They illustrate, though, that even when an investor observes no excess IPO returns, the underlying market conditions may be quite different. I now discuss an equilibrium that is feasible when the above equilibria are not, and that can explain why firms underprice.

In this equilibrium, high-quality firms sell a fraction of the firm for less than its inferable market value, and low-quality firms reveal their type. The intuition behind this separation is that the marginal cost of underpricing is higher for low-quality firms than for high-quality firms. Imitating low-quality firms face not only the loss of firm value through underpricing (as do high-quality firms) but also the possible loss of some imitation expenses. The program that ensures that high-quality firms choose to signal with (α_1^U, P_1^U), that low-quality firms reveal their quality by not operating, and that the market's beliefs are rational is

$$\alpha_1^U P_1^U + (1 - \alpha_1^U)[r(V^L - C) + (1 - r)V^H] \leq V^L, \tag{17}$$

$$\alpha_1^U P_1^U + (1 - \alpha_1^U)V^H \geq V^L, \tag{18}$$

$$\alpha_1^U P_1^U + (1 - \alpha_1^U)V^H \geq \max_{\hat{\alpha}_1, \hat{P}_1} \quad \hat{\alpha}_1 \hat{P}_1 + (1 - \hat{\alpha}_1)$$

$$[rV^H + (1 - r)$$

$$(V^L - C)]$$

$$\text{s.t.} \quad \hat{\alpha}_1 \hat{P}_1 \geq 2C - V^L,$$

$$\hat{P}_1 \leq V^L - C, \tag{19}$$

$$\alpha_1^U P_1^U \geq 2C - V^L, \tag{20}$$

$$P_1^U \leq V^H. \tag{21}$$

Constraint (17), the self-selection constraint, states that low-quality firms do not imitate high-quality firms' operations and IPO pricing (α_1^U, P_1^U). Constraints (18) and constraints (19), the incentive compatibility constraints, state that high-quality firms prefer offering fraction α_1^U for price P_1^U to both selling the entire firm for V^L, or choosing an alternative price to fund operations. Since P_1^U will be shown to be less than V^H in this equilibrium, and since the previous constraint

ensures that low-quality firms neither underprice nor operate, these constraints state that high-quality firms choose to signal by pricing below their true and inferrable value. Constraint (20) is the minimum proceeds constraint, and constraint (21) is the maximum market price for firms publicly known to be high quality. (Note that an announcement equilibrium necessarily satisfies these constraints.) Market beliefs are rational since low-quality firms choose to reveal their quality, and high-quality firms choose to price their IPO at (α_1^U, P_1^U).

Figure 1 illustrates the set of all feasible underpricing equilibria (α_1^U, P_1^U) that satisfy the program in (17–21). The exogenous variables were chosen so that no announcement equilibrium is feasible. In the figure, this implies that the self-
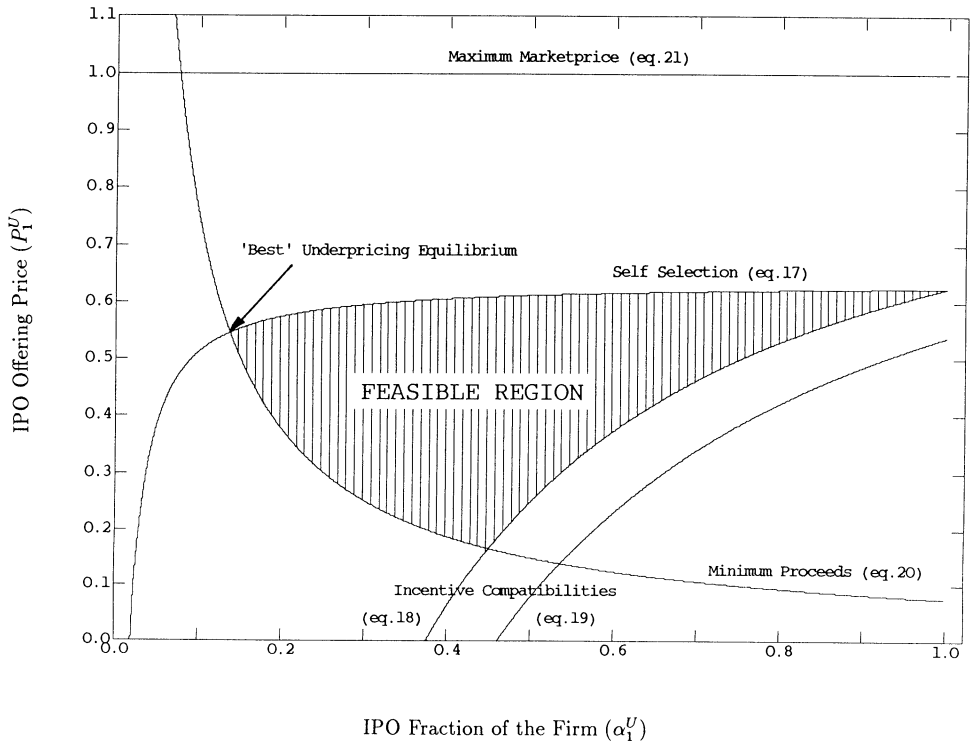

IPO Fraction of the Firm (α_1^U)

Figure 1. Five restrictions on the two endogenous variables, the proportion of the firm sold at the IPO (α_1^U) and the IPO offering price (P_1^U), define the set of feasible underpricing equilibria. The pair (α_1^U, P_1^U) must lie (1) below the self-selection constraint (equation (17)) to prevent low-quality firms from imitating; (2) above the minimum proceeds constraint (equation (20)) to permit high-quality firms to operate; (3) above an incentive compatibility constraint to induce high-quality firms to signal instead of selling the firm at the IPO for V^L (equation (18)); (4) above another incentive compatibility constraint to induce high-quality firms to signal instead of non-signalling but operating (equation (19)); and (5) below the maximum market price constraint (equation (21)). Since constraint (5) here is redundant, high-quality firms underprice. In this example, the value of high-quality firms (V^H) is held constant at 1, the value of low-quality firms (V^L) at 0.625, real imitation costs (C) at 0.35, and the probability of natural revelation after the IPO (r) at 50%. In the "best" underpricing equilibrium, high-quality firms sell about 14% of the firm for an IPO price of about 54% of the firm's true and inferable value, thereby raising just sufficient funds to operate; and low-quality firms just prefer not to imitate.

selection constraint (equation (17)) dominates the maximum price constraint (equation (21)).

As in the pooling equilibrium, the only underpricing equilibrium of interest is that which leaves high-quality firms best off.[9] Lemma 3 has already identified the conditions under which high-quality firms need not underprice to separate themselves. It can be shown that, when the announcement equilibrium is not feasible, high-quality firms are better off selling as small a proportion of the firm as necessary to begin operations at their IPO and underpricing just enough to satisfy the self-selection constraint. Therefore, in the best underpricing equilibrium, indicated by an arrow in Figure 1, constraints (17) and (20) are binding. Lemma 5 summarizes the restrictions on the exogenous variables for the feasibility of the best underpricing equilibrim. Let $P_1^U \equiv \dfrac{(2C - V^L)[r(V^L - C) + (1 - r)V^H]}{(r - 2)(V^L - C) + (1 - r)V^H}$ and $\alpha_1^U \equiv \dfrac{2C - V^L}{P_1^U}$.

LEMMA 5 (UNDERPRICING EQUILIBRIUM): *When the announcement equilibrium is not feasible, a (best) underpricing equilibrium always exists where*

1. *the market believes that only high-quality firms sell a proportion α_1^U of the firm for P_1^U and operate:*

$$\rho_1[H \,|\, (\alpha_1^U, P_1^U, \text{O})] = 1,$$

$$\rho_1[L \,|\, (\hat{\alpha}_1, \hat{P}_1, \hat{o})] = 1 \quad \text{for all } (\hat{\alpha}_1, \hat{P}_1, \hat{o}) \neq (\alpha_1^U, P_1^U, \text{O}),$$

$$\rho_2[H \,|\, (\alpha_1^U, P_1^U, \text{O}), \text{NR}] = 1,$$

$$\rho_2[L \,|\, (\hat{\alpha}_1, \hat{P}_1, \hat{o}), \text{NR}] = 1 \quad \text{for all } (\hat{\alpha}_1, \hat{P}_1, \hat{o}) \neq (\alpha_1^U, P_1^U, \text{O}),$$

$$\rho_2[H \,|\, (\alpha_1, P_1, o), \text{RH}] = \rho_2[L \,|\, (\alpha_1, P_1, o), \text{RL}] = 1 \quad \text{for all } (\alpha_1, P_1, o),$$

2. *and only high-quality firms sell proportion α_1^U of the firm for P_1^U and operate in stage 1:*

$$\sigma_1[(\alpha_1^U, P_1^U, \text{O}) \,|\, H] = 1,$$

$$\sigma_1[(1, V^L, \text{NO}) \,|\, L] = 1,$$

$$\sigma_2[(1 - \alpha_1^U, V^H) \,|\, H, (\alpha_1^U, P_1^U, \text{O}), R] = 1 \quad \text{for } R \in \{\text{RH, NR}\},$$

$$\sigma_2[(0, V^L) \,|\, L, (1, V^L, \text{NO}), R] = 1 \quad \text{for } R \in \{\text{RL, NR}\}.$$

Proof: See the Appendix.

The next section shows that high-quality firms are better off in the pooling than in the underpricing equilibrium whenever both are feasible. It is therefore difficult to determine under what conditions high-quality firms would be best off in an underpricing equilibrium. For example, too high a probability of detection

---

[9] All other underpricing equilibria require an excessive amount of signalling by high-quality firms. And they are not immune to a Cho and Kreps (1987) argument. That is, the market should believe that a firm is high quality when it proposes an out-of-equilibrium offering that satisfies the constraints of the best underpricing equilibrium. For a low-quality firm would never choose to pursue such an out-of-equilibrium strategy, which—by construction—leaves it worse off.

makes underpricing unnecessary because an announcement equilibrium exists in which low-quality firms voluntarily reveal their identity. Too low a probability of detection makes it very expensive (relative to high-quality firms' total proceeds in the pooling equilibrium) for high-quality firms to send a sufficiently strong underpricing signal to separate to the market.

Two other interesting results about the offering characteristics of underpriced issues can be derived, however. When either the probability of detection decreases or the value of high-quality firms increases, high-quality firms must expend more resources on signalling to fulfill the self-selection constraint to prevent low-quality firms from imitating. Therefore, in an underpricing equilibrium, the impact of changes in r and V^H on the IPO return is

LEMMA 6: *Underpricing firms offer higher IPO returns the higher V^H is and the lower r is.*

Proof: The IPO return, $V^H/P_1^U - 1$, is strictly decreasing in P_1^U and strictly increasing in V^H. Differentiation of the definition of P_1^U with respect to r yields

$$\frac{\partial}{\partial r} P_1^U = \frac{2(2C - V^L)(V^L - C)(V^H - V^L + C)}{[(r - 2)(V^L - C) + (1 - r)V^H]^2} \geq 0. \qquad (22)$$

Inspection of P_1^U further reveals that a decrease in r is equivalent to an increase in V^H.   Q.E.D.

Similar results can be derived for the fraction of the firm that is sold in the IPO of an underpricing equilibrium.

LEMMA 7: *Underpricing firms sell a smaller fraction α_1^U of the firm at the IPO the higher r and V^L are, and the lower V^H and C are.*

Proof: This result follows from differentiation of the definition of α_1^U. (See the Appendix for the derivation.)   Q.E.D.

Lemma 7 applies of course both to the fraction of the firm sold at the IPO and the ratio of the IPO proceeds over SO proceeds (because the unsold remainder of the firm after the IPO is sold in stage 2).

D. Preference and Efficiency of Equilibria

Within the pooling and the underpricing equilibria, I have restricted attention to equilibria in which high-quality firms are better off than in other equilibria of the same type. I now show that either each of the three equilibria is feasible where no other is, or high-quality firms are better off in each of the three equilibria at least for some values of the exogenous variables.[10] First I prove that the announcement and pooling equilibria are mutually exclusive, and thus that neither precludes the other.

LEMMA 8: *Whenever the announcement equilibrium is feasible, the pooling equilibrium is not feasible, and vice versa.*

---

[10] Cho and Kreps (1987) rationality would also allow me to exclude equilibria that do not satisfy this dual criterion.

Proof: Inspection of equations (9) and (14) shows that the regions intersect only for $P_1^P = V^H$ ($\Leftrightarrow h = 1$). Since the comparative statics of the pooling equilibrium imply that the right-hand side of (14) is strictly increasing in P_1^P, and since $P_1^P \leq V^H$ for $h \leq 1$, the regions of equilibrium feasibility for the pooling and the announcement equilibrium do not overlap. In general, there is a gap between them where neither is feasible.   Q.E.D.

Moreover, together with Lemma (5), this result implies that the underpricing equilibrium exists where neither of the other two equilibria exist. Without proof, I assert that high-quality firms are always better off in an announcement than in the underpricing equilibrium. The final lemma proves that the underpricing equilibrium—which is always feasible when the pooling equilibrium is feasible— is never preferred to the pooling equilibrium by high-quality firms (nor, of course, is the underpricing equilibrium preferred by low-quality firms). This implies that one would expect to see underpricing only when pooling is not feasible.

LEMMA 9: *High-quality firms are better off in the best pooling equilibrium than in the best underpricing equilibrium whenever both equilibria are feasible.*

Proof: See the Appendix.

Figure 2 illustrates high-quality firms' total proceeds from their IPO and SO, in each of the equilibria for varying probabilities of natural revelation (r) holding other exogenous variables constant. The pooling equilibrium is feasible for low values of r, the announcement equilibrium is feasible for high values of r, and the underpricing equilibrium fills the gap between the other two equilibria. The figure shows that high-quality firms prefer the pooling over the announcement equilibrium when both are feasible.

Figure 3 displays the regions over varying probabilities of natural revelation (r) and the market priors about firm quality (h) for which the three equilibria are feasible. Since the pooling equilibrium and announcement equilibrium are both preferred by high-quality firms to the underpricing equilibrium, only the region where neither the pooling equilibrium nor the underpricing equilibrium are feasible is labeled *underpricing*. The figure shows that one would expect to find underpricing when both r and h are low.

Finally, the overall efficiency of the equilibria is briefly discussed. In both the pooling and the underpricing equilibria, high-quality issuers are forced to accept less than the market value of their firm for their IPO. However, when compared to the first-best full-information solution, low-quality firms waste resources on otherwise useless imitation activities in the pooling equilibrium. In the underpricing equilibrium no resources are wasted on imitation, but high-quality firms must surrender part of their wealth to investors. Therefore, the economy can be considered efficient in both the announcement and the underpricing games, but not the pooling game.

E. Efficient Signalling: Underpricing versus Advertising

This paper has focused on underpricing as a signalling mechanism, taking as given such parameters as project types, revelation probabilities, the market's
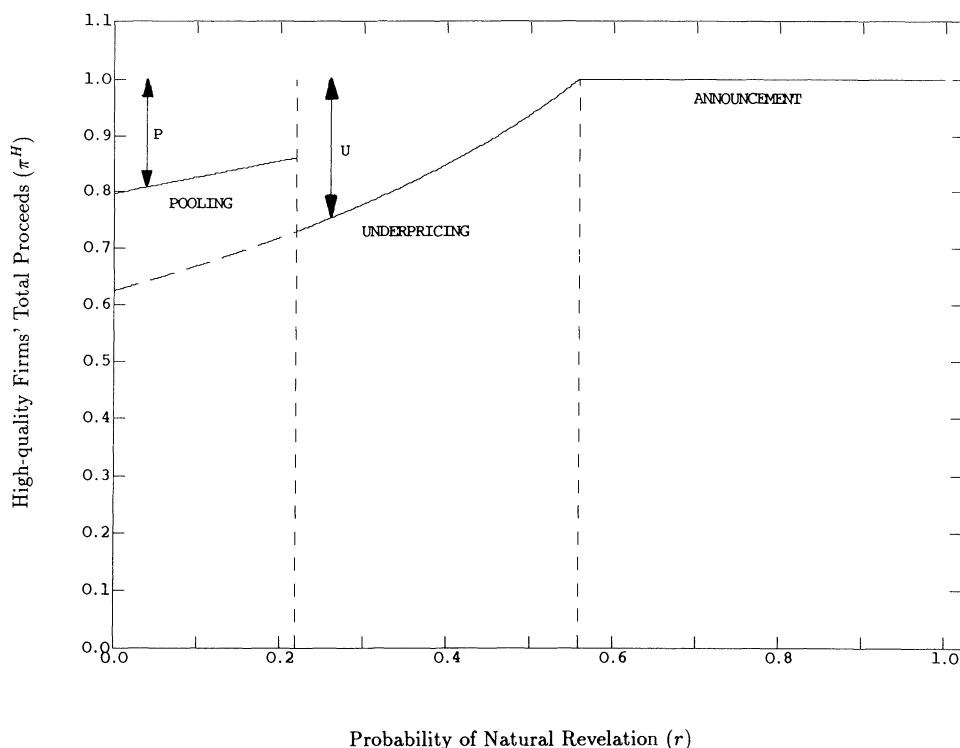
Probability of Natural Revelation (r)

Figure 2. High-quality firms' total expected proceeds (π^H), i.e., the sum of IPO and expected seasoned offering (SO) proceeds, in the three equilibria for varying probabilities that nature reveals the firm's quality between the IPO and the SO (r). When r is very high, an announcement equilibrium is feasible. Operating high-quality firms can receive their true value (V^H)—not only at their SO but even at their IPO—because low-quality firms, fearing that imitation would be discovered before their SO, prefer not to operate. For intermediate values of r, only an underpricing equilibrium is feasible. By pricing their IPOs below V^H, high-quality firms provide low-quality firms with the additional incentive to reveal themselves. The cost of this underpricing to high-quality firms is the distance U. For low values of r, two equilibria are feasible: high-quality firms can either underprice, or they can pool with low-quality firms at their IPO. If high-quality firms pool, they sell their firms for the average value of unidentified firms at their IPO and, if their quality is not discovered, also at their SO. Due to pooling, high-quality firms expect to lose P. The figure shows that when the pooling equilibrium is feasible, the IPO underpricing necessary to induce low-quality firms is so severe, that high-quality firms prefer to pool. For this example, the value of high-quality firms (V^H) is held constant at 1, the value of low-quality firms (V^L) at 0.625, real imitation costs (C) at 0.35, and the proportion of high-quality firms (h) at 60%.

priors and specific borrowing abilities and issue timing. Yet, in real life, firms have a variety of options at their disposal. For example, high-quality firms can deliberately over-invest in activities which are more expensive for low-quality firms to imitate, or owners can trade off increased revelation probabilities against later funding by committing themselves (ex ante) to a longer lockup period.[11] Similarly, bank lines of credit, delayed compensation schemes for managers

---

[11] A lockup period is the time after the IPO during which insiders agree not to sell their securities.
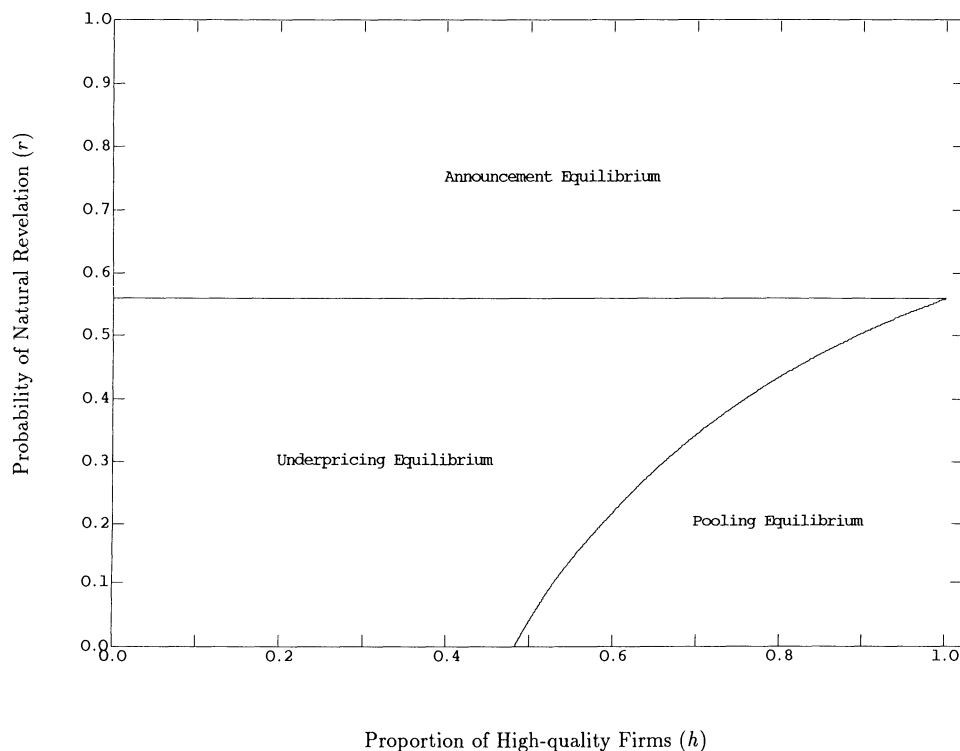
Proportion of High-quality Firms (h)

Figure 3. Conditions that favor each equilibrium. The announcement equilibrium is feasible when r, the probability that nature reveals the firm's true value after the IPO but before the seasoned offering (SO), is very high. That is, low-quality firms need no signalling cost inducement to reveal themselves at the IPO when imitation is very likely to be revealed before the seasoned offering. The pooling equilibrium is feasible when both the probability of nature revealing the firm's true quality—and therefore the probability of discovery of low-quality firm's imitation—is low, and the proportion of high-quality firms in the pool (h)—and therefore the average value of unidentified firms in the pool—is high. Under such conditions, both types of firms prefer to operate and pool. The underpricing equilibrium is feasible when the announcement equilibrium is not feasible. Yet, since both high-quality and low-quality firms prefer to pool when the pooling equilibrium is feasible, only the region where exclusively the underpricing equilibrium is feasible is labeled "underpricing." In this example, the value of high-quality firms (V^H) is held constant at 1, the value of low-quality firms (V^L) at 0.625, and real imitation costs (C) at 0.35.

endowed with insider information, and a continued stake in the firm by the owner may not only reduce the need for equity capital, but also certify quality.

Further, high-quality firms could also choose advertising or a combination of advertising and underpricing as an alternative signalling mechanism (see also Milgrom and Roberts (1986)). Ritter (1987) points out that IPO firms incur substantial costs in addition to underpricing. A portion of their underwriting, legal, accounting, and printing expenses may be an indirect advertising signal. Some authors (for example, Beatty and Ritter (1986) and Carter and Manaster (1988)) have suggested that the underwriter's reputation can be an important factor in the market's beliefs about firm quality.

In a modified model, advertising would offer one major advantage. When a firm advertises, investors observe the signalling expenses directly. In contrast, underpricing is only observable relative to the value of an unidentified firm.[12] That is, pricing a high-quality firm slightly below V^H already imposes a cost on its owner, whereas this price still allows low-quality firms to profit. Since signalling expenses must overcome low-quality firms' self-selection constraint, underpricing is a more costly and consequently less efficient signal.[13]

However, a number of factors favor underpricing in the IPO context. First, there is a ban on conventional marketing activities imposed by the SEC.[14] Second, young firms are often strapped for the cash necessary to pay for advertising. Third, underpricing could also simultaneously serve non-signalling functions. It could, for example, reduce the probability of a lawsuit or other hostile action by disappointed investors against owners or managers in unfavorable states of nature. Furthermore, advertising may be more difficult to verify than underpricing. Investors may have to expend resources to find out if the resources were really transferred, and be convinced that firm owners did not arrange any kickback schemes.[15] Ultimately, since the signalling opportunities of firms seem too abundant and complicated to analyze, the most efficient choice of signal(s) must be an empirical matter.

III. Predictions of the Model

I now discuss how the model is consistent with documented empirical regularities and derive further predictions that could serve as a basis for a test of the model. The first implication of the model was the original motivation for this essay.

[12] An important feature of this model is that investors actually know V^H, V^L and C, and thus can calculate the necessary pricing to signal high-quality (and therefore the necessary underpricing). In real life, my interpretation is that investors first form a (commonly known) prior about the firm's quality, and then observe the pricing of the firm relative to their priors. That is, each firm signals relative to these priors, and therefore investors can interpret how much money a firm leaves on the table at a given price.

[13] Technically, this model is easily modified to allow firms to (observably) advertise or—equivalently—donate money to charity; another term would be subtracted from both firm's IPO proceeds. The technical result is that advertising becomes a more efficient signal than underpricing. (Any action that observably costs money at the IPO is a superior signal.) It can also be shown that when there is no minimum-proceeds constraints, the model would collapse due to an adverse selection problem. High-quality firms would prefer to sell an infinitely small fraction of the firm for an infinitely negative price. Indeed, wasting a specified sum $\alpha_1 P_1$ while effectively no proportion of the firm is sold is equivalent to advertising.

[14] Schneider, Manko, and Kant (1986) state in a manual for IPO firms distributed by Packard Press:

> Prior to the initial filing of the registration statement, no public offering, either orally or in writing, is permitted. [Securities Act of 1933, §5(c), 15 U.S.C. §77e(c) (1976)]. For this purpose, the concept of offering has been given an expansive interpretation.... Publicity about the company or its products may be considered an illegal offering, in the sense that it is designed to stimulate an interest in the securities, even if the securities themselves are not mentioned (p. 19f.).

[15] For the same reason, underpricing issuers should have an interest in allocating their issue to a wide cross-section of investors. Rock's model offers a similar prediction. Distribution of an issue to fewer informed and more uninformed investors reduces the winner's curse.

IMPLICATION 1: *There exist market conditions in which some firms choose to underprice.*

It is my interpretation of the model that "normal" levels of IPO underpricing result when some groups of firms play an underpricing game, while others play pooling or announcement games. *Hot-issue markets* occur when a significant fraction of firms—possibly related by industry, size, location, or some other common characteristic—jointly switch to the underpricing game.[16]

A second well-documented empirical regularity is a strong positive relationship of underpricing and high ex ante uncertainty. Beatty and Ritter (1986), following Rock (1986), attribute this relationship to the differential impact of uncertainty on informed and uninformed investors. This model offers a different interpretation. Uncertainty about firm value is related to h, the market's prior that the firm is high quality. This prior is related to underpricing because a decrease in h reduces the incentive of firms to play a pooling equilibrium. Clearly, h is less a measure of the spread than of the mean of investors' priors. However, the proxies for ex ante uncertainty that, for example, Beatty and Ritter use—the number of uses of proceeds and the inverse of gross proceeds—are likely to be highly correlated with investor's estimates of firm quality.

IMPLICATION 2: *The lower the probability that a firm is high quality, the higher the probability that it underprices.*

Ritter (1984) describes a hot-issue market in 1980 in the natural resources industry. One plausible explanation is that the market believed that the second oil shock could have attracted a sufficiently large number of opportunistic, low-quality oil exploration firms to force a large number of high-quality oil firms into an underpricing equilibrium.[17]

These two implications cannot yield a good test of the model since they were well documented before this model was developed. The following implications are to the best of my knowledge not established in the empirical literature and could therefore serve as a basis for a test.

IMPLICATION 3: *IPO firms issue a substantial amount of claims in seasoned offerings.*

If IPO firms did not reissue, the argument of this paper would of course be false or irrelevant. The fact that firms choose to issue more than once may simply be because they wish to take advantage of lower issuing costs of SOs relative to IPOs. Therefore, this implication does not yield a powerful test to discriminate between the winner's curse and this model. The next section provides some evidence that many IPO firms do indeed issue again.

To yield a more powerful test, further predictions must be derived that neither have been documented nor seem supportive of Rock's model. The following implications may qualify. Implication 4 uses the results in Lemma 6 and Lemma 7.

---

[16] For a description of hot-issue markets, see Ibbotson and Jaffe (1975) and Ritter (1984). In brief, a hot-issue market is a situation where the average IPO underpricing increases dramatically.

[17] The absence of non underpriced oil-exploration firms is not inconsistent with this observation. Firms with low prospects of finding oil may have chosen not to participate in a signalling IPO market.

IMPLICATION 4: *The proportion of the firm that underpricing firms sell at the IPO (relative to the SO) increases when the value of high-quality firms or the cost of imitation increases, or when the probability of detection or the value of low-quality firms decreases. IPO returns increase when the value of high-quality firms increases, or when the probability of detection decreases.*[18]

Of course, when the equilibrium type switches with the parameters, this prediction fails to hold.

Another set of tests of this model can be constructed by considering the IPO after-market uncertainty. Only in the pooling equilibrium will insider information about firm quality eventually surface, and, hence, add to whatever uncertainty the firm's projects generate after the IPO. The challenge of any tests based on this after-market uncertainty is of course the extraction of the after-market uncertainty that is due to insider knowledge *before* the IPO (henceforth referred to as *residual uncertainty*).[19] With this measure, further tests can be constructed. In the underpricing equilibrium firms have already signalled their quality. Consequently:

IMPLICATION 5: *Underpriced issues have low residual uncertainty.*

The residual uncertainty measure can also be used to determine whether a non-underpricing IPO market segment plays a pooling game (high residual uncertainty) or an announcement game (low residual uncertainty). Therefore, the comparative statics of the pooling equilibrium (in Lemma 2) imply:

IMPLICATION 6: *From a pooling equilibrium (conditional on observing no IPO underpricing and high residual uncertainty), an underpricing equilibrium is more likely to ensue when there is an exogenous decrease in the value of high-quality firms (V^H) or the proportion of high-quality firms in the pool (h), or when there is an exogenous increase in the probability of detection (r);*[20]

and the comparative statics of the announcement equilibrium (in Lemma 4) imply:

IMPLICATION 7: *From an announcement equilibrium (conditional on observing no IPO underpricing, and low residual uncertainty), an underpricing equilibrium is more likely to ensue when there is an exogenous increase in the value of high-quality firms (V^H) or the cost of imitation (C), or when there is an exogenous decrease in the probability of detection (r) or the value of low-quality firms (V^L).*

A final test does not follow directly from the model but does not require a proxy for residual uncertainty. Recall again that, in an underpricing equilibrium, the quality of firms is known, while in the pooling equilibrium it is unknown. If the market were to consider information about the issuing of a SO worse news

---

[18] I am grateful to Peter Pashigian for pointing out this prediction.

[19] Since market makers protect themselves against insider information by increasing the bid-ask spread, the immediate IPO after-market bid-ask spread may be a suitable proxy (see also Miller and Reilly (1987)).

[20] Note that Implication 2 is really a special case of Implication 6. However, there was no need to condition Implication 2 on residual uncertainty because h has relevance only in the pooling equilibrium.

when the information asymmetry and the residual uncertainty is high, one could speculate about the correlation of the IPO and SO discount:

IMPLICATION 8: *The value of outstanding shares falls less upon news of a SO when a firm has played an underpricing equilibrium.*

IV. Preliminary Evidence on Reissuing Activity of IPO Firms

In this section evidence for the hypothesis that IPO firms reissue substantially is presented. My sample is 1028 IPO firms in the 1977–1982 period reported by Howard & Co.'s *Going Public: The IPO Reporter*. Of these firms, 926 began trading on NASDAQ, and two on the American Stock Exchange within six months of going public. This data base has been used in papers by Beatty and Ritter (1986) and Ritter (1984, 1987) and was generously made available to me.

Panel A of Table II lists for each year in 1977–1982 the number of IPOs, the average initial return (realized from buying the IPO from the underwriter and selling [at the closing bid price for NASDAQ firms] on the first day of trading), and the average gross proceeds. All dollar amounts have been discounted to 1982 real dollars (using the CPI). Where appropriate, the first number in a cell is the mean, the subsequent number (in parentheses) the standard deviation, and the two numbers below (separated by three dots) the range of the series. For the entire period, average IPO proceeds were $7.1 million 1982 dollars, and investors received on average a 26% return per IPO on their investment.

The model provides little guidance as to what constitutes a seasoned offering. For reasons of data availability, I have used a restrictive definition of seasoned offerings, which biases the results against the prediction that firms reissue substantially by considering only public seasoned equity offerings as SOs. This ignores all other claims a firm or its owners may sell, including risky and convertible debt, equity issues that are not offered to the public, and open-market sales by insiders (subject to Rule 144 restrictions).

The SO data for these firms were collected from several volumes of Zehring & Co.'s *Corporate Finance Sourcebook (CFS)*, covering the period from January 1, 1977 to December 31, 1987.[21] Panel B of Table II summarizes these data. Rows 1 and 2 show that for the entire period 288 of the firms reissued a total of 395 SOs. Rows 3 and 4 list statistics for firms that had their IPO in the year indicated

---

[21] This introduced further biases against the hypothesis: (1) *CFS* omits non-underwritten issues; (2) *CFS* may exclude smaller underwriters' issues, (3) the SO data ends in 1987; and (4) firms may have merged or been taken over by other firms or investors. In such a sale, one would nevertheless expect the signalling benefits to accrue to the owners. To check for upward bias due to reported but nonexistent offerings, the *CFS* data was compared to the SEC's 1985 *Registered Offering Statistics (ROS) Tape*. The *ROS* tape appears to be an unreliable source of information, in particular for information beyond issue date and issue proceeds. At least 95% of all SO listings in the *CFS* can be found on the *ROS* tape. However, the *ROS* tape indicates at least twice as many SOs that are not included in the *CFS*. Since I cannot determine if omission of issues is due to special issue features or oversight by the *CFS*, I bias the data against my hypothesis by displaying statistics only for equity issues listed in the *CFS*.

Finally, name changes were collected from the CRSP NASDAQ and NYSEAMEX files. In case of doubt about a company's identity, I always ruled against inclusion of such SOs.

Table II
Descriptive Statistics for Issuing Firms Categorized by IPO Year

Panel A lists characteristics for initial public offerings (IPOs) from 1977 to 1982 reported in *Going Public: The IPO Reporter*. Panel B lists characteristics of the seasoned equity offerings (SOs) for these IPO firms as reported in the *Corporate Finance Sourcebook*. Here each column displays the statistics for subsequent SOs for all firms whose IPO took place in the column's listed period. Total SO proceeds are firms' total proceeds over all their seasoned offerings. The total SO proceeds and total SO proceeds/IPO proceeds statistics are only for firms that had reissued by December 31, 1987. For the last two rows in both panels, the first cell entry is the mean, the number following (in parentheses) is the standard deviation of the series, and the line below is the range of the series. All dollar series have been normalized to 1982 CPI dollars.

Panel A: Initial Public Offerings (IPOs)

Year of IPO	1977–1982	1977	1978	1979	1980	1981	1982
Number of IPOs	1028	32	48	77	234	439	198
Issue Proceeds (in millions 1982 dollars)	7.1 (10.4)	7.4 (10.6)	7.4 (8.7)	7.2 (7.1)	6.7 (10.6)	7.6 (10.7)	6.5 (10.9)
	0.1 … 128.1	0.5 … 55.9	0.9 … 38.1	0.8 … 42.1	0.4 … 110.6	0.2 … 128.1	0.1 … 73.3
Initial Return	0.26 (0.62)	0.21 (0.46)	0.26 (0.42)	0.24 (0.56)	0.51 (0.89)	0.17 (0.50)	0.21 (0.47)
	-0.69 … 7.75	-0.31 … 2.00	-0.38 … 1.63	-0.44 … 2.75	-0.40 … 7.75	-0.50 … 4.00	-0.69 … 3.50

Panel B: Corresponding Seasoned Equity Offerings (SOs)

Year of IPO	1977–1982	1977	1978	1979	1980	1981	1982
Number of IPO Firms Reissuing	288	6	21	32	55	116	58
Total Number of SOs	395	9	38	46	84	150	68
Total SO Proceeds (in millions 1982 dollars)	25.9 (36.9)	43.2 (54.7)	44.1 (89.9)	19.9 (12.9)	25.6 (24.5)	25.8 (34.0)	21.4 (23.7)
	0.2 … 411.1	7.1 … 151.3	0.5 … 411.1	0.2 … 45.6	0.5 … 85.9	0.4 … 237.2	1.6 … 148.0
Total SO proceeds/IPO proceeds	3.4 (4.5)	4.7 (5.8)	3.4 (3.1)	3.6 (5.6)	2.9 (2.5)	3.0 (3.6)	4.3 (6.7)
	0.1 … 40.4	0.5 … 13.2	0.3 … 10.8	0.1 … 26.0	0.3 … 13.6	0.1 … 21.3	0.2 … 40.4

by the column title *and* reissued. Row 3 shows that the total SO proceeds, defined for each reissuing IPO firm as the sum of all its seasoned equity offering proceeds, was on average $26.2 million (1982 dollars), about three times their average IPO proceeds. Since firms that do not reissue are excluded, the sum of SO proceeds could give a distorted picture of firms' reissuing activity. If only larger firms reissued, one would expect the average SO proceeds to exceed the average IPO proceeds. Therefore row 4 displays for each reissuing firm the ratio of the total SO proceeds over their IPO proceeds. Here, too, the conclusion is that IPO firms that reissue do so substantially. The mean ratio of SO proceeds over IPO proceeds for reissuing firms over the entire period is in excess of 3.

Figure 4 provides a different view of firms' reissuing activity. It displays the frequency of SOs and the distribution of SO proceeds for all firms as a function of the time (in 180 calendar day intervals) that has passed since the IPO. Within the first three years after the IPO, these firms issued 265 of the 395 SOs for a total gross proceeds of about $5.0 billion—two-thirds of both the total number and total proceeds of all SOs. Subsequently, the issuing frequency and proceeds
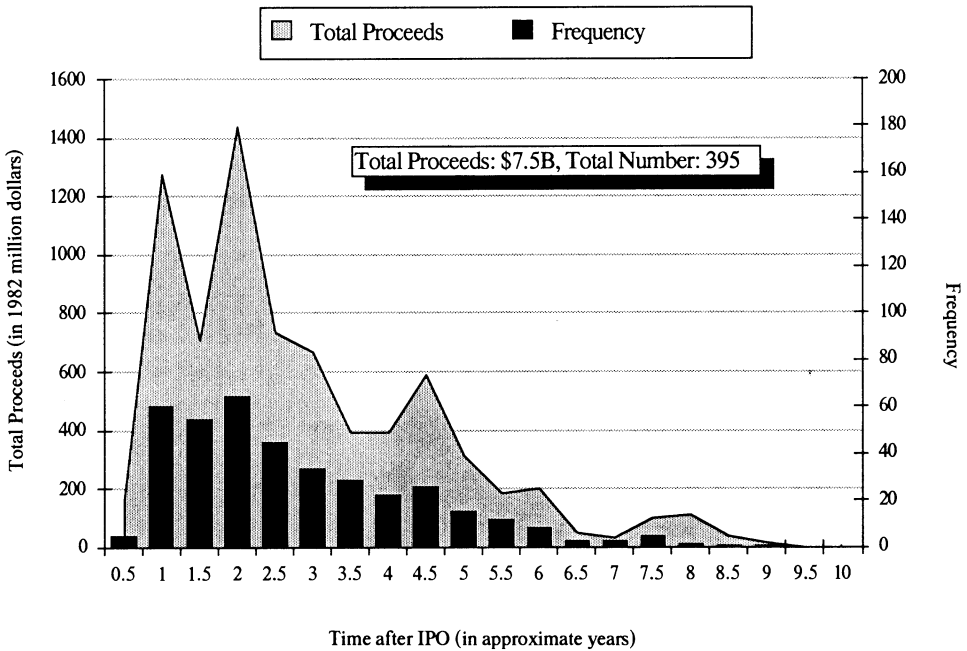

Time after IPO (in approximate years)

Figure 4. Total number and total proceeds of all seasoned equity offerings by all IPO firms as a function of the time passed since the IPO. Note that a single firm with more than one seasoned offering can contribute to several bars in the chart. To illustrate Figure 4: within approximately half a year (180 days to be exact) after their IPO, an unknown number of firms issued five seasoned offerings with total proceeds of $165 million (in 1982 dollars). The figure shows that IPO firms' reissuing activity changes. It is high soon after the IPO, begins to decline after about two years, and levels off after about six years. In terms of total issuing proceeds, IPO firms issued about the same total dollar amount of seasoned equity as they raised in IPOs. This was accomplished by only the third of IPO firms that actually reissued.

are considerably lower. I interpret Figure 4 to be supportive evidence for the hypothesis that IPO firms choose a timing for SOs that is related to the IPO.

In sum, despite unfavorably biased data, I find that both the issuing frequency and issuing proceeds of *CFS*-reported SOs are not insignificant for the 1977–1982 IPO firms. I conclude (a) that the timing of seasoned equity offerings was related to the initial public offering, and (b) that initial public offerings could have indeed been used to advertise for seasoned equity issues.[22]

V. Conclusion

The purpose of this paper has been to provide an alternative explanation of IPO underpricing that avoids some of the shortcomings of explanations based on a winner's curse. In particular, the model predicts that—although underpricing firms demand mechanisms that indirectly reduce IPO underpricing by increasing the probability that the market discovers the firm's quality—underpricing firms do not demand less IPO underpricing per se. A higher price at a seasoned offering (SO) eventually compensates firms for the intentionally low IPO price. That is, the model strongly suggests that IPO firms pursue a multiple issue strategy when they choose both the price and the proportion of the firm they offer at their IPO.

The reason why IPO underpricing results in a higher SO price is an information asymmetry between firm owners and investors. High-quality firm owners can signal their superior information to investors because their marginal cost of underpricing is lower than the marginal cost of underpricing for low-quality firm owners. To imitate high-quality firms, low-quality firms would not only have to incur the signalling costs but also expend the resources to imitate the observable real activities and attributes of high-quality firms. The market may discover the true quality between the IPO and SO and therefore force an imitating firm to bear some of the imitation expenses whose only purpose was to deceive investors. Higher signalling costs then increase the attractiveness of low-quality firms' alternative—revealing themselves as low-quality firms. In other words, IPO underpricing can drive the additional wedge between the costs and benefits of low-quality firms' imitation tradeoff to induce low-quality firms to reveal themselves. On a more general level, imitation costs are an alternative justification for advertising and underpricing without Nelson's (1974) and Milgrom and Roberts' (1986) assumption that buyers learn from their initial purchase.

I have argued that the empirical implications of the model are consistent both with rigorously documented empirical regularities and popular beliefs on Wall Street, and I have produced additional predictions that permit testing of the model. Finally, I have provided some empirical support for one central implication of the model: many IPO firms from the 1977–1982 period indeed chose to issue a substantial amount of public, underwritten seasoned equity.

[22] Finally, I wish to note that the sample period may be inadequate for a study of the predictions of this model. The oil industry, which as Ritter (1984) documents was responsible for the hot-issue market in 1980, was struck by a largely unexpected recession after 1981. Therefore, these oil firms may not issue a substantial amount of SOs. This is why—contrary to the implication of this model—firms that went public in 1980 reissued less than firms during other years. The model is not general enough to capture common ex-post changes in firm value. In future research, the model could be extended by permitting the owner's belief of the true value of the firm to be a random variable.

Appendix

A. *Proof of Lemma 5*

First, I show that, when the announcement equilibrium is not feasible α_1^U is a proper proportion, $\alpha_1^U \in [0, 1]$, or

$$0 \leq \frac{(r - 2)(V^L - C) + (1 - r)V^H}{r(V^L - C) + (1 - r)V^H} \leq 1. \tag{23}$$

The upper constraint is obviously true. The lower constraint yields

$$r \leq \frac{V^H - 2(V^L - C)}{V^H - V^L + C}. \tag{24}$$

Comparing the announcement equilibrium condition (14) to this bound, the following statement can be verified:

$$\frac{V^H - 2(V^L - C)}{V^H - V^L + C} \geq \left(\frac{V^H}{V^H - V^L + C}\right)\left(\frac{V^H - V^L}{V^H + V^L - 2C}\right). \tag{25}$$

Therefore, whenever the announcement equilibrium is not feasible, α_1^U is a proper proportion. Similarly, one can show that $P_1^U \leq V^H$. From (23) it follows that the denominator of $\dfrac{(2C - V^L)[r(V^L - C) + (1 - r)V^H]}{(r - 2)(V^L - C) + (1 - r)V^H}$ ($\equiv P_1^U$) is positive. After some rearrangement of $P_1^U \leq V^H$, one finds that

$$(2C - V^L - V^H)[r(V^L - C) + (1 - r)V^H] \leq (-2)V^H(V^L - C). \tag{26}$$

It is easy to verify that even for $r \to 0$ this condition is true.

Next, I assert that (α_1^U, P_1^U) satisfies binding self-selection (equation (17)) and minimum proceeds (equation (20)) constraints by construction. To complete the proof, it must be shown that a high-quality firm has no incentive to deviate. Let π^U denote the total proceeds of a high-quality firm that adheres to the under-pricing equilibrium

$$\pi^U \equiv \pi^U(\alpha_1^U, P_1^U)$$

$$= \alpha_1^U P_1^U + (1 - \alpha_1^U)V^H$$

$$= (2C - V^L + \left(\frac{2(V^L - C)}{r(V^L - C) + (1 - r)V^H}\right)V^H. \tag{27}$$

It must be shown that

$$\pi^U \geq \max\left\{V^L, (2C - V^L) + \left(\frac{2V^L - 3C}{V^L - C}\right)[rV^H + (1 - r)(V^L - C)]\right\}, \tag{28}$$

where the optimal $\hat{P}_1 = V^L - C$ and $\hat{\alpha}_1 \hat{P}_1 = 2C - V^L$ were substituted into (19). Simplifying $\pi^U \geq V^L$ by subtraction of V^L from both sides and multiplication by $r(V^L - C) + (1 - r)V^H$, one finds that $\pi^U \geq V^L$ is equivalent to

$$2(V^L - C)V^H - 2(V^L - C)[r(V^L - C) + (1 - r)V^H] \geq 0. \tag{29}$$

Therefore $\pi^U \geq V^L$. The second alternative to deviate—operating but not underpricing—is never preferred if

$$\left(\frac{2(V^L - C)}{r(V^L - C) + (1 - r)V^H}\right) V^H$$

$$\geq \left(\frac{2V^L - 3C}{V^L - C}\right) [rV^H + (1 - r)(V^L - C)]. \quad (30)$$

Simplifying, one obtains

$$r^2 + r + \frac{(V^L - C)V^H[2(V^L - C) - (2V^L - 3C)]}{(V^H - V^L + C)^2(2V^L - 3C)} \geq 0. \quad (31)$$

The term $2V^L - 3C$ is positive because a necessary condition for possible deviation is that high-quality firms are able to raise the minimum proceeds at $\hat{\alpha}_1 = 1$, requiring that $1(V^L - C) > 2C - V^L$. Therefore the constant term in (31) is positive, and the entire parabola lies above the x-axis. This completes the proof that high-quality firms have no desire to deviate. Q.E.D.

It may be startling to find that, even when the probability of detection is arbitrarily small, the underpricing equilibrium exists. However, note that the best alternative to a deviating high-quality firm is to accept V^L (or worse, $V^L - C$). As a result, as $r \to 0$, both high- and low-quality firms receive V^L, sufficient to permit low-quality firms to reveal themselves and high-quality firms to not deviate.

B. Proof of Lemma 7

Differentiating α_1^U yields

$$\frac{\partial \alpha_1^U}{\partial r} = \frac{2(V^L - C)(V^L - C - V^H)}{(rV^L - rV^H + V^H - rC)^2} \; (\leq 0), \quad (32)$$

$$\frac{\partial \alpha_1^U}{\partial V^L} = \frac{2(r - 1)V^H}{(rV^L - rV^H + V^H - rC)^2} \; (\leq 0), \quad (33)$$

$$\frac{\partial \alpha_1^U}{\partial C} = \frac{2(1 - r)V^H}{(rV^L - rV^H + V^H - rC)^2} \; (\geq 0), \quad (34)$$

$$\frac{\partial \alpha_1^U}{\partial V^H} = \frac{2(1 - r)(V^L - C)}{(rV^L - rV^H + V^H - rC)^2} \; (\geq 0). \quad (35)$$

C. Proof of Lemma 9

Let

$$\pi^P(r) \equiv (2C - V^L) + \left(\frac{U - 2C + V^L}{U}\right)[r(V^H - U) + U], \quad (36)$$

and

$$\pi^U(r) \equiv (2C - V^L) + \left(\frac{2(V^L - C)}{r(V^L - C) + (1 - r)V^H}\right) V^H. \quad (37)$$

The lemma states that

$$\pi^P(r) \geq \pi^U(r) \tag{38}$$

for

$$U \geq V^L, \tag{39}$$

$$0 \leq r \leq r^*, \tag{40}$$

where

$$r^* \equiv \left(\frac{U - V^L}{U - (V^L - C)}\right)\left(\frac{U}{U - (2C - V^L)}\right). \tag{41}$$

First I show that, for $r = 0$ and $r = r^*$, condition (38) is satisfied. Substituting $r = 0$, one finds $\pi^P(r = 0) \geq \pi^U(r = 0)$ since

$$(U - V^L) + 2(V^L - C) \geq 2(V^L - C). \tag{42}$$

Next, evaluating $\pi^P(r = r^*) - \pi^U(r = r^*)$, some tedious algebra reveals that

$$\pi^P(r^*) - \pi^U(r^*) = \frac{(V^H - U)(U - V^L)(2C - V^L + U)(V^H - V^L + C)}{(V^L - C - U)(V^H V^L + UV^L - UV^H - 2CV^H - U^2)}. \tag{43}$$

The numerator can be signed (positive) by inspection. To show that the denominator is positive, it must be shown that $(V^H V^L + UV^L - UV^H - 2CV^H - U^2) \leq 0$. The quadratic equation in U on the left-hand side is maximized at $U = (V^L - V^H)/2$. Hence for the valid range of parameters the expression is maximized at $U = 0$, and $(V^H V^L + UV^L - UV^H - 2CV^H - U^2) < V^H(V^L - 2C)$. By (1) the latter expression is always negative. Therefore, total pooling proceeds are preferred to total underpricing proceeds at $r = r^*$.

It remains to show that $\pi^U(r)$ lies below $\pi^P(r)$ over the interval $[0, r^*]$. Inspection of $\pi^U(r)$ reveals that it is a hyperbola of the form

$$\pi^U(r) = \frac{a}{br + c} \tag{44}$$

with $a \geq 0$, $b \leq 0$, $c \geq 0$, and $br + c \geq 0$. Differentiating (44), one obtains $(\pi^U)'(r) = -\frac{ab}{(br + c)^2} \geq 0$ and $(\pi^U)''(r) = \frac{2ab^2}{(br + c)^3} \geq 0$. Since the function $\pi^U(r)$ is convex over the interval $[0, r^*]$, it lies below the join of the endpoints $(0, f(0))$, and $(r^*, f(r^*))$. Since $\pi^P(r)$ is linear in r, and since its endpoints lie above the join of $\pi^U(r)$'s endpoints, $\pi^U(r)$ lies below $\pi^P(r)$ over the entire interval.   Q.E.D.

REFERENCES

Allen, Franklin and Gerald R. Faulhaber, 1987, Signaling by underpricing in the IPO market, Working Paper, Wharton School, University of Pennsylvania.

Baron, David P., 1982, a Model of the demand for investment banking advising and distribution services for new issues, *Journal of Finance* 37, 955–976.

Barry, Christopher B., Chris J. Muscarella, John W. Peavy, III, and Michael R. Vetsuypens, 1988, Venture capital and initial public offerings, Working Paper, Texas Christian University and Southern Methodist University.

Beatty, Randolph P., and Jay R. Ritter, 1986, Investment banking, reputation, and the underpricing of initial public offerings, *Journal of Financial Economics* 15, 213–232.

Carter, Richard and Steven Manaster, 1988, Initial public offerings and underwriter reputation, Working Paper, Iowa State University and University of Utah.

Cho, In-Koo and David M. Kreps, 1987, Signalling games and stable equilibria, *Quarterly Journal of Economics* 102, 179–221.

Corporate Finance Sourcebook, 1982–1988 (Zehring & Co., Wilmette, IL).

Going Public: The IPO Reporter, 1977–82. Published weekly (Howard & Co., Philadelphia, PA).

Grinblatt, Mark and Chuan Yang Hwang, 1989, Signalling and the pricing of new issues, *Journal of Finance* 44, 393–420.

Harris, Milton, 1987, *Dynamic Economic Analysis* (Oxford University Press, New York).

———— and Artur Raviv, 1986, A sequential signalling model of convertible debt call policy, *Journal of Finance* 41, 1263–1281.

Ibbotson, Roger G., 1975, Price performance of common stock new issues, *Journal of Financial Economics* 2, 235–272.

———— and J. F. Jaffe, 1975, Hot issue markets, *Journal of Finance* 30, 1027–1042.

Kreps, David M. and Robert Wilson, 1983, Sequential equilibrium, *Econometrica* 51, 1799–1819.

Leland, Hayne E. and David H. Pyle, 1977, Informational asymmetries, financial structure and financial intermediation, *Journal of Finance* 32, 371–387.

Mikkelson, Wayne H. and M. Megan Partch, 1986, Valuation effects of security offerings and the issuance process, *Journal of Financial Economics* 15, 31–60.

Milgrom, Paul and John Roberts, 1986, Price and advertising signals of product quality, *Journal of Political Economy* 94, 796–821.

Miller, Robert E. and Frank K. Reilly, 1987, An examination of mispricing, returns and uncertainty for initial public offerings, *Financial Management*, 33–38.

Nanda, Vikram, 1988, Why firms go public, Working Paper, University of Chicago.

Nelson, Phillip, 1974, Advertising as information, *Journal of Political Economy* 81, 729–754.

Ritter, Jay R, 1984, The 'hot issue market' of 1980, *Journal of Business* 57, 215–239.

————, 1987, The costs of going public, *Journal of Financial Economics* 19, 269–281.

Rock, Kevin, 1986, Why new issues are underpriced, *Journal of Financial Economics* 15, 187–212.

Schneider, Carl W., Joseph M. Manko, and Robert S. Kant, 1981, 1986, *Going public: Practice, procedure and consequences* (Packard Press, Financial Publications Division). Reprinted from *Villanova Law Review* 27, 19–21.