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Sequential Sales, Learning, and Cascades

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ABSTRACT

When IPO shares are sold sequentially, later potential investors can learn from the purchasing decisions of earlier investors. This can lead rapidly to "cascades" in which subsequent investors optimally ignore their private information and imitate earlier investors. Although rationing in this situation gives rise to a winner's curse, it is irrelevant. The model predicts that: (1) Offerings succeed or fail rapidly. (2) Demand can be so elastic that even risk-neutral issuers underprice to completely avoid failure. (3) Issuers with good inside information can price their shares so high that they sometimes fail. (4) An underwriter may want to reduce the communication among investors by spreading the selling effort over a more segmented market.

CONSIDER A SCENARIO IN which an issuer is selling a new security of uncertain value, for example, an IPO (initial public offering) of stock or high-yield debt, through an underwriter. The S.E.C. has banned variable-price sales. While the value of this new security is highly uncertain to individual market participants, investors hold perfectly accurate information when aggregated. Moreover, there are many (potential) investors, and a small number of these investors can jointly determine the value of the firm (or its project) with high precision. It would seem that in this scenario underpriced offerings would succeed and overpriced offerings would fail.

Nevertheless, this paper shows that, if the distribution channels of invest ment banks are limited, underpriced offerings can fail and overpriced offer ings can succeed. With limited distribution channels, it takes the under writer 'time to approach interested investors. Therefore, later investors can observe how well an offering has sold to date -- or at least how successful it has sold relative to offerings previously undertaken by this underwriter.

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Hence, investors approached after some time can infer information from investors who were approached earlier. An individual investor can interpret a successful initial sales effort to imply that earlier investors had favorable information about the offering, in turn giving him additional incentive to invest. Analogously, slow initial sales discourage subsequent investing. This conditioning of information on the decisions of earlier investors is a positive informational externality.

As a result, offering proceeds depend on the ordering of information among investors. Holding aggregate information constant, if early investors are favorably inclined, an issuer will in general receive more proceeds. Still, one might expect that this would not be a significant problem with infinitely many investors; eventually the influence of a few random initial signals would give way to the overwhelming influence of the information that is held by that of many other investors. This conclusion is incorrect. Even though in the aggregate investors already have perfect (and free) information, the inferred value by an investor approached arbitrarily late need not be close to the true or aftermarket value of the offering.

The inferred value would converge to the true value if investors could observe the signals held by investors approached earlier. However, it is not realistic to assume that investors can observe these signals. They can only observe (and believe) earlier investors' actions from early sales. Then, an investor who has observed previously high or previously low demand will rapidly base the decision to purchase exclusively on earlier sales and ignore his own, private information. Given that one individual finds it in his best interest to ignore private information -investing with bad or abstaining with good information -and instead relies completely on the information inferred from previous sales in his purchasing decision, all subsequent investors will face the same decision and act alike. As a consequence, if a few early investors believe that the offering is overpriced, they can swamp the informa tion held by all other investors and doom the offering to fail. Or, if a few early investors believe that the offering is a bargain, they can create almost unlimited demand for this issue. I shall refer to this phenomenon as the "cascade" effect.[1]

Cascades are not necessarily bad for an issuer. They leave him at less of an informational disadvantage because they reduce the information aggregation among investors: if late investors ignore their own information and act like

[1] In reality, underwriters appear to be aware of the effect of some investors' decisions on those of other investors. *Fortune Magazine-in Money* and Markets/Cover Story "Inside the Deal That Made Bill Gates $350,000,000" by Bro Uttal (July 21, 1986), p. 32-reports in a case study of the Microsoft IPO that

> Eric Dobkin, 43, the partner in charge of common stock offerings at Goldman Sachs felt queasy about Microsoft's counterproposal. For an hour he tussled with Gaudette, using every argument he could muster. Coming out $1 too high would drive off some high-quality investors. Just a few significant defections could lead other investors to think the offering was losing its luster.

the previous investors, then their actions are not informative to other in vestors. This paper will show that this increases the issuer's expected wealth. (At times, an issuer can even be as well off with cascades (and a fixed price) as in a variable -price auction!) The issuer's desire to prevent communication among investors has empirical implications. In particular, I interpret an underwriter as an institution that distributes an offering widely, i.e., to investors who find it more difficult to communicate among themselves.

Path dependence and cascades provide an explanation for 1PO underpricing without a winner's curse when all submissions are not simultaneously added up and pro -rated, but are served in sequential order. In fact, the demand curve created by the cascade model is likely to ensure that by the time the first investor who could be rationed is approached, the success or failure of an offering already has been determined. This is not to argue that this marginal investor does not face a winner's curse; only that an issuer's pricing decision is more concerned with convincing early instead of late investors.

Finally, I consider the effect of inside information -an important factor in the IPO market-on IPO pricing. When both the issuer and investors observe private signals about the firm's aftermarket value distribution, positive inside and outside information is correlated. In this situation, an issuer with private information of high future cashflows can risk an unusually high price: A high price increases not only the probability of offering failure (which occurs when many outsiders have negative information), but it in creases it more so for a lower quality issuer. Thus, the marginal cost of higher pricing - increased incidence of offering failure-is higher for a low-quality issuer, and a se parating equilibrium is feasible.

Path dependence, and the effect of long lines, have been considered in the bank -run literature (Chari and Jagannathan (1988), Diamond and Dybvig (1983)).[2] Herd behavior is also a concern in other contexts. In a principal agent model, Scharfstein and Stein (1990) argue that managers imitate the investment decisions of other managers to appear to be informed. Froot, Scharfstein, and Stein (1991) analyze in a market microstructure model how short-horizon traders can all try to learn the same information-even if it is noise-instead of information about (long-horizon) fundamentals. Bannerjee (1992) independently models a cascade effect. However, he does not consider how a seller reacts when faced with a cascade situation among buyers. Finally, there is also some support in the behavioral literature for path dependence. Shiller and Pound (1989) discuss a behavioral explanation in the IPO and other contexts, which they term the "impresario hypothesis," under

[2] However, aside from the different context, bank-run models differ in two assumptions: (1) reasons other than information can determine investment choice (leading to a signal extraction problem to distinguish between negative information and simply new preferences to consume); (2) agent's welfare depends on the actions of other agents (the early termination of an illiquid investment is costly). Such positive externalities are addressed in section E. Also in these models, typically only their outcomes-a multiplicity of equilibria (some with, others without bank-runs)-are considered, rather than the dynamics of investors' actions. This cannot shed light on the frequency of different equilibria.

which impresarios try to create lines to foster subsequent demand. This paper differs primarily by interpreting the decision to create long lines to be perfectly rational, and not the outcome of irrational rumor or contagion.

The paper is laid out as follows: Section I outlines the basic argument: the influence of early sales on investors' decision and on the issuer's pricing. Section 11 discusses why rationing and the winner's curse, although similar in setting and predictions, can often be irrelevant. Section III shows how inside information can be signaled by high pricing which induces more frequent offering failures. Section IV discusses IPO pre -selling activity and the incentives for an issuer to pay early investors on the side. And Section V concludes.

I. A Model

Assume that the efficient aftermarket value of a share of an IPO firm, denoted V, is unknown to both an issuer (a firm or an investment banker) and *n* investors. All investors and the issuer are rational, risk-neutral, expected wealth-maximizers. An investor purchases a share of an offering if and only if its expected aftermarket value, given all his information, is not less than the offering price.[3] The issuer has enough shares to offer each approached investor one share, and no investor has sufficient resources to purchase more than one share. Each investor's decision is either to buy or to abstain.[4] The issuer can only once offer shares to the public at a fixed, common price.

It is common knowledge that the issuer's reservation value is V^P, and that he is selling because a diversified, capital-unconstrained market would value a share between V^L and V^H, where $V^P \leq V^L < V^H$.[5] Specifically, I assume that all market participants have a diffuse uniform prior distribution on the after-market value $\tilde{V} \sim U[V^L, V^H]$.[6] The difference between V^L and V^P could be interpreted as the capital requirements of the issuer; it is the loss incurred by the owner if the offering fails and he has either to prepare a new offering or arrange for private borrowing. Note that this scenario posits

[3] All basic results are robust to alternative tie-breaking assumptions (e.g., if indifferent, an investor could purchase a share depending on the outcome of a coin flip).

[4] This assumption can be justified with limited wealth, as Rock (1982, 1986) assumes for informed investors. It can also be justified by assuming that investors decide mainly whether to participate or to abstain in an IPO, instead of deciding only on the amount of participation. When investors cannot order an epsilon or infinite amount, their information cannot be perfectly inferred by subsequent investors, and the cascade effect appears.

[5] If V^P can exceed V^L, there are no constraints on the set of optimal prices.

[6] Beatty and Ritter (1986) also assume a uniform distribution to derive implications from Rock's (1982, 1986) model. Although some results change if the underlying distribution differs, the cascade effect on which this paper concentrates is robust. Bikhehandani, Hirshleifer, and Welch (1990) show that it arises in any distribution with limited support.

constant returns to scale (each share's public value is independent of total sales).[7]

It is convenient to define θ as the (unknown) type of the issuer, where θ *î* [0, 1] is a linear transformation of the value range $[V^L, V^H]$. An issuer's price (and/or aftermarket value) can therefore be expressed not only as a price p in terms of the underlying value (p *C* [V^L, V^H]) but also as a unique θ *E* [0, 11, where p = (1 – θ) V^L + *q* V^H . *That* is, a type 0 offering is worth V^L, a type 0.5 offering is worth *(*$V^L + V^H$)/2, and a type 1 offering is worth V^H, Also, let θ^P be the same linear transformation of V^P. Since $V^P \leq V^L$, $\theta^P \leq 0$.

Now assume that each investor can freely observe one independently drawn signal s ∈ {H, L}. Knowledge of each signal is private to the individ ual investor, unless specified otherwise (in a later section, both the issuer and investors are endowed with their own private information). These private signals can be interpreted as information that investors hold about the aftermarket value of shares. In fact, the value inferred when all signals are aggregated ("investors' beliefs") can be defined *to be* the aftermarket value. The signal and the actual aftermarket value (or type) of the project, 0, are related in a simple fashion: the probability that any one investor observes H is θ. To illustrate, if the aftermarket value of the project is one quarter into the range of possible aftermarket values, then on average, for every investor with an *H* signal there are three investors with *L* signals. This specification makes it convenient to derive the posterior mean of $\tilde{\theta}$ given *n* signals of which *k* are of type *H.*

LEMMA 1: *Assume that the prior ~ is distributed uniform [0, 11. Each signal s is a drawing from a Bernoulli variable {H, L} with probability q. Then, the posterior expected value of q given k in n H signals is*

$$E(q \mid k, n) = \frac{k+1}{n+2} \qquad (1)$$

Proof: See the Appendix.

In other words, given only one signal realization, the firm's expected aftermarket value (or type) is 2/3 if the signal is *H; 1/3* if the signal is *L.* Note also that if investors could freely communicate, their average signals could determine the aftermarket value of the firm arbitrarily precisely.[8] I specify the communication among investors when required. However, I always assume that each investor can observe how many investors the issuer (i.e., the firm, or his underwriter) has approached earlier.

[7] Subsection E shows that the cascade results hold when there are locally increasing returns to scale.

[8] As stated earlier, the model could be interpreted to allow for either endogenous or exogenous aftermarket valuation: the large number of signals among investors, when perfectly aggregated, could be the efficient aftermarket value of the offering; or, signals could *be* informative about an underlying true value that is revealed soon after the IPO.

A. Perfect Communication

First consider the benchmark case of a risk-neutral issuer, facing a market in which investors freely communicate. Let k denote the number of H in n signals. Lemma 2 derives the probability that an offering succeeds, if the issuer sets a price that attracts each and every investor if there are at least k investors with good information:

LEMMA 2: *Under the assumptions outline in Lemma 1, the (issuer's) ex-ante probability of observing k H signals is*

$$\text{prob (k H signals} \mid \text{n investors)} = \frac{1}{n+1} \tag{2}$$

and the ex-ante probability of observing k or more H signals is

$$\text{prob(k or more H signals} \mid \text{n investors)} = 1 - \frac{k}{n+1} \tag{3}$$

Proof: See the Appendix.

For example, with 4 investors, the ex-ante probability that an issuer is faced with a market that has received 3 or 4 H signals is 0.4. With this lemma, the optimal offering price is easy to derive. The highest price at which each investor would invest given k in n H signals is (see Lemma 1):

$$P^{\max}(k, n) = \frac{k+1}{n+2} \tag{4}$$

The issuer's problem is to maximize expected utility. The maximization problem over price can be expressed as a maximization problem where the issuer guesses the number of investors who hold H signals. Each proceed-maximizing price P (by convention, quoted in 0 units) corresponds to exactly one k for which (all) investors purchase the offering if at least k investors hold an H signal. (In other words, the issuer optimizes against the unknown total signal of investors.) If the issuer does not choose a price to ensure selling either all or no shares, his problem is[9].

$$\max_k \text{prob}(k \text{ or more } H \text{ signals} \mid n)\ P^{\max}(k, n)n$$
$$+ \text{prob (less than } k \text{ H signals} \mid n)\, \theta^P n$$
$$= n \max_k \left(1 + \frac{k}{n+1}\right)\left(\frac{k+1}{n+2}\right) + \left(\frac{k}{n+1}\right)\theta^P. \tag{5}$$

The first term, for chosen k, is the issuer's proceeds from pricing the offering so that it succeeds if k investors hold good information, multiplied by the probability that at least k investors observe an H signal. The second term is the probability of failure, multiplied by his private value of unsold

[9] I am ignoring integer constraints here, and am adding the border constraints later.

shares. The solution is an optimal number of investors $k^{\star}$ that the issuer should attract. For a risk-neutral issuer, the optimal interior choice is

$$k^{\star} = \frac{n}{2} + \left(1 + \frac{n}{2}\right)\theta^{P} \tag{6}$$

According to (4), this translates into an optimal price of $P^{\star} = (1 + \theta^{P})/2$. For example, for 10 investors, if $\theta^{P} = 0$, the issuer should set the price to sell the offering if there are 5 or more high signals. This requires a price of ½. Substituting the optimal k into (5), the expected wealth of an optimizing issuer is

$$n \max\left\{\frac{1}{n+2}, \left(\frac{1}{4}\right)\left[\frac{n(\theta^{P}+1)^{2}}{n+1}\right]\right\} \tag{7}$$

where the first expression is the proceeds from ensuring success by pricing extremely low, and the second expression is the optimal interior solution where the issuer is willing to sell sometimes.[10] In the above example, if $\theta^{P} = 0$, the issuer expects proceeds of $0.\overline{\quad}$ per share. The average underpricing of successful offerings if the solution is interior is (in θ space)

$$\int_{\theta=P}^{1}\left(\frac{\theta - P}{P}\right)\left(\frac{1}{1-P}\right)d\theta = \frac{1-P}{2P} = \frac{1-\theta^{P}}{2(1+\theta^{P})} \tag{8}$$

All successful offerings are underpriced. I have proven Theorem 1[11]:

THEOREM 1: *With perfect investor communication, a risk-neutral uninformed issuer never prices above the value of the average type (q = 1/2); the average ex-post underpricing (observed initial returns) of successful offerings is not bounded above; no successful offerings are overpriced; offerings are at least as likely to succeed as they are to fail; and, for a large number of investors, the expected utility of the issuer is at most 1/4 per share.*

Furthermore, consistent with the empirical evidence, observed ex -post IPO underpricing is strictly increasing in the common uncertainty about an issue.

*Proof :*See the Appendix.

Thus, the strong documented IPO underpricing/risk relationship (e.g., Beatty and Ritter (1986)) is consistent with this model.

[10] 1/(n + 2) is always larger than the issuer's private utility from not selling any shares ($\theta^{P} < 0$). Also, the interior solution dominates if $k^{\star} \geq 1$ (equation (6) shows that at $k^{\star} = 0$, $-\theta^{P} = n\frac{1}{n+2}$).

[11] In this setting, average underpricing for successful offerings is a natural outcome, as is always the case when an informed buyer faces an uninformed seller. Like Rock's winner's curse, the cascade effect induces simply a specific downward sloping demand curve. (Yet, if the seller is informed and investors are not, there are serious adverse selection problems, possibly leading to market collapse. Section III discusses a case where both the issuer and investors hold some unique information.)

B. Simple Path Dependence: Perfect Communication only from Early to Late Investors

Now assume that each investor observes only his own and the private information of investors who were previously approached by the underwriter. With probability θ, the first approached investor holds *H* information. If this occurs, the issuer receives proceeds P if P $\leq$ 2/3 and 0 otherwise. Analogously, the second investor observes a signal, and invests conditional on the realization of his own and the previous investor's signal. Note that the ultimate proceeds to the issuer are path-dependent: With two investors and a price of 2/5, if the first and second investors draw an *H* and L signal, respectively, the issuer ends up with total proceeds of 2(2/5). If the drawings are reversed, the issuer receives only (2/5).

Figure I summarizes the decisions that the first two investors face in standard game tree format. For example, if the first investor observes an L signal, he purchases if the price does not exceed 1/3. The second investor acts according to the price, his own signal and all information held by the first investor. Figure 2 describes the equilibrium proceeds to the issuer as a function of the price and information of investors (where π are the issuer's proceeds). The figure shows how the issuer's proceeds depend on the ordering of information among the first three investors.

Simple path-dependent expected proceeds as a function of P and *n* cannot be analytically derived. However, with infinitely many investors (henceforth casually referred to as a "large market"), an issuer's expected wealth is the same as in the case where investors perfectly communicate (since knowledge of many earlier investors' signals conveys the expected aftermarket value of the company with high precision to future investors). Thus, an issuer behaves on a large market as if investors perfectly communicated.

THEOREM 3: *With simple path dependence (where investors can observe the signals of prior potential investors), for an infinite number of investors, the outcome is identical to the outcome when all investors perfectly communicate (see Theorems 1-2).*

Extensive simulations show that even for small *n*, an issuer is best off pricing as if he were in the perfect-communication scenario. However, the issuer's expected wealth can be larger, intuitively because an (uninformed) issuer is at less of an informational disadvantage against early investors when setting his price. When there are only a few investors, the first few investors are comparatively uninformed and constitute a larger proportion of the market.

C. Cascades: Observation of Early Investors by Late Investors

Simple path dependence is unrealistic because an investor cannot observe previous investors' information. Instead, an investor can only observe -- and conceivably verify -- whether earlier investors purchased or abstained. This can lead to unstoppable cascades. Once any investor, index him by *M*, with

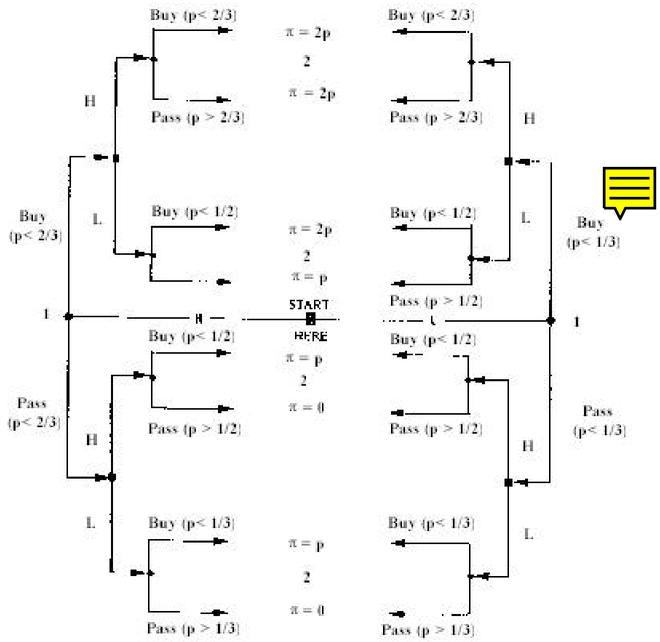

Figure 1. The simple path dependence game tree with two buyers. Square boxes indicate actions taken by nature, round boxes indicate decisions to be made by an investor. The tree shows that: (1) nature chooses the private signal *(H* or *L)* of investor 1; (2) investor 1 decides to *purchase* (which is optimal if the *price* is not above 0.33/0.67 if the signal is *L/H) or* to *pass* (which is optimal if the price is above 0.33/0.67 if the signal is *L/H); (3)* nature chooses the private signal of investor 2; (4) observing both his private signal and the signal of investor 1 (i.e., the node at which he is located in the game tree), investor 2 decides to purchase (which is optimal if the price is not above 0.25/0.5/0.75 and the two signals are *LL / LH* or *HL / HH),* or to *pass* (which is optimal if the price is above 0.25/0.5/0.75 and the two signals are *LL / LH* or *HL / HH).*

an H realization finds it not in his interest to invest, no subsequent investor will. Each subsequent investor will realize that M's decision not to invest given previous observed investment choices should not be interpreted as M actually having received an L signal. Yet, although $M + 1$ cannot learn

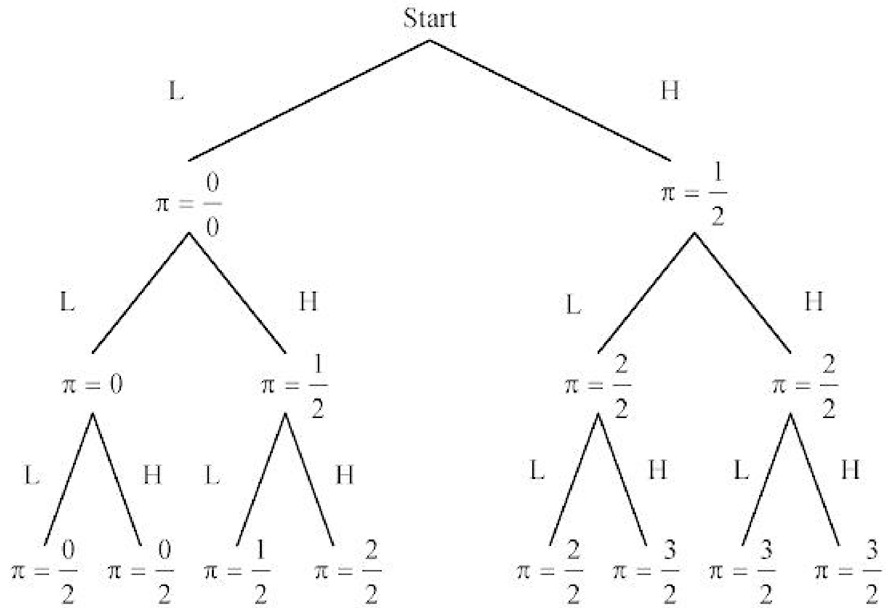

Figure 2. The simple path dependence equilibrium outcome tree when the price is P = 1/2. This tree details the (cumulative) proceeds of an issuer *(w)* as a function of the signals (H or *L)* of the first three investors. For example, if these investors hold *H, L,* and *L,* respectively, two investors will purchase shares. Note that proceeds are path-dependent: an issuer receives more if the first three investors hold signals *HLL* than when they hold *LLH.*

from M's action, *M* + 1 still faces exactly the same decision as *M*. This will of course cause *M* + 1 to also abstain regardless of his signal. By induction, this argument holds for all later investors. Analogously, if an investor with an *L* signal finds it in his interest to invest, so will all later investors.

To illustrate this argument, consider what happens if the issuer prices at P = 1/2 and the first two investors observe (in this order) the signals *H* and *L*. The first investor infers the project value to be 2/3 and purchases. Consequently, the second investor infers project value to be 1/2, and although he received an *L* signal, he believes that a share is not overpriced, and purchases, too. The third investor, regardless of his signal, can not infer the signal of the second investor. Yet, like the second investor he will purchase even with an *L* signal. This produces a positive cascade. Now consider what happens if the first three investors observe (in this order) the signals, *L*, *L*, and *H*. They infer share values of 1/3, 1/4, and 2/5, respectively. The third investor thus abstains, regardless of his signal. The fourth investor can not infer if the third investor held an *L* or an *H* signal, but will also choose not to invest regardless of his signal. Therefore, all subsequent investors abstain, too, and a negative cascade ensues.

Figures 3 and 4 illustrate the cascade model. Like Figure 1, Figure 3 describes the decisions that the first two investors face in standard game tree format. Figure 3 differs from Figure 1 in that the two dotted lines indicate that the second investor does not know at which of the two nodes he moves: In the cascade scenario, he cannot observe the actual signal of his predecessor, only the purchasing decision. Like Figure 2, Figure 4 describes the equilibrium proceeds to the issuer as a function of the price and information

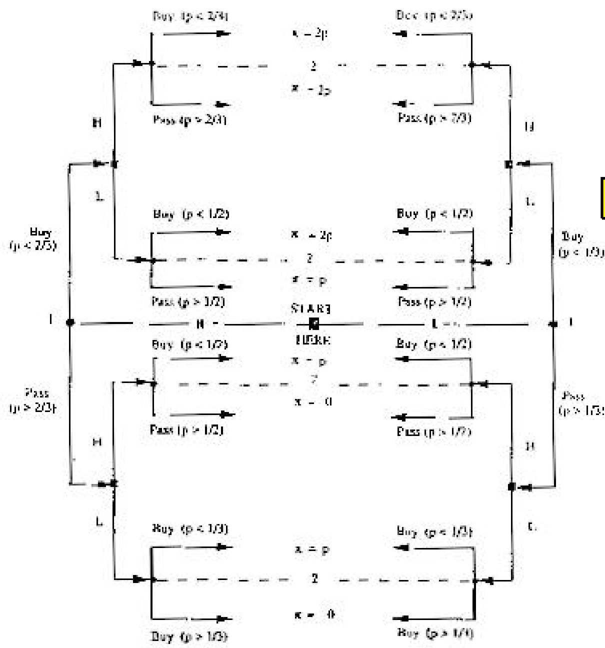

Figure 3. The cascades game tree with two buyers. For a general description, refer to the description of Figure 1, the dotted lines, which each connect two nodes of investor 2, indicate that investor 2 cannot infer the signal of investor 1. However, investor 2 can still observe whether player 1 chose to purchase or abstain. (Only for some but not all prices, if investor I acts rationally, this information allows investor 1 to infer at which node he makes his buy or pass decision, i.e., his information set would be the same as that in Figure 1 where he can observe the signal of Investor 1.)

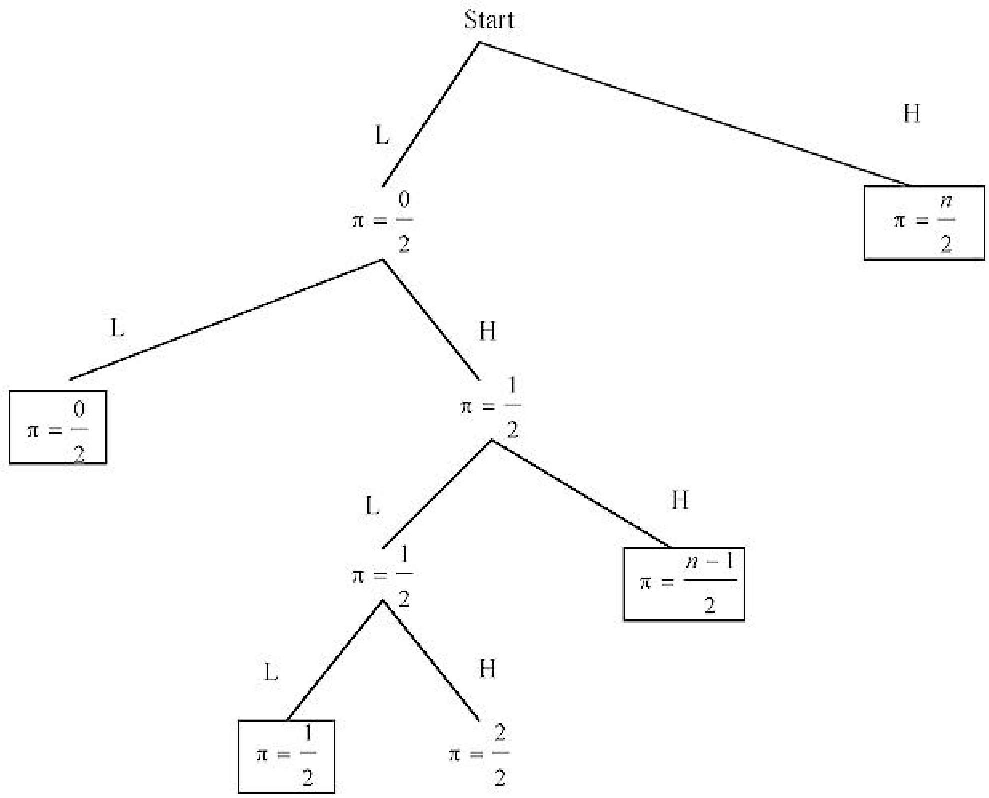

Figure 4. The cascade equilibrium outcome tree when the price is $P = ½$. This tree details the (cumulative) proceeds of an issuer (ir) as a function of the signals (H or *L)* of the first four investors. A boxed node indicates that all subsequent investors will act alike, and total proceeds can be determined (as a function of n the total number of investors). For example, if the first two investors hold the signals *LL* a negative cascade ensues and no further investor will purchase.

of, investors. Figure 4 indicates that the tree can end abruptly (boxed nodes), either with all subsequent investors purchasing in a positive cascade, or with all subsequent investors abstaining in a negative cascade. Theorem 4 sum marizes this argument:

THEOREM 4: *With cascades, for any given price $P > 1/3$ even with an infinite number of investors (and therefore with infinite selling opportunities), the probability that an offering of ultimate value $q < 1$ (and in particular an underpriced issue) fails completely (no investor purchases) is strictly positive. For any given price $P < 2/3$, the probability that an offering of ultimate value $q > 0$ (and in particular an overpriced issue) succeeds perfectly (all investors purchase) is strictly positive.*[12]

[12] Theorem 8 shows that for $P = 1/2$, the probability of a cascade approaches one rather rapidly.

Thus, it cannot be concluded that in the limit (for arbitrarily many, competitive investors with heterogeneous information), a well-informed market aggregates information well. An efficient market does not prevent offering failure given underpricing, or guarantee offering failure given overpricing. However, for a price of 1/3, an issuer can ensure success and each investor ignores all information. A second important observation is that if the aftermarket price perfectly aggregates information, successful offerings cannot only be underpriced, but can also be overpriced relative to the aftermarket price. (Yet, Theorem 5 shows that at the optimum (fullsubscription) price, underpricing occurs on average.) This distinguishes cascades. Ex-post overpriced offerings can never fully succeed in the other information scenarios. [13]

Unfortunately, I cannot solve analytically for proceeds, π, as a function of price (P), the number of investors (n), and the true type (θ),[14] -- even with my specific distributional assumptions. However, the optimal price, expected underpricing, and proceeds can be derived.

THEOREM 5: *An uninformed risk-neutral issuer optimally chooses the fullsubscription price ($q = 1/3$), and all offerings succeed. Successful offerings can ex-post be either over- or underpriced. The expected IPO underpricing (initial return) is between 0% and 50%.*

Proof: See the Appendix.

The intuition is that the price reduction to induce each investor to participate is minor compared with the risk of a completely failed offering for any price above the full-subscription price. In other words, at the optimum price, cascades raise the elasticity of demand. It should also be clear that if the issuer is risk-averse, the full-subscription price with its safe proceeds is even better (when compared to the uncertain simple path dependent proceeds).

Figure 5 graphs the expected proceeds per investor under the three information scenarios as a function of price, holding capital requirements constant at $\theta^P = 0$ and the number of investors at a constant $n = 10$. It is noteworthy that both path-dependent expected proceeds functions are highly irregular, whereas the perfect communication scenario's expected proceed function appears to be regular. The graph shows that the optimal price in both the simple path dependence and perfect communication scenarios is 1/2. In contrast, the optimal price is 1/3, in the cascade scenario, and the figure clearly shows the sharp decline in IPO proceeds if this price were raised. The figure further shows that the issuer is best off in the cascade scenario (with optimal proceeds of $n/3$).

Finally, note that the cascade model is also consistent with the documented

[13] This is a feature that this model shares with Rock (1986). However, many other models cannot explain why some new issues are ex-post overpriced.

[14] From the view of the issuer, θ is of course unknown. For simulations, however, θ is drawn from a uniform distribution, and therefore for each simulation iteration, θ must be specified.

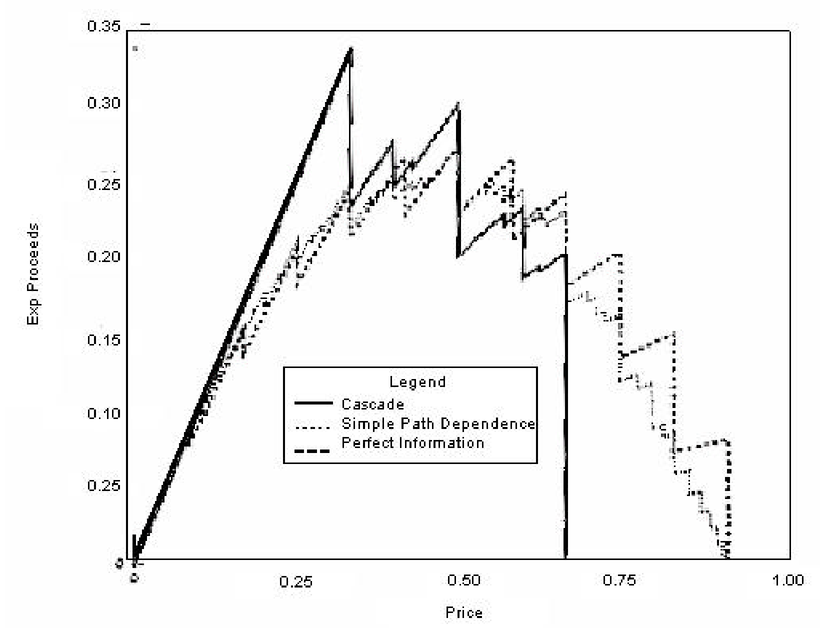

Figure 5. Expected proceeds per investor as a function of the price in the perfect communication, simple path dependence and cascades scenarios. The cascade proceeds are somewhat "chaotic," with sharp irregular drops. At a price of *1/3,* the issuer maximizes expected proceeds (which are higher than those achieved in the other two scenarios). The perfect communication proceeds are quite regular and peak at a price of 1/3. The simple path dependence proceeds track the perfect communication proceeds, albeit in a chaotic fashion similar to the cascades proceeds.

positive relationship between IPO underpricing and ex-ante risk (see Beatty and Ritter (1986)).

Theorem 6: IPO underpricing increases when a mean-preserving spread is added to the common prior.

Proof: See the Appendix.

D. Risk Aversion and Path-Dependent Pricing

I now briefly discuss the value of path-dependent pricing: is an issuer better off if he can adjust the offering price according to early sales? When there are many investors, a risk-neutral issuer is clearly better off: after varying the price such that only investors with H signals buy, the issuer can then create a cascade to induce all subsequent investors to purchase the offering arbitrarily close to the aftermarket value. That is, in the limit, an issuer can expect to receive $n(1/2)$ (the expected value of the project). However, with sufficiently strong risk-aversion, the issuer may actually prefer to create a cascade immediately, and forego path-dependent pricing.

THEOREM 7: *For asymptotically infinitely many investors, a highly risk-averse issuer can be better off pricing low enough to create a cascade immediately, rather than changing the offering price in response to past sales.*

Proof: See the Appendix.

Thus, allowing the offer price to change would not help issuers if they are highly risk-averse. This can explain why there has been remarkably little lobbying against the S.E.C.'s fixed-price rules on IPO pricing.

E. Economies of Scale

The model so far has assumed that an issuer loses proceeds only for unsold shares. However, a project may not be scalable. For example, if economies of scale are extreme, i.e., if the entire NPV is lost unless sufficiently many investors subscribe, then the issuer has an additional reason to desire full-subscription. Therefore, given that the best strategy in a cascade scenario for scalable projects was to price at the full-subscription price, it must also be in the issuer's best interest to ensure full-subscription if the project is not scalable.[15] Thus, the model is robust with respect to (local) positive economies of scale.

F. Model Implications

F.1. Observed Offering Characteristics

This section lists a number of implications that can be derived from the simple model discussed so far. *Of course, all implications rely on the specific distributional assumptions discussed in the previous section: the uninformed knows only that the project's value is uniformly distributed, and each investor can observe one binary signal.*

The first challenge of an empirical test is to find a proxy for how segmented investors' communication is. The "locality" of an issue distribution can be one. (Of course, an empirical researcher may find a better proxy.) In *national (or international) distributions*, where underwriters approach investors in many different states, information is likely to be more segmented. In contrast, in *local distributions* organized by *regional* underwriters, often held in one state or city, investors can communicate more easily. (A second challenge, classifying the degree of inside information, is discussed later.)

The first implication summarizes restrictions of the perfect communication and simple path dependence scenarios derived in Theorems 1-3:

[15] Yet, it should be noted that while the proposed equilibrium exists, so do other pareto-inferior equilibria. In particular, one equilibrium always exists in which no investor purchases. This is rational because investors fear that no other investors will purchase. The issuer can overcome this problem by (1) guaranteeing refunds if the offering does not sell the required minimum and (2) pricing at the full-subscription price.

IMPLICATION 1: *In local issue distributions, an uninformed issuer optimally prices at less than half the expected offering value. Since only (ex-post) underpriced offerings succeed, the typical observed IPO underpricing is high (can exceed 50%). Ex-post, IPO uncertainty and underpricing are positively related.*

Unfortunately, the degree of inside information also plays an important role in smaller distributions. Since the degree of inside information is diffi cult to empirically control for (see Section III), Implication I will be difficult to test.

Theorem 5 showed that, in a cascade situation, even a risk -neutral issuer is best off with the full-subscription price (i.e., the price where each investor invests regardless of his own information).

IMPLICATION 2: *In national issue distributions, an uninformed issuer optimally chooses the full-subscription price, and both ex-post overpriced and ex-post underpriced offerings succeed. Unlike in locally distributed offerings, even highly rationed national offerings can experience aftermarket price drops. Average IPO underpricing is between 0% and 50%, and increases with ex-ante uncertainty.*

Casual observation of the IPO market confirms Implications 1 and 2. IPO underpricing is low and offering failures are rare in the national, high -quality segment of the IPO market (large offerings that are underwritten by well -known investment bankers, where inside information (discussed in a later section) may be less pronounced). Almost invariably, there are more investors than there are shares, and, nevertheless, issuers do not raise prices. Failures and higher IPO underpricing are more common in offerings under written by regional underwriters.[16]

Further implications can be derived if there are costs to a national dis tribution through an underwriter (e.g., underwriter fees). Since such costs do not affect the optimal pricing choice given the antecedent choice of national distribution, the model can predict differences between national and local offerings (if, unlike national issuers, local issuers cannot prevent communication):

IMPLICATION 3: *In national offerings, the offering price does not change with the risk-aversion or capital requirements of the issuer. Thus, in national offerings, there is no relationship between either the risk-aversion and capital requirements of the issuer and the offering price.*[17]

Conversely,

IMPLICATION 4: *In local offerings, the offering price decreases with the riskaversion and the capital requirements of the issuer. Thus, there is a strong*

[16] **Rock (1982, 1986) must assume risk-aversion by the issuer. It is not clear that large issuers and their firm-committed investment banks are really sufficiently risk-averse to price at the full-subscription price.**

[17] **Section III shows that national offerings can fail when there is inside information.**

negative relationship between either. the risk-aversion or capital requirements of the issuer and the offering price. Moreover, local offerings display higher average ex-post underpricing, never ex-post overpricing, can fail, but fail less often when the issuer is more risk-averse or capital-constrained.

When a risk-averse or capital-constrained issuer is faced with a market in which there are no cascades, he prefers safer proceeds and therefore reduces the offering price. In one sense, the above implications can be made more precise: There is not only a predictable influence of risk-aversion, but there is also a prediction regarding the relative prices in national and local markets:

IMPLICATION *5. A risk-neutral or capital- unconstrained issuer prices lower in a national offering (with cascades) than in a local offering (with simple path dependence or perfect investor communication). The reverse holds for a highly risk-averse or capital-constrained issuer.*

And, the more risk-averse or capital-constrained the issuer, the higher are the benef its to undertaking the offering nationally.

IMPLICATION 6: *If national distributions have fixed costs, then a more risk-averse or capital-constrained issuer is more inclined to choose a national (rather than a local) distribution.*

Of course, implications about either market segmentation (investor com munication), risk-aversion, or capital requirements are difficult to test." Finally, I can hypothesize why issuers do not attempt to change fixed-price offering distribution rules to a flexible-pricing scheme (e.g., the lobbying the S.E.C. or by conducting a dutch auction). According to Theorem 7:

IMPLICATION 7: *A highly risk-averse issuer is as well off in a fixed-price scenario (accepting cascade proceeds) as in a variable-price scenario.*

F.2. Offering Distribution and the Role of the Underwriter

Another implication of the cascades model (which is robust to the introduc tion of some inside information) is that

IMPLICATION 8: *An issuer is better off with cascades than with either simple path dependence,*[19] *or perfect communication among investors. Therefore, the issuer has an incentive to prevent communication.*

The intuition behind this implication is that cascades occur when the market aggregates less information, and thus when the issuer is less at a

[18] An exception may be the Bank and Savings and Loan IPO markets (although regulation may complicate such a test). These IPOs are commonly sold in local distributions, sometimes through rights offerings, to customers and local stakeholders. Moreover, the bank's portfolio of assets can provide a measure of risk aversion.

[19] This can be proven by simulations for small *n*, and analytically for large *n*. See also Theorems 1-3 and 5.

disadvantage when setting the price. Moreover, with cascades, an issuer has no incentive to lower the price further (below the full-subscription price) to attract more investors. Persistently long lines are optimal when investors cannot communicate. Note also that it can be shown that investors are no worse off (in an ex-ante sense) in the cascade scenario than in the perfect communication scenario. Consequently, it is *not* socially desirable to prevent cascades. [20]

The desire of issuers to prevent communication provides further implications. The model predicts that it is consistent that issuers both underprice and seek out more (noncommunicating) investors.

IMPLICATION 9: *Since underwriters reduce the cost of placing an offering across a large market (by their maintaining of a national base of investors), an important role of an underwriter may be to provide issuers w th investors that cannot communicate with one another.* [21]

By preventing communication, an underwriter can reduce the failure rate and increase the expected wealth of the issuer. After all, at an average discount of 15%, why do underwriters have to market shares at all, instead of selling all shares at the annual meetings of the American Finance Association? There are, of course, other ways for the issuer to find segmented markets. For example, an issuer can choose a technology that prevents interested investors from coming into close contact with one another. In the extreme, an issuer can even decide to place factories far apart, or to minimize the number and scope of "sociable" dog-and-pony show events for potential investors. [22]

11. The Irrelevance of the Winner's Curse

The scenario of this model mirrors that of Rock's model of the winner's curse, except that investors can perfectly observe earlier demand. Yet, Rock (1986, p. 206) himself, in his discussion of best-effort offerings, states that

[20] This has policy implications. Shareholder organizations currently demand that investors should have access-just as the firm-to a list of shareholders. Such concerns have been embodied in legislation, e.g., in the Shareholder Democracy Act of 1985, in particular to allow corporate raiders equal access. While the context is different, this model suggests that shareholder democracy is not always in the best economic interest (even of shareholders). An issuer can be worse off (if forced to divulge the identity of their shareholders) and investors no better off if the communication among issue buyers were to be improved.

[21] Another explanation for the desire to distribute an offering widely are the corporate control consequences. Small shareholders are easier to "handle" than large shareholders.

[22] Subsequent anecdotal evidence in Bruck (1988, p. 179) has supported this prediction:

> This investment bankers and bond salesmen at Paine Webber wanted to feature Icahn at their road shows. But since Icahn wanted to travel as little as possible, they staged an enormous road show at the Waldorf… According to Joseph Adams (of Drexel Burnham Lambert), 'it was a mistake to feature Icahn… (one should] keep it to relatively small groups. Otherwise, if one person has a valid concern and stands up and voices it, all of a sudden two hundred fifty people who never would have thought of it are worried.'

> Suppose that, instead of the orders *all* being received on one day and filled by lot, the orders arrive over a period of many days and are filled in order of arrival. . . . If the issuer closes the offer as soon as the last share is subscribed, the rationing is 'invisible' because the unfilled orders can't be seen. Nevertheless, the unfilled orders exist; they belong to the disappointed buyers who arrive after all the shares are sold.
> Invisible rationing exerts the same downward pressure on the offering price as the more overt kind. Uninformed investors who arrive in time suspect their success is due as much to lack of interest on the part of informed investors as it is to good luck.

To clarify, the first investor to face rationing in a sequentially served offering is an investor who would be approached only if earlier investors did not subscribe. It is the concern of (all) investors to be among these later investors that forces an issuer to underprice to keep uninformed investors in the market. If an investor knew that she was approached early, or if a project could be scaled arbitrarily to ensure allocations for all interested investors, then underpricing would not be necessary.

My model differs from Rock's model in that I assume that all investors are aware of how many investors arrived earlier. I argue that this can be more realistic for three reasons: (1) The typical best-efforts offering does take some time. Therefore, investors on the first few days are certain that they are never rationed. (Section IV discusses firm -commitment offerings.) (2) Underwriters have an incentive to suggest privately to early investors that they are approached early-even in Rock's scenario (since the first few investors would then be more likely to purchase if they are uninformed). (3) Casual observation suggests that even small investors can find information from brokers about how well an issue has sold to date.

Still, there can be rationing in my model, and thus a winner's curse. However, as far as the issuer is concerned, rationing bias against investors is irrelevant:

THEOREM 8: *Even at a price of 1/2,*[23] *as the number of investors, n, increases, the probability that the last investor is still using his own information decreases rapidly with n. In the limit, for an infinite number of investors, this probability approaches 0.*

Proof: See the Appendix.

Since the model assumes that *all* investors observe the earlier sales success -which is a strong simplification- it is important to know how robust the cascade effect is. With as few as 10 share allocations, the probability of the 11th investor (the first marginal investor facing a winner's curse) to still consider her own information is less than 0.2%. With 100 share allocations

[23] Intuitively, this is the price resulting least likely in a cascade. With a price below or above 1/2,offerings are more likely to fall into a cascade. A price above (below) 1/2 induces a cascade immediately if the first investor observes an *L(H)* signal.

and 101 investors, this probability drops to 10^{-26}. Therefore, I can argue that it must be quite common that an issue's proceeds are a foregone conclusion - - success or failure -- by the time any marginal investors arrive. At the time, the value of shares can be inferred with such a high precision, that there is hardly any winner's curse. This is because even the first investor facing a winner's curse has more information available than the first investor purchasing the issue.

In sum, as in Rock, an issuer wants to ensure full-subscription. However, underpricing serves not to reduce investors' fear to be among the final marginal investors who face the winner's curse. Instead, issuers desire full-subscription so that early investors purchase shares. Indeed, an issuer prices the offering at the full-subscription price even if neither he nor investors care about the marginal investor, or if the project can be freely scaled to avoid rationing.

III. Inside Information

The cascade model in the preceding section suffered from two drawbacks. First, issuers did not hold inside information, and IPO underpricing was due to the issuer's uncertainty about demand. Welch (1989) argues that this is a problematic assumption, because issuers (underwriters) would have strong incentives to pool offerings. This would reduce the overall uncertainty, which in turn would reduce IPO underpricing. Second, offerings in the cascade model never failed. However, empirical evidence suggests that even national offerings - particularly smaller offerings in the low-end segment where inside information is more pronounced-fail occasionally and/or are withdrawn due to "adverse market-conditions." To the best of my knowledge, there are no other papers in the IPO literature than can explain such issue failure.[24] Introducing inside information into the cascade model can solve both of these problems.[25]

Because an issuer has a unique knowledge of the production facilities and product market conditions, I now assume that the issuer (the "insider") observes a private signal. This signal is an independent drawing but similar to the signals (which model information about capital markets) that investors (the "outsiders") hold. First note that when the issuer has information that the firm is of higher quality (value), it is also more likely that investors believe the firm is of high quality. Positive inside information about value is thus correlated with positive outside information. Mathematically, the probability that an issuer is faced with a market of n investors of which k $(n - k)$

[24] Rock (1986) concentrates his discussions on the "full-subscription price,"

[25] This model ignores alternative signaling mechanisms, in particular the IPO underpricing itself when there are seasoned equity offerings (Allen and Faulhaber (1989), Chemmanur (1990), Grinblatt and Hwang (1989), and Welch (1989)). 7`his model tries to explain why issues failures can occur. In real life, an issuer could of course employ several signaling mechanisms simultaneously.

hold *H* (*L*) signals, given that this issuer has access to m private signals of which i were type *H* is:

LEMMA 3:

$$prob(k \text{ of } n \text{ outside } H \text{ signals} \mid i \text{ of } m \text{ inside } H \text{ signals}) = \frac{\binom{n}{k}\binom{m}{i}}{\binom{n+m}{k+i}}\left(\frac{m+1}{n+m+1}\right) \quad (9)$$

Proof: See the Appendix.

For example, when an issuer has privately observed exclusively *H* signals ($i = m$), the probability that all outside signals are *H* (that $k = n$) is $(m + 1)/(n + m + 1)$. This converges to 1 as m increases. On the other hand, if this issuer has privately observed no H in m signals ($i = 0$), then the probability of receiving only *H* signals is

$$\binom{n+m}{n}^{-1}\left(\frac{m+1}{n+m+1}\right) \quad (10)$$

which converges to 0 as *m* increases.

Given this positive correlation of inside and outside signals, the intuition behind a separating equilibrium is straightforward. A high-quality firm loses less (in expected proceeds) than a low-quality firm when risking a higher price: although a higher price increases the incidence of failure for both types, an imitating low-quality firm faces an even higher probability that investors also hold negative information and will reject the offering. The marginal difference in project failure probability makes a higher price a valid signal. I now discuss a simple example to convey the intuition of an equilibrium in which a high-quality firm can separate.

A. An Example

In this example, the issuer's private valuation is $\theta^P = 0$.[26] He can privately observe one piece of information ($m = 1$). The market consists of only two investors ($n = 2$), holding one piece of information each. All three signals are independently distributed draws [*H*, *L*] of equal precision from the true 0 value of the firm. (For convenience, I shall sometimes refer to an issuer holding an *H* [*L*] signal as a high-quality or *G* [low-quality of *B*] firm.) The

[26] That is, the issuer values an unsold share at the same price as the market values the very lowest type v^L.

issuer's Bayesian prior on demand depends on his own private information, and according to Lemma 3 is:

$$prob(LL \mid G) = prob(LH \mid G) = prob(HL \mid G) = 1/6 \quad prob(HH \mid G) = 1/2 \tag{11}$$

$$prob(HH \mid B) = prob(HL \mid B) = prob(LH \mid B) = 1/6 \quad prob(LL \mid B) = 1/2 \tag{12}$$

where the order of signals refers to the ordering of approached investors.

A. 1. Simple Path Dependence

Assume now that there is simple path dependence, and all investors can observe the issuer's signal. The first two panels in Table I detail the issuer's expected proceeds for all possible proceeds-maximizing prices. To illustrate, consider a low quality issuer *(B)* who prices at 1/4. If the first investor holds an *L* signal, knowing that the issuer also holds an *L* signal, he computes the firm's expected value to be $(k + 1)/(n + 2) = 1/4$,[27] I assume and invests. If the second investor also holds an *L* signal, he similarly computes the firm-value to be $(k + 1)/(n + 2) = 1/5$, and abstains. The probability that the first and second investors hold *L* signals, given that issuer is *B*, is 1/6. Repeating this procedure for the other three states *(LH, HL, HH),* and taking the probability-weighted sum of the proceeds gives the issuer expected proceeds of 0.375 (which can be found in the last column.)

Table I shows that if the issuer's signal is known, a high-quality issuer *(G)* optimally speculates that at least one of the two investors is *H*. A low-quality issuer prefers to play it safe, and sets the full-subscription price.

Now consider the situation when investors do not know the issuer's signal, but can infer it because *G* and *B* firms price at 1/5 and 3/4 with expected proceeds of 0.4 and 0.875, respectively. Do these prices satisfy incentive compatibility? The third panel in Table I shows the low-quality issuer's expected proceeds if he mimicks a high-quality issuer. The benefit of the mimicking strategy is that investors infer than the issuer holds an H signal. Therefore, investors are more likely to buy for the same price and signal. However, the probability of reaping the benefits-market conditions favorable enough to have many investors purchase-has not changed for the low-quality firm (see Row 4). Thus, at a price of 3/4, the expected proceeds to a low-quality firm are only 1/3. Since this is less than 2/5, a *B* issuer is better off pricing at 1/5. A *G* issuer on the other hand also prefers to receive 0.875 to 0.4, and thus prices at 3/4.

I can define a simple separating equilibrium (called a proposition instead of a theorem, since it holds only in this numerical example):

[27] When economically indifferent, I assume that a low-quality firm does not imitate a high-quality firm. This improves the exposition without sacrificing generality.

PROPOSITION 1: *With simple path dependence, a signaling equilibrium exists in which the high-quality firm (G) prices at 0.75, fails completely with probability 1/3, succeeds partially with probability 1/6 and fully with probability 1/2. G expects proceeds of 0.875. A low-quality firm (B) prices at 0.2, always succeeds completely, and expects to receive 0.4.*

A high-quality firm receives less than the 0.917 it would receive if it could costlessly and credibly communicate its type. The difference between 0.917 and 0.8 is the cost of signaling.

A.2. Casades

I now show that in this example with signaling and inside information, both types of issuers prefer cascades over path dependence. Table 11 dupli cates Table I in a world with cascades.[28]

A comparison of Tables I and II shows that the low -quality firm's incentive compatibility constraint is weakened, because a truthful *B* firm expects to receive 0.5 (at a price of 0.25) instead of 0.4 (at a price of 0.2). This allows the existence of a separating equilibrium in which a high-quality firm can price lower (P = 0.6 instead of P = 0 .75). This makes *G* better off, because the lower price greatly reduces his probability of failure. In sum, according to Table II.

PROPOSITION 2: *With cascades, a signaling equilibrium exists in which the high-quality firm. (G) prices at 0.6 and expects to raise 0.9. G fails completely with probability 1/6, succeeds partially with probability 1/6, and fully with probability 2/3. The low-quality firm (B) prices at the full-subscription price of 0.25, and expects to raise 0.5.*

A.3. Implications

Some implications can be derived from this example of inside information. It must of course be emphasized that these implications rest not on a model of an arbitrary number of investors, *n*, but on a specific number, and on a specific kind of inside information (i.e., exactly one signal of comparable precision to that of outside investors). Among the intuitively more generalizable implications are:

IMPLICATION 10: *When inside information is an important factor, national offerings are less likely to fail than local offerings.*

[28] The required changes were: (1) At a price of 1/4, the known low-quality firm always attracts both investors (i.e., even in state *LL*). Thus it can expect proceeds of 0.5 instead of 0.375. (2) At a price of 3/5, the known low-quality firm never attracts any investors. (3) At a price of 1/2, the known high-quality firm always attracts both investors. Therefore, its expected proceeds at P = 1/2 are 1. (4) At a price of 4/5, the known high-quality firm never attracts any investor.

Table I

The Path-Dependent Pricing Strategies under Inside Information

This table details investors' behavior and the issuer's expected proceeds (from both investors) at all potentially profit-maximizing prices, given the true and given the inferred type of the firm. In each panel, the second line lists all possible distributions of outside signals. The third line lists the true probability of each firm to be faced with a particular distribution of outside signals. Subsequent lines list -- for all prices -- the optimal decisions of the two investors (given the inferred type and observed signal(s); the expected share value is always $(k + 1)/(n + 2)$ where k is the number of H signals in n draws). To illustrate, "buy/abstain" means that the first investor optimally decides to purchase and the second investor optimally decides to abstain. The expected proceeds of the issuer, in column 6, are obtained by multiplying each state probability by the number of investors purchasing and the assumed issue price, and summing over all states. From the table, the separating equilibrium is for *B* to price at 1/5, and G to price at 3/4.

True and Inferred Firm Value	*B* (Issuer holds one *L* signal)				
Investors Observe:	*L/L*	*L/H*	*H/L*	*H/H*	
True Probability:	1/2	1/6	1/6	1/6	
	Optimal Investor Behavior				E Proc
Price = 1/5	buy/buy	buy/buy	buy/buy	buy/buy	0.400
Price = 1/4	buy/abstain	buy/buy	buy/buy	buy/buy	0.375
Price = 1/3	abstain/abstain	abstain/buy	buy/buy	buy/buy	0.277
Price = 2/5	abstain/abstain	abstain/buy	buy/buy	buy/buy	0.333
Price = 1/2	abstain/abstain	abstain/abstain	buy/abstain	buy/buy	0.250
Price = 3/5	abstain/abstain	abstain/abstain	abstain/abstain	abstain/buy	0.100
Price = 2/3					
Price = 3/4					
Price = 4/5					

Table I – *Continued*

True and Inferred Firm Value	*G* (Issuer holds one *H* signal)				
Investors Observe:	*L/L*	*L/H*	*H/L*	*H/H*	
True Probability:	1/6	1/6	1/6	1/2	
	Optimal Investor Behavior				*E* Proc
Price = 1/8					
Price = 1/4					
Price = 1/3					
Price = 2/5	buy/buy	buy/buy	buy/buy	buy/buy	0.800
Price = 1/2	buy/abstain	buy/buy	buy/buy	buy/buy	0.917
Price = 3/5	abstain/abstain	abstain/buy	buy/buy	buy/buy	0.900
Price = 2/3	abstain/abstain	abstain/abstain	buy/abstain	buy/buy	0.778
Price = 3/4	abstain/abstain	abstain/abstain	buy/abstain	buy/buy	0.875
Price = 4/5	abstain/abstain	abstain/abstain	abstain/abstain	abstain/buy	0.400

False but Inferred Firm Value	*G* (Issuer falsely believed to hold one *H* signal)				
Investors Observe:	*L/L*	*L/H*	*H/L*	*H/H*	
True Probability:	1/2	1/6	1/6	1/6	
	Optimal Investor Behavior				*E* Proc
Price = 1/5					
Price = 1/4					
Price = 1/3					
Price = 2/5	buy/buy	buy/buy	buy/buy	buy/buy	0.800
Price = 1/2	buy/abstain	buy/buy	buy/buy	buy/buy	0.750
Price = 3/5	abstain/abstain	abstain/buy	buy/buy	buy/buy	0.500
Price = 2/3	abstain/abstain	abstain/abstain	buy/abstain	buy/buy	0.333
Price = 3/4	abstain/abstain	abstain/abstain	buy/abstain	buy/buy	0.375
Price = 4/5	abstain/abstain	abstain/abstain	abstain/abstain	abstain/buy	0.133

Table II

The Cascade Pricing Strategies under Inside Information

This table details investors' behavior and the issuer's expected proceeds (from both investors) at all potentially profit-maximizing prices, given the true and given the inferred type of the firm. In each panel, the second line lists all possible distributions of outside signals. The third line lists the true probability of each firm to be faced with a particular distribution of outside signals. Subsequent lines list-for all prices-the optimal decisions of the two investors (given the inferred type and observed signal(s); the expected share value is always $(k + 1)/(n + 2)$ where k is the number of *H* signals in n draws). To illustrate, "buy/abstain" means that the first investor optimally decides to purchase and the second investor optimally decides to abstain. The expected proceeds of the issuer, in column 6, are obtained by multiplying each state probability by the number of investors purchasing and the assumed issue price, and summing over all states. From the table, the separating equilibrium is for B to price at 1/4 and G to price at 3/5.

True and Inferred Firm Value		*B* (Issuer holds one *L* signal)			
Investors Observe:	*L/L*	*L/H*	*H/L*	*H/H*	
True Probability:	1/2	1/6	1/6	1/6	
	Optimal Investor Behavior				E Proc
Price = 1/5	buy/buy	buy/buy	buy/buy	buy/buy	0.400
Price = 1/4	buy/buy	buy/buy	buy/buy	buy/buy	0.500
Price = 1/3	abstain/abstain	abstain/buy	buy/buy	buy/buy	0.277
Price = 2/5	abstain/abstain	abstain/buy	buy/buy	buy/buy	0.333
Price = 1/2	abstain/abstain	abstain/abstain	buy/abstain	buy/buy	0.250
Price = 3/5	abstain/abstain	abstain/abstain	abstain/abstain	abstain/abstain	0.000
Price = 2/3					
Price = 3/4					
Price = 4/5					

-

Table 11- *Continued*

True and Inferred Firm Value	*B* (Issuer holds one *L* signal)				
Investors Observe:	*L/L*	*L/H*	*H/L*	*H/H*	
True Probability:	1/6	1/6	1/6	1/2	
	Optimal Investor Behavior				E Proc
Price = 1/5					
Price = 1/4					
Price = 1/3					
Price = 2/5	buy/buy	buy/buy	buy/buy	buy/buy	0.800
Price = 1/2	buy/buy	buy/buy	buy/buy	buy/buy	1.000
Price = 3/5	abstain/abstain	abstain/buy	buy/buy	buy/buy	0.900
Price = 2/3	abstain/abstain	abstain/abstain	buy/abstain	buy/buy	0.778
Price = 3/4	abstain/abstain	abstain/abstain	buy/abstain	buy/buy	0.875
Price = 4/5	abstain/abstain	abstain/abstain	abstain/abstain	abstain/abstain	0.000

False but Inferred Firm Value	*G* (Issuer falsely believed to hold one *H* signal)				
Investors Observe:	*LIL*	*LIH*	*HIL*	*HIH*	
True Probability:	1/2	1/6	1/6	1/6	
	Optimal Investor Behavior				E Proc
Price = 1/5					
Price = 1/4					
Price = 1/3					
Price = 2/5	buy/buy	buy/buy	buy/buy	buy/buy	0.800
Price = 1/2	buy/buy	buy/buy	buy/buy	buy/buy	1.000
Price = 3/5	abstain/abstain	abstain/buy	buy/buy	buy/buy	0.500
Price = 2/3	abstain/abstain	abstain/abstain	buy/abstain	buy/buy	0.333
Price = 3/4	abstain/abstain	abstain/abstain	buy/abstain	buy/buy	0.375
Price = 4/5	abstain/abstain	abstain/abstain	abstain/abstain	abstain/abstain	0.000

and

IMPLICATION 11: *With inside information, national offerings can fail. These failed offerings are offered by issuers who held, ex-ante, positive inside information.*

This has policy implications. Occasional offering failures are not socially undesirable. An issuer that fails may have had a better ex-ante outlook. That is, while negative offering failure consequences are undeniable, they are costly but useful signals that allow the higher quality firms to separate.

Finally, the implications about the relative benefits of local versus national distributions (5-6) can be retained:

IMPLICATION 12: *When inside information is an important factor, risk-neutral issuers prefer national offerings given equal cost. Since fewer offerings fail in national offerings, more risk-averse or capital-constrained issuers are more likely to issue nationally.*

The result that issuers prefer cascades over simple path dependence thus holds even when the issuer has some inside information. Therefore, under writers can also play a useful role in preventing communication among investors when there is inside information. [29]

IV. Pre-Selling Activity

The above model is idealized in that it ignores the ability of underwriters to "line up" interested investors before an offering opens. This occurs primarily in firm-commitment offerings. Benveniste and Spindt (1989) model this pre-selling activity, assuming that regular informed investors neither face a winner's curse nor communicate with one another. Consequently, a highly risk-averse underwriter can extract information from investors (at a considerable cost although he can drive investors' surplus to zero) by committing (1) to underprice if demand is strong and (2) to not overprice the issue if demand is weak.

The model set out in this paper endows investors with more ability to communicate, and underwriters with less power over informed investors. In particular, the cascade model can also apply to the pre-market if potential investors can "eye" one another for (a) who has provided an indication of interest when indicating early interest, or (b) who actually purchases when the ultimate purchasing decision arises. In fact, even firm-commitment offerings are sometimes withdrawn "due to adverse market conditions" before the offering is effective. [30]

[29] Finally, it can be shown that the pricing decisions of issuers are distinctive in the separating equilibrium, and that an educated guess is that the average ex-post underpricing in the path dependence separating scenario is higher than in the separating cascades scenario. Also, the demand of risk-neutral issuers to purchase information is downward-sloping.

[30] The Microsoft IPO mentioned in footnote 1 was indeed a firm-commitment offering, for which Goldman Sachs had lined up investors.

Another problem worth mentioning is the assumed inability of underwrit ers to withhold all information about their initial sales success (if they are not at liberty to lie). If the number of offerings is finite, then in the last offering, underwriters have strong incentives to advertise early sales success. Consequently, the market would interpret failure to do so to imply sluggish early sales. Of course, given that the underwriter cannot commit to not disclosing the last offering's early sales results, there is no incentive to commit in the second-to-last offering, and so on (see also Selten (1978) for a similar finite horizon unraveling argument). Also, even if the underwriter could withhold some information, it would probably be of little help, since it is in the interest of (institutional) investors to cooperate and inform one another of their ultimate (and potentially verifiable) purchasing decisions.[31]

A final problem is that there are strong incentives for firms and underwrit ers to bribe a few early investors, or to lie about an issue's lackluster early sales. Yet, since underwriters are legally liable for any statements made to investors, and since they have a reputation to protect, it can be in the underwriter's best interest to tell the truth. Systematic cheating would probably be uncovered eventually, and cost the underwriter dearly: an nounced investor interest in future offerings would probably be generally construed as artificial and make future offerings more difficult to sell. Even the popular press indicates that underwriters are aware of the need to accurately report outside demand.[32]

V. Conclusion and Future Directions

This paper has argued that the pricing decisions of issuers can reflect informational cascades-where later investors rely completely on the pur chasing decisions of earlier investors and ignore their own information. The pricing and purchasing decisions of agents and patterns of issue failure are cross-sectionally distinctive across different informational scenarios, and can therefore be used to construct empirical tests. The paper also contends that cascades reduce the applicability of the winner's curse when offerings are sold over a period of time: issuers price with an eye toward early rather than late investors. Unlike in models of rationing, projects that can be scaled to fit investor demand are still underpriced. Next, the paper has claimed that inside information can cause issue failures. These failures are by issuers with good, not bad, inside information. Finally, I have argued that underwriters can play an important role in the creation of cascades. Roughly, the interpre -

[31] Information about investors' actions is probably the most verifiable and thus most credible form of communication ("actions speak louder than words"). Note that the relationship between an investor and a selling investment bank is far more adversarial than the relationships among investors.

[32] In the *Fortune* magazine story mentioned in footnote 1, Uttal reports that, in a compromise in the earlier-mentioned Microsoft EPO, it was agreed by Dobkin (of Goldman Sachs) to set a high price despite objections, but "Dobkin would report [to the market] Monday on which investors had dropped out."

tation is that one role of underwriters is to find investors in segmented markets (nationwide), since those investors are less likely to communicate with one another. Selling to separated investors, the issuer is at less of an informational disadvantage when pricing the issue. Yet, most importantly, this paper has provided a dynamic rational explanation for a phenomenon -- herd behavior -- that is often mentioned but rarely explained by financial practitioners and academics. [33]

Future empirical research may examine not only to what extent cascades exist in the IPO market, but also in other fixed-price markets. In particular, the markets for insurance and bank syndication appear to function in a similar sequential nature. The model could also be used to explain sales of such products as movie tickets (where national box-office rankings and long lines indicate demand), real estate and computer sales or even product sales in supermarkets (where shoppers infer information from the length of time that an item remains on the shelves). Future theoretical research may determine how risk-averse sellers optimally adjust prices to take advantage of both the cascade effect and their ability to adjust prices.

Appendix A

Proof of Lemma 1: This lemma is a special case of DeGroot (1970, p. 160), Theorem 1. However, since I use some of the derived forms in other lemmata and theorem proofs, it is repeated here. By Bayes Theorem,

$$\text{Prob}(\theta \mid k\, H \text{ signals}) = \frac{\text{prob}(\text{kHsignals} \mid \theta)\,\text{prob}(\theta)}{\text{prob}(kH\text{signals})} \tag{13}$$

$$= \frac{\binom{n}{k}\theta^k (1-\theta)^{n-k} f(\theta)}{\int_{\theta=-\infty}^{\infty} \binom{n}{k}\theta^k (1-\theta)^{n-k} f(\theta)\,d\theta} \tag{14}$$

$$= \frac{\theta^k (1-\theta)^{n-k}}{\int_{\theta=0}^{1} \theta^k (1-\theta)^{n-k}\,d\theta} \tag{15}$$

Multiplying by θ and integrating out θ gives the posterior expected value of θ given k H signals.

$$E(\theta \mid k \text{ H signals}) = \frac{\int_{\theta=0}^{1} \theta^{k+1} (1-\theta)^{n-k}\,d\theta}{\int_{\theta=0}^{1} \theta^k (1-\theta)^{n-k}\,d\theta} \tag{16}$$

$$= \frac{\int_{\theta=0}^{1} \theta^{k+1} (1-\theta)^{n-k}\,d\theta}{\int_{\theta=0}^{1} \theta^k (1-\theta)^{n-k}\,d\theta} \tag{17}$$

[33] For some rare -- and recent -- exceptions, see the discussion of Bannerjee (1992), Froot, Scharfstein, and Stein (1991), and Scharfstein and Stein (1990) in the introduction.

Consider the denominator. Integration by parts of the indefinite integral (with $u' = \theta^k$ and $v = [1-\theta]^{n-k}$) yields

$$\int \theta^k (1-\theta)^{n-k} d\theta = \frac{\theta^{k+1}(1-\theta)^{n-k}}{k+1} + \int \frac{\theta^{k+1}}{k+1}(n-k)(1-\theta)^{n-k-1} d\theta \tag{18}$$

Since the eventual integration is from [0, 1], the nonintegral term can be omitted.

$$\int_{\theta=0}^{1} \theta^k (1-\theta)^{n-k} d\theta = \int_{\theta=0}^{1} \frac{\theta^{k+1}}{k+1}(n-k)(1-\theta)^{n-k-1} d\theta \tag{19}$$

$$= \int_{\theta=0}^{1} \frac{\theta^{k+2}}{(k+1)(k+2)}(n-k)(n-k-1) \times (1-\theta)^{n-k-2} d\theta \tag{20}$$

Continued integration *by* parts to reduce the $(1 - \theta)$ factor to 1 yields

$$\int_{\theta=0}^{1} \theta^k (1-\theta)^{n-k} d\theta = \int_{\theta=0}^{1} \frac{(n-k)!}{(k+1)(k+2)...n} \theta^n (1-\theta)^0 d\theta \tag{21}$$

$$= \frac{(n-k)!\,k!}{n!} \int_{\theta=0}^{1} \theta^n d\theta = \frac{1}{(n+1)\binom{n}{k}} \tag{22}$$

The same procedure can be applied to the nominator of (17):

$$\int_{\theta=0}^{1} \theta^{k+1}(1-\theta)^{n-k} d\theta$$

$$= \int_{\theta=0}^{1} \frac{n-k}{k+2} \theta^{k+1}(1-\theta)^{n-k-1} d\theta \tag{23}$$

$$= \int_{\theta=0}^{1} \frac{(n-k)!}{(k=2)(k+3)...n} \theta^{n+1}(1-\theta)^0 d\theta$$

$$= \frac{k+1}{(n+1)\binom{n}{k}} \int_{\theta=0}^{1} \theta^{n+1} d\theta \tag{24}$$

$$= \frac{k+1}{(n+2)(n+1)\binom{n}{k}} \tag{25}$$

Dividing expression (25) by (22) gives the hypothesized expression in the lemma.

Proof of Lemma 2: It is to be shown that

$$\text{prob}(k\ H \text{ signals} \mid n \text{ investors}) = \int_{\theta=0}^{1} \binom{n}{k} \theta^k (1-\theta)^{n-k} d\theta \neq \frac{1}{n+1} \tag{26}$$

Since in *(22),* the identity

$$\int_{\theta=0}^{1} \theta^k (1-\theta)^{n-k} d\theta = \frac{1}{(n+1)\binom{n}{k}} \tag{27}$$

was established, both the density and the cumulative distribution function stated in the lemma follow immediately.

Proof of Theorem 2: It is to be shown that adding a mean-preserving spread to the prior distribution on IPO value increases ex-post IPO underpricing. Since successful offerings are distributed uniformly between P and V^H, the observed ex-post IPO underpricing for an arbitrary offer price is

$$U(p) \equiv \int_{p}^{V^H} \left(\frac{V-p}{p}\right)\left(\frac{1}{V^H - p}\right) dV = \frac{V^H - p}{2p} \tag{28}$$

The transformation $p(\theta) = (1-\theta)V^L + \theta V^H$ translates P $^\star = (1 + \theta^P)/2$ into V space:

$$p^\star = \left(1 - \frac{1+\theta^P}{2}\right)V^L + \left(\frac{1+\theta^P}{2}\right)V^H \tag{29}$$

Substituting (29) into (28), the observed ex-post IPO underpricing simplifies to

$$U^\star \equiv U(p^\star) = \frac{(1-\theta^P)(V^H - V^L)}{2[(1+\theta^P)V^H + (1-\theta^P)V^L]} \tag{30}$$

Ex-post underpricing is thus lowest (zero) when $V^H = V^L$, i.e., when there is no uncertainty about firm value. Ex-post underpricing can exceed 50%, e.g., if $\theta^P = -0.5$ and $V^L = 0$.

I can now add a mean-preserving spread to the prior distribution. Substituting $V^H + \varepsilon$ and $V^L - \varepsilon$ for V^H and V^L, yields

$$U^\star(\varepsilon) = \frac{(1-\theta^P)(V^H - V^L) + 2(1-\theta^P)\varepsilon}{2[V^H(1+\theta^P) + V^L(1-\theta^P) + 2\theta^P\varepsilon]} \tag{31}$$

Differentiating underpricing with respect to ε yields

$$\frac{\partial U^\star}{\partial \varepsilon} = \frac{(V^H - V^L)(1-\theta^P)}{[V^H(1+\theta^P) + V^L(1-\theta^P) + 2\varepsilon\theta^P]^2} \tag{32}$$

which is Positive if $\theta^P > (-1)$.[34] Since this is the case if the solution is interior, ex-post IPO underpricing strictly increases with ex-ante uncertainty as long as the solution is interior.

Proof of Theorem 5: The proof proceeds in steps. I first assume that the issuer's private valuation, θ^P, is zero. Then, I show, sequentially, an issuer is better off with a price other than 1/3 (and proceeds of *n* /3), this price must lie (weakly) below 2/3, 3/5, 1/2, 2/5 and finally 1/3. The proof works by considering the combination of the highest price with the minimum probability of offering failure (i.e., at the lowest price) in a given range. I will show that an issuer is always better off choosing P = 1/3 than with the price/failure combination in each range, and since the ranges cover all prices above 1/3, I conclude that an issuer is best off at P = 1/3. Finally, since at P = 1/3, an issuer ensures full-subscription, he will also choose P =1/3 for any $\theta^P \leq 0$.

P >2/3: Clearly, no investor ever purchases the issue, and thus, no price larger than 2/3 can be optimal.

2/3 ≥ *P* > 3/5: The following states induce a "negative cascade," i.e., a cascade where no investor purchases the issue: *LL* (the implied value given *LLH is 2/5), LH* and *HL* (the implied value given *HLH* or *LHH is 3/5* -although in the latter case, the first (and only the first) investor purchases). The probability that an issuer is faced with states *LL, LH,* and *HL* was. derived in equation 36)

$$\text{prob}(LL) = \int_{\theta=0}^{1} (1-\theta)^2 \, d\theta = \frac{1}{3} \tag{33}$$

$$\text{prob}(LH) = \int_{\theta=0}^{1} (1-\theta)\theta d\theta = \frac{1}{6} \tag{34}$$

$$\text{prob}(HL) = \int_{\theta=0}^{1} \theta(1-\theta) d\theta = \frac{1}{6} \tag{35}$$

1 now show that even with the largest possible price in the domain 3/5 ≤ *P* < 2/3 -- i.e., 2/3 -- an issuer can do no better than choosing a price of 1/3 (i.e., with proceeds of *n* /3). Since states *LL, LH,* and *HL* are mutually exclusive, the expected proceeds ($E\pi$) can at most be

$$E\pi < \text{prob}(LL) * 0 + \text{prob}(LH) * 0 + \text{prob}(HL) * 1 * \frac{1}{3} + \text{prob}(HH) * n * \frac{2}{3} \tag{36}$$

[34]It can be shown that the inferior solution's expected proceeds expected dominates the boundary's proceeds if $k^\star \geq 1$, since at $k^\star = 0$, the two solutions are identical. Thus, if $\theta^P \geq -n/(n+2)$, the interior solution dominates. Obviously, this is greater than −1.

(If state HL occurs, the issuer of course receives 2/3 from the first investor before facing the negative cascade.) Expected proceeds are definitely less than the RHS of (36), since nonpurchases and even negative cascades are possible for some states subsequent to state HH ... (e.g., a negative cascade results in state HHLL since the expected value given HHLLH is 4/7 which is less than 3/5). Substituting into (36),

$$E\pi < \tfrac{1}{3} * 0 + \tfrac{1}{6} * 0 + \tfrac{1}{6} * \tfrac{2}{3} + \tfrac{1}{3} * \tfrac{2}{3} * n = \tfrac{1}{9} + \tfrac{2}{9} * n \tag{37}$$

For n > 1, the RHS lies below n /3. Thus, an issuer is better off at a price of 1/3 than at any price strictly exceeding 3/5.

3/5 ≥ P > 1/2: The following states certainly create a negative cascade: LL (th implied value given *LLH* signals is 2/5), *LHL* and *HLL* (the implied value given *HLLH* is 1/2). In the latter case, only the first investor purchases a share. The probabilities that an issuer is faced with *LL*, *HLL*, and *LHL* are

$$\text{prob}(LL) = \int_{\theta=0}^{1} (1-\theta)^2 \, d\theta = \frac{1}{3} \tag{38}$$

and that he is faced with two *L*s and one *H* are

$$\text{prob}(HLL) = \int_{\theta=0}^{1} \theta(1-\theta)^2 \, d\theta = \frac{1}{12} \tag{39}$$

$$\text{prob}(LHL) = \int_{\theta=0}^{1} (1-\theta)\theta(1-\theta) d\theta = \frac{1}{12} \tag{40}$$

Again, the maximum expected proceeds to an issuer are

$$E\pi < \text{prob}(LL) * 0 + \text{prob}(LHL) * 0 + prob(HLL) * 1 * \tfrac{3}{5} + \text{prob}(\overline{LL \text{v} LHL \text{v} HLL}) * n * \tfrac{3}{5} \tag{41}$$

where the strict inequality is justified, e.g., by omissions of negative cascades implicit in $\text{prob}(\overline{LL \text{v} LHL \text{v} HLL})$ (for example, *LHHLL*...). Substituting,

$$E\pi < \tfrac{1}{3} * 0 + \tfrac{1}{12} * 0 + \tfrac{1}{12} * \tfrac{3}{5} + \tfrac{1}{2} * \tfrac{3}{5} * n = \tfrac{1}{20} + \tfrac{6}{20} * n \tag{42}$$

Since the RHS is less than *n* /3 for n ≥ 2, an issuer is better off with the price of $\tfrac{1}{3}$ than with a price larger than $\tfrac{1}{2}$.

$\tfrac{1}{2} \geq P > \tfrac{2}{5}$ *:* The state *LL* still suffices to induce a negative cascade ($E[\theta \mid LLH] = \tfrac{2}{5}$). The probability of *LL* is

$$\text{prob}(LL) = \int_{\theta=0}^{1} (1-\theta)^2 \, d\theta = \frac{1}{3} \tag{43}$$

Thus, with a largest possible price of $\tfrac{1}{2}$, expected proceeds are less than

$$E\pi < \text{prob}(LL) * 0 + \text{prob}(\overline{LL}) * \tfrac{1}{2} * n = \tfrac{1}{3} * 0 + \left(\tfrac{2}{3}\right) * \left(\tfrac{1}{3}\right) * n = n/3 \tag{44}$$

since I have omitted combinations such as *LHLLL*... (which also induce a negative cascade). It follows that no price above 2/5 can be an expected proceeds maximizer.

2/5 ≥ P > 1/3 : Now, state *LLL* suffices to create a negative cascade ($E\theta \mid LLLH = 2/6$). The probability of this state is

$$\text{prob}(LLL) = \int_{\theta=0}^{1} (1-\theta)^3 \, d\theta = \frac{1}{4} \tag{45}$$

and, again, the expected proceeds must lie below

$$E\pi < \text{prob}(LLL) * 0 + \text{prob}(\overline{LLL}) * \tfrac{2}{5} * n = \tfrac{1}{4} * 0 + (\tfrac{2}{3}) * (\tfrac{2}{5}) * n$$
$$= (3n)/10 < n/3 \tag{46}$$

This completes the proof for n ≥ 5. No price above 1/3 allows an issuer to increase expected proceeds for n ≥ 3. For $2 \le n \le 4$, simple enumeration shows that safe proceeds at P = 1/3 are preferred to the expected proceeds at the other potential proceed-maximizing prices. (For n = 1, the issuer is indifferent between setting the price at 1/3, or 2/3.)

The final statements of the theorem are that both underpriced and overpriced offerings succeed. This is obvious because no offering fails. The average underpricing is the expected value of successful offerings divided by the price. Translating $\theta^\star = 1/3$ back into V space, we find that, since an offering's price is $p^\star = V^L + (V^H - V^L)/3$ and expected value is $V^L + (V^H - V^L)/2$, the IPO underpricing (U) at the optimal price is

$$U^\star = \frac{EV - P}{P} = \frac{[V^L + (V^H - V^L)/2] - [V^L + (V^H - V^L)/3]}{V^L + (V^H - V^L)/3}$$
$$= \frac{V^H - V^L}{2V^H - 4V^L} \tag{47}$$

Therefore, since $0 \le V^L < V^H$, $0 < U \le 0.5$.

Proof of Theorem 6. The IPO underpricing in the cascade scenario is given in equation 47. *Adding a mean-preserving spread,* i.e., increasing V^H by an ε and decreasing V^L by an ε, the new IPO underpricing is

$$U^\star(\varepsilon) = \frac{(V^H + \varepsilon) - (V^L - \varepsilon)}{2(V^H + \varepsilon) + 4(V^L - \varepsilon)} = \frac{V^H - V^L + 2\varepsilon}{4V^L + 2V^H - 2\varepsilon}. \tag{48}$$

Differentiating U with respect to ε yields

$$\frac{\partial U^\star}{\partial e} = \frac{3(V^H + V^L)}{2(V^H + 2V^L - e)^2} \tag{49}$$

which is always positive. Thus, a larger ε (mean-preserving spread) always increases IPO underpricing.

Proof of Theorem 7: Since all shares are selling in either scenario, θ^P is irrelevant. The safe cascade utility is

$$U(n\tfrac{1}{3}) \tag{50}$$

At best, a path-dependent strategy provides asymptotically the same proceeds as the firm's worth (i.e., the market-clearing price shouldn't exceed the aftermarket value of the firm). Ex-ante, an issuer can then expect

$$\int_{\theta=0}^{1} U(n\theta)d\theta \tag{51}$$

An example suffices to show that for a highly risk-averse issuer, the former utility is preferred. Let U(x) = log(x). Then, the integration shows that an issuer ran expect infinitely negative utility, clearly less than in the cascades case where proceeds are safe.

Proof of Theorem 8: Note that at P = 1/2, a necessary condition for a cascade to not have resulted, is that the number of L and H investors has never differed by more than one. (With two different signals, one signal "the other way" does not suffice to make this investor choose according to the private signal.) It is also a sufficient condition if the "tie-breaking" assumption is that an investor who is indifferent between purchasing and abstaining acts different from the action of the previous investor.[35] Thus, the probability of not ending up in a cascade is a simple martingale process with two final states, described by the Markov transition matrix.

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1-\theta & 0 & \theta & 0 & 0 \\ 0 & 1-\theta & 0 & \theta & 0 \\ 0 & 0 & 1-\theta & 0 & \theta \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{52}$$

The exact probabilities of being in a nonterminal state (state 2, 3, and 4) after n investors have participated can be read from the matrix after multiplying $TT \ldots T = T^n$ by the initial state vector (0, 0, 1, 0, 0). The middle row of the T^n matrix for $n \in \{2, 4, 6, 8, 10, \ldots\}$ is

$$\left[\sum_{i=1}^{n/2}(1-\theta)^{i+1}(2\theta)^{i-1}, 0, [2\theta(1-\theta)]^{n/2}, 0, \sum_{i=1}^{n/2}\theta^{i+1}[2(1-\theta)]^{i-1}, \right] \tag{53}$$

Multiplied by the initial state vector, the probability of not being in a cascade is $[2\theta(1-\theta)]^{n/2}$. The log of this probability, n/2 lim 2θ (1 –θ), clearly

[35] This prevents a cascade when there is one more H signal than there are L signals. With the tie-breaking assumption in the text, this would induce the next investor to purchase even if his/here signal is L.

approaches $-\infty$; thus, the probability of not having ended up in a cascade approaches 0 as $n \to \infty$ for any given θ. The integral (using (22))

$$\int_{\theta=0}^{1} [2\theta(1-\theta)]^{n/2} d\theta = 2^{n/2} \frac{1}{(n+1)\binom{n}{n/2}} \tag{54}$$

describes the ex-ante probability of the game not ending up in a cascade with n investors. It is now easy to show that this probability is both monotonously decreasing in n, and approaches 0 as $n \to \infty$.

Proof of Lemma 3: The lemma is proven by deriving the posterior probabil -ity of θ given the observed inside signals, finding the probability of k outside H signals as a function of θ and integrating out the nuisance parameter θ.

prob(k of n outside H signals | i and m inside H signals)

$$= \int_{\theta=0}^{1} \binom{n}{k} \theta^k (1-\theta)^{n-k} f^{\text{post}}(\theta; i, m) dp \tag{55}$$

where f^{post} denotes the posterior probability of θ given i in m H signals. This posterior probability was derived in (22) as

$$f^{\text{post}}(\theta; i, m) \equiv \frac{\binom{n}{k} \theta^i (1-\theta)^{m-i}}{\int_{\theta=0}^{1} \binom{n}{k} \theta^i (1-\theta)^{m-i} d\theta} = \binom{n}{k} (m+1)[\theta^i (1-\theta)^{m-i}] \tag{56}$$

where I have used the fact (from (22)) that

$$\int_{\theta=0}^{1} \theta^i (1-\theta)^{m-i} d\theta = \frac{1}{(m+1)\binom{n}{k}} \tag{57}$$

Substituting (56) into (55), and using (57) again with arguments (n + m) and i + k, *(9)* results.

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