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The Optimal Concentration of Creditors

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ABSTRACT

Our model assumes that creditors need to expend resources to collect on claims. Consequently, because diffuse creditors suffer from mutual free-riding (Holmstrom (1982)), they fare worse than concentrated creditors (e.g., a house bank). The model predicts that measures of debt concentration relate positively to creditors' (aggregate) debt collection expenditures and positively to management's chosen expenditures to resist paying. However, collection activity is purely redistributive, so social waste is larger when creditors are concentrated. If borrower quality is not known, the best firms choose the most concentrated creditors and pay higher expected yields.

COORDINATION FAILURE AMONG MULTIPLE CLAIMANTS, be they creditors or owners, is a subject well studied in the academic literature. Such coordination failure can lead to takeover failures (Grossman and Hart (1980)) or bank-runs (e.g., Diamond and Dybvig (1983), Morris and Shin (1999)), or can generally reduce the probability of successful renegotiation to a proposed reorganization plan when renegotiation requires simultaneous assent by many claimants (e.g., Berglöf, Roland, and van Thadden (2000)). In many of these models, the coordination failure aids the dispersed claimants—multiple claimants' cooperation is only purchased with an offer that is attractive enough for each and every claimant to choose to collaborate. Thus, coordination failure can suggest that dispersed creditors or owners can receive higher settlements than their hypothetically more concentrated but otherwise identical counterparts.

Zingales (1995) uses this insight to show that an entrepreneur may prefer to sell a firm to dispersed owners in an IPO, who in turn can later obtain a higher price for the shares from a potential acquirer than this entrepreneur could have obtained by herself. However, dispersion can also have more subtle effects, as modeled, for example, in Bolton and Scharfstein (1996).

Our view is that dispersed creditors are first and foremost unable to be proactive, given that they cannot easily coordinate. Thus, even though they are at an advantage when positive assent to a relief plan is required from every creditor, they are at a *disadvantage* when active opposition to management's relief plan is required. In this case, mutual free-riding incentives weaken the overall outcome for dispersed claimants. Good examples of how dispersion can facilitate bondholder expropriation are Gertner and Scharfstein (1991) and Bernardo

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and Talley (1996), in which management can use exchange offers to expropriate wealth from uncoordinated creditors.

In our model, creditors do not automatically receive their due but have to negotiate with the entrepreneur in the event of financial distress. (Note that in this paper, we use "management," "equity," and "entrepreneur," interchangeably.) Collection costs can stem from the costs of filing a claim, following up through the bankruptcy process, investigating the firm's true resources, communicating and negotiating with and pressuring management, hiring lawyers, bringing motions to the court, etc. Ex post, management will want to establish procedures, which will make it difficult for its creditors to prove and recover their claims. Management can also hire lawyers to outright oppose termination and/or absolute priority rule. Indeed, a casual inspection of bankruptcy records offers many examples of small creditors who did not find it in their interest to go through the legal hoops necessary to file, much less collect, relatively modest claims. Furthermore, civil liability claims are commonly dismissed by the bankruptcy court altogether.

Because creditors in our model must proactively seek to enforce their claims, lobbying and collection activities allow more proactive claimants to achieve better outcomes for themselves—even if these activities are purely redistributive when incurred. Our main focus is the role of creditor dispersion in determining the collective creditor actions. In his seminal paper, Holmstrom (1982) points out that team members have incentives to free-ride, because they bear all effort costs but enjoy only a fraction of their own marginal contribution to output. Team effort is socially desirable, so a socially good solution with little free-riding occurs when the number of team members is small. In contrast, in our model, lobbying and collection expenses are only redistributive and thus socially unproductive. Consequently, it is free-riding among creditors to reduce overall creditor "team collection effort" that is socially desirable.

Team free-riding not only reduces deadweight rent-seeking, but also compromises creditors' collection abilities—a given number of creditors determines both the ex post distribution of cash flows in distress and the socially inefficient costs of claim collection. Our model posits that, given a fixed level of debt, a distressed firm with a million uncoordinated small creditors is less likely to be forced to pay its obligations than a firm with one creditor or a firm with creditors that have a coordinating organ, for example, a trustee for financial bonds. The strongest application of our model applies to idiosyncratic, small credit such as small trade credit (Biais and Gollier (1997), Petersen and Rajan (1997)). To a lesser extent, our model could also apply to highly dispersed public debt that is not fully coordinated (though in formal bankruptcy, public debt often becomes more coordinated through the appointed creditor committee) or even to civil nonclass claims brought by product customers and other stakeholders.

Both deadweight lobbying and collection are lower when there are more creditors on the team, which allows us to derive an ex ante optimal concentration of creditors. An entrepreneur who chooses a large number of creditors ex ante assures herself of better bargaining ability against creditors in case of financial distress ex post. Although this minimizes deadweight lobbying costs, we show that in equilibrium, the ex post ability to expropriate creditors costs the entrepreneur a higher interest rate when raising the debt ex ante. In contrast, an entrepreneur who chooses a single creditor ex ante will be forced to extensively (and expensively) negotiate with this creditor in the event of financial distress, and this single creditor will likely be relatively more successful in enforcing her claim. Although this maximizes deadweight lobbying costs, in equilibrium, such an entrepreneur will also enjoy a lower interest rate when raising the debt ex ante. Putting this all together, the model shows that measures of debt dispersion (the number of creditors) correlate positively with the entrepreneur's retention of the firm in bankruptcy (fewer creditors $\Rightarrow$ worse outcome for management in financial distress), and correlate negatively with the in-equilibrium claims collection costs (fewer creditors $\Rightarrow$ more collection efforts, costs, and waste).

In this, our simplest framework, the only deadweight cost of credit is the in-equilibrium spending on conflict. Thus, by itself, this "number of creditors" tradeoff in financial distress—in which more creditors in financial distress have lesser ability to wrestle the firm from management—has an ex ante first-best outcome, in which the number of creditors is infinitely large (dispersed). Zero deadweight collection costs would be incurred in financial distress, and perfectly dispersed creditors receive proper ex ante compensation (higher interest rates) for their anticipated perfect ex post expropriation.

However, this model is too naïve. Management that commits itself to fully expropriate creditors in financial distress would likely suffer ex ante from anticipated agency and signaling costs. For example, empowering creditors in financial distress may induce management to work harder to avoid it. Or, if only the manager knows that the firm is of high quality, choosing fewer creditors can signal higher confidence that the firm will not go bankrupt and incur ex post collection waste. As a result, many firms will find it in their interest to choose a small number of creditors, thereby trading off in-bankruptcy collection deadweight costs against prebankruptcy deadweight agency or signaling costs.

Our paper develops a signaling model in more detail, and yields an especially interesting implication. At times, we identify the most concentrated debt, that is, a single creditor, as a house bank. Although banks doubtlessly perform other functions, they do tend to assume debt in a more concentrated fashion than public or trade creditors. Hence, lack of dispersion is a good characterization of *one* feature of a house bank. In the signaling version of our paper, when concentration (the most efficient signal) is exhausted, the intrinsically highest-quality creditors choose to pay excess rents to banks to ensure separation. Thus, the signaling variant of our model can predict that bank debt earns a higher expected (not just promised!) yield than public debt.

Creditor concentration is best seen as one effect among others. There are many substitute and complement mechanisms to control agency/signaling concerns. These mechanisms can range from different types of credit arrangements, to debt contract features (seniority, timing, etc.), other creditor characteristics, shareholder concentration, choice of shareholder types, formal and informal corporate governance mechanisms, different formal and informal contracting schemes, type of asset choice, formal and informal disclosure mechanisms, bonding mechanisms, financial constraints (Aghion and Bolton (1992), Pagano and Röell (1998)), and so on. Moreover, the effects of dispersion have also received theoretical attention in other contexts. For example, there are studies that focus on the roles of concentration among shareholders (e.g., Pagano and Röell (1998) and Burkart, Gromb, and Panunzi (1997)), and of concentration among bank relationships (e.g., Petersen and Rajan (1995), Greenbaum, Kanatas, and Venezia (1989)).

We shall now proceed as follows: Section I describes the conflict game played between N creditors and a management team that is in financial distress. This section solves the dynamic optimization from the perspective of management. The result of this section is that there is a monotonically positive relation between debt concentration and in-equilibrium waste. Section II grafts onto this base model a signaling scenario in which higher-quality managers signal their confidence by choosing fewer creditors. We also show that after concentration signaling is exhausted (i.e., the firm has only one creditor, a "house" bank), entrepreneurs must resort to yield signaling. The section also outlines variant models (agency, continuation, marketing) that similarly lead to an interior optimal creditor concentration. Section III discusses our implications, contrasts them with Bolton and Scharfstein (1996), and describes some evidence that is relevant to our argument. Section IV concludes.

I. The Cost of a Given Number of Creditors

A. Assumptions

We begin with a simple "creditor concentration" model. Our primary intent is to derive the in-equilibrium collection waste as a function of the number of creditors. Our web site contains a table of symbols and a numerical example that illustrates the model.

In stage 1 of the game, the entrepreneur owns in-place assets worth V_{Old}. To adopt a project that provides 0 with probability π and V_{New} with probability $(1-\pi)$, the entrepreneur must raise risky external financing I ($\Rightarrow I > V_{\text{Old}}$). We also assume that the project is intrinsically worthwhile, that is, $(1-\pi)\cdot V_{\text{New}} > I$. This financing can be in the form of debt raised from an endogenously determined number of creditors, N.

If the project later succeeds, creditors are paid and there is no issue of concern for us. However, if the project later fails, the firm still owns its project in place, V_{Old}. Although creditors presumably are due to receive what the absolute priority rule (APR) promises them, collection costs (such as courts, lawyers, and "legal maneuvers") will allow management to reduce the claims of creditors in financial distress by up to X. The idea that financial distress is not free or ex ante completely contracted away (Schwartz and Watson (2000)), and, moreover, that part of the function of lawyers is to influence courts and obtain rents, is reasonably realistic (Glaeser and Shleifer (2001)). However, the specific details of court and collection conflict are extremely complex, and thus our paper relies on a flexible, parameterized "black box." To "fight" for X, both creditors and management can devote effort. The exact allocation of X to management (equity) is determined by the *contest success function*

$$\alpha(L_e, L_d) = \left(\frac{L_d}{N}\right) \times (1 - L_e), \tag{1}$$

where α is the fraction of the contested amount X that debt recovers, if creditors spend $L_d \in [0, 1]$ in aggregate on debt collection and management spends $L_e \in [0, 1]$ on payment avoidance. (We shall call these activities "lobbying," to reflect the fact that they involve a broad range of activities). Define L_d to be the aggregate of all creditors' efforts, $L_d = \sum_{i=1}^{N} l_i$, where N is the number of creditors, and $l_i \in [0, 1]$ is each individual creditor's collection effort. When $\alpha(L_e, L_d) < 1$, APR is partly violated in favor of equity. Thus, α can be thought of as the probability of holding onto APR, the fraction of the disputed amount X that is allotted to debt in financial distress, or both. The combination of a parametrized X with a contest success function can cover a wide range of possible allocation scenarios. Moreover, the success probability is asymmetric: If equity expends the maximum amount $L_e = 1$, APR is violated with probability 1, irrespective of L_d; however, when debt expends the maximum amount, $L_d = 1$ (which requires all creditors to expend the maximum amount, that is, $l_i = 1, i = 1, \ldots, N$), APR may still be violated as long as $L_e > 0$.

Both equity and creditors are assumed to pay for their own lobbying expenses. An amount x of lobbying (collection) effort costs $c_d \cdot x^2$ for creditors and $c_e \cdot x^2$ for equity. As required by law, management must reimburse creditors in the same class equally. Thus, creditor dispersion will play a role through a variation of the team problem identified in Holmstrom (1982): Each individual creditor must absorb the full cost, but will benefit only from $1/N$ of the results, of his collection efforts.

The reader should not take the contest success function in (1) too literally. An earlier version of our paper entertained a symmetric contest success function ($\alpha(L_e, L_d) = L_e^{\lambda}/(L_d^{\lambda} + L_e^{\lambda})$) and came up with virtually identical results. Similarly, other variations of the model show that it is robust to other symmetric contest success functions, to reimbursement of creditor expenses by the firm, and to redundant efforts among creditors. The critical ingredient for our analysis is, given equal aggregate lobbying expense, creditors recover less when they are diffuse rather than concentrated.

Neither management nor creditors can commit to not act opportunistically in the event of financial distress. Capital markets are perfectly competitive, and the firm is acting strategically. All participants are risk-neutral optimizers, and there is no asymmetric information in the financial distress game. (Any ex post asymmetric information is assumed to be fully captured by the known contest success function. We will later introduce an ex ante signaling component.)

B. The Financial Distress Game

B.1. The Creditors' Problem

First consider the problem of a single creditor among N creditors in the event that the firm enters financial distress. Under full APR, he receives V_{Old}/N because $V_{\text{New}} = 0$ and this creditor has first claim to the firm's remaining assets, which are assumed to be insufficient to cover the required investment. Under maximum violation, the creditor receives $V_{\text{Old}}/N - X/N$. He benefits from both his own lobbying, denoted l_d, and the lobbying of other creditors, denoted l_o. Thus, one single creditor maximizes with respect to l_d

$$\alpha(L_e, l_d + l_o) \cdot \left(\frac{V_{\text{Old}}}{N}\right) + [1 - \alpha(L_e, l_d + l_o)] \cdot \left(\frac{V_{\text{Old}}}{N} - \frac{X}{N}\right) - c_d \cdot l_d^2$$
$$\equiv \frac{V_{\text{Old}} - [1 - \alpha(L_e, l_d + l_o)] \cdot X}{N} - c_d \cdot l_d^2, \tag{2}$$

where $\alpha(L_e, l_d + l_o) = (\frac{l_d + l_o}{N}) \cdot (1 - L_e)$.

This creditor's first-order condition is

$$(1 - L_e) \cdot \left(\frac{X}{N^2}\right) = 2 \cdot c_d \cdot l_d. \tag{3}$$

Note that all creditors are equal. Thus, a minimal equilibrium symmetry condition is $l_o^{\star} = (N-1) \cdot l_d^{\star}$ and aggregate creditor collection effort is $L_d^{\star} \equiv N \cdot l_d^{\star}$.

B.2. The Management's Problem

Unlike creditors, management does not suffer from a free-riding problem. Under APR, management receives 0. The entrepreneur maximizes with respect to L_e, given financial distress (i.e., $V_{\text{Old}} - I + (1-\pi) \cdot V_{\text{New}}$ are sunk costs, and we are only investigating the bankruptcy payoffs, which occurs with probability π):

$$\alpha(L_e, L_d) \cdot 0 + [1 - \alpha(L_e, L_d)] \cdot X - c_e \cdot L_e^2. \tag{4}$$

Her first-order condition is

$$\frac{X \cdot L_d}{N} = 2 \cdot c_e \cdot L_e. \tag{5}$$

B.3. The Joint Solution

Solving the two first-order conditions, we find that the in-distress equilibrium choices are

$$L_e^{\star} = \frac{X^2}{4 \cdot c_e \cdot c_d \cdot N^2 + X^2}; \quad L_d^{\star} = \frac{2 \cdot c_e \cdot X \cdot N}{4 \cdot c_e \cdot c_d \cdot N^2 + X^2}. \tag{6}$$

In equilibrium,

$$\alpha^{\star}(L_d^{\star}, L_e^{\star}) = \frac{8 \cdot c_e^2 \cdot c_d \cdot X \cdot N^2}{\left(4 \cdot c_e \cdot c_d \cdot N^2 + X^2\right)^2}, \tag{7}$$

which is decreasing in N. Therefore, APR violations in favor of equity are more likely as the number of creditors increases. Deadweight waste W is

$$\begin{aligned} W^{\star}(N) &\equiv c_d \cdot \sum_{i=1}^{N} l_i^{\star 2} + c_e \cdot L_e^{\star 2} \\ &= c_d \cdot \left(\frac{L_d^{\star 2}}{N}\right) + c_e \cdot L_e^{\star 2} \\ &= \frac{c_e \cdot X^4 + 4 \cdot c_d \cdot c_e^2 \cdot X^2 \cdot N}{\left(4 \cdot c_e \cdot c_d \cdot N^2 + X^2\right)^2}. \end{aligned} \tag{8}$$

Therefore,

$$\frac{\partial W^{\star}}{\partial N} = \frac{4 \cdot c_d \cdot c_e^2 \cdot X^2 \cdot \left[(1 - 4 \cdot N) \cdot X^2 - 12 \cdot c_d \cdot c_e \cdot N^2\right]}{\left(4 \cdot c_d \cdot c_e \cdot N^2 + X^2\right)^3} < 0 \tag{9}$$

for $N \geq 1$. The waste in this conflict game is smaller when there is more asymmetry in the strength between the debt and equity contestants, that is, as N increases. Here, creditors are weakest when their number is high. As $N \to \infty$, we get first-best: $W^{\star}(N) \to 0$.[1] Note that given financial distress, if X is large, such creditors might not receive very much, if anything, at all—creditors would effectively become more of a residual claimant than equity!

Interestingly, the model implies that investors have an incentive to concentrate debt in financial distress, even though this is socially wasteful. Although vulture investors do succeed on occasion, much empirical and anecdotal evidence suggests that concentrating debt may often be more costly than resisting management's expropriation—dispersed creditors may be difficult to locate and buy out. Of course, our stylized model really requires only some monotonic mapping of ex ante concentration into expected (not uniformly actual) ex post concentration in financial distress.

C. The Ex Ante Price of Debt

As in all models of competitive credit provision, in equilibrium the entrepreneur internalizes ex post inefficiencies. Thus, without any other considerations that could induce the entrepreneur into restricting the number of creditors, having as many creditors as possible maximizes the entrepreneur's firm value.

[1] In the model, N is a control mechanism that translates into an effective aggregate collection strength. If c_d were a choice variable, issuers could choose specific creditors with high c_d, instead of more creditors.

To obtain credit of I, which is assumed to be necessary to finance the project, an entrepreneur has to offer debt face value FV that satisfies

$$I = \pi \cdot \left[\alpha^\star \cdot V_{\text{Old}} + (1-\alpha^\star)\cdot(V_{\text{Old}} - X) - c_d \cdot \left(\frac{L_d^{\star 2}}{N}\right)\right] + (1-\pi)\cdot FV^\star. \quad (10)$$

The first term is the expected payoff to creditors in bankruptcy, and the second term is the promised payoff to creditors outside of bankruptcy. In bankruptcy, the claimants can recover V_{Old}, the assets in place (because the value of the new project V_{New} is worthless), net of both their in-equilibrium reduction due to managerial ex post opportunism and their own fighting costs. (We are also assuming the side conditions that $FV^\star \leq V_{\text{Old}} + V_{\text{New}}$, so that the firm is able to pay off the debt in the nonbankrupt state, and only up to 100% of the firm value can be sold.) Solving for $FV^\star$, the in-equilibrium solution for the face value of debt, we obtain

$$FV^\star = \frac{I - \pi \cdot \left[V_{\text{Old}} - (1-\alpha^\star)\cdot X - \frac{c_d \cdot L_d^{\star 2}}{N}\right]}{1-\pi}. \quad (11)$$

D. The Entrepreneur's Optimal Choice

The entrepreneur chooses the number of creditors, N, to maximize the equity's value (E) ex ante, that is,

$$E \equiv \pi \cdot \left[\alpha^\star \cdot 0 + (1-\alpha^\star)\cdot X - c_e \cdot L_e^{\star 2}\right] + (1-\pi)\cdot\left(V_{\text{Old}} + V_{\text{New}} - FV^\star\right). \quad (12)$$

In the event of financial distress, $E + I = V_{\text{Old}} - W^\star(N)$; on the other hand, if the project is successful, $E + I = V_{\text{Old}} + V_{\text{New}}$. The first-order condition of E with respect to N is a long algebraic expression, and it is easier to derive the sign of the comparative statics from the insight that entrepreneurs internalize all waste in a competitive capital market, that is, from equation (8):

$$E^\star = V_{\text{Old}} + \left[-I + (1-\pi)\cdot V_{\text{New}} - \pi \cdot W^\star(N)\right]. \quad (13)$$

The main result of Section I is that as $N \to \infty$, $E^\star$ converges to the first-best, $V_{\text{Old}} - I + (1-\pi)\cdot V_{\text{New}}$. Absent other considerations, with enough creditors, there is no wasteful bargaining expense given financial distress. The interesting comparative statics are:

PROPOSITION 1: *The entrepreneur's objective, the ex ante equity value $E^\star$:*

1. *Increases in N.*
2. *Increases in c_d.*
3. *Decreases in c_e for* $c_e > \frac{X^2}{4 \cdot c_d \cdot N^{\star 2}}$.

4. Decreases in X.

(All proofs are in the Appendix.)

II. Creditor Concentration and Financial Distress Conflict in a Capital Structure Model: A Signaling Model

Like other capital structure theories, our model merely identifies the deadweight costs of bankruptcy as the waste of socially inefficient claims collection, and it relates this specific cost of debt to the number of creditors. There is no drawback to the use of multiple creditors. Creditors are maximally expropriated given financial distress, but compensated ex ante for being ex post expropriated. To obtain an equilibrium in which some firms are willing to incur these financial distress costs in equilibrium, there must also be some advantages to the otherwise disadvantageous debt choice to a finite number of creditors. This section discusses different mechanisms that yield this outcome. We start with a signaling model similar to Ross (1977) with two different kinds of firms: good, high-quality firms (G) with a lower probability of bankruptcy (π_G), and bad, low-quality firms (B) with a higher probability of bankruptcy (π_B).

Signaling works if there is a differentially higher cost for low-quality firms to send the signal. To deter imitation, high-quality firms therefore like ex ante lower expected corporate payoffs to themselves if they enter financial distress. These payoffs are lower if (1) litigation waste given financial distress is higher, and (2) the entrepreneur's relative (postlitigation) share of the firm is lower. Having fewer creditors accomplishes both objectives. Thus, signaling through creditor concentration is likely to be a relatively efficient separation mechanism.

We intentionally set the problem up so that the signaling equilibrium is easy to construct. Because signaling equilibria are well understood, we shall be casual with regard to formal equilibrium definitions, and focus on the Pareto-dominant signaling equilibrium. For the sake of brevity, we shall also treat integer constraints on the number of creditors rather casually.

In a separating equilibrium, the low-quality entrepreneur prefers revelation to imitation. Revelation provides the low-quality entrepreneur with her full-information first-best proceeds of

$$V_{\text{Old}} - I + (1 - \pi_B) \cdot V_{\text{New}}. \tag{14}$$

To achieve this, the entrepreneur must offer highly dispersed (public) debt. Imitation would provide a potentially cheating entrepreneur with

$$\pi_B \cdot \left[\alpha^\star \cdot 0 + (1 - \alpha^\star) \cdot X - c_e \cdot L_e^{\star 2}\right] + (1 - \pi_B) \cdot (V_{\text{New}} + V_{\text{Old}} - \mathsf{FV}_G), \tag{15}$$

where FV_G, given in equation (11), indicates that an out-of-equilibrium imitating low-quality firm can receive the high-quality firm's price of credit (based on the good firm's distress probability π_G, not the imitator's true distress probability π_B). A reasonable signaling equilibrium emerges in which the difference in profits between a cheating and a truthful low-quality firm, that is, the gain from imitation (GFI), is

$$\begin{aligned} \mathsf{GFI} &\equiv \pi_B \cdot \left[(1-\alpha^\star)\cdot X - c_e \cdot L_e^{\star 2}\right] + (1-\pi_B)\cdot(V_{\text{New}} + V_{\text{Old}} - \mathsf{FV}_G) \\ &\quad - [V_{\text{Old}} - I + (1-\pi_B)\cdot V_{\text{New}}] \\ &= \left(\frac{\pi_G - \pi_B}{1-\pi_G}\right)\cdot\left\{V_{\text{Old}} - X\cdot\left[1 - \left(\frac{L_d}{N}\right)\cdot(1-L_e)\right]\right\} - \pi_B \cdot c_e \cdot L_e^2 \\ &\quad - \pi_G \cdot \left(\frac{1-\pi_B}{1-\pi_G}\right)\cdot c_d \cdot \left(\frac{L_d^2}{N}\right) + \left(\frac{\pi_B - \pi_G}{1-\pi_G}\right)\cdot I, \end{aligned} \tag{16}$$

which is just below zero. Note that $\partial\mathsf{GFI}/\partial N$ is a complex expression, but the Appendix proves that it is always positive. Even without signaling, a larger number of creditors is preferred. Here, the low-quality firm's outcome does not depend on N if it confesses its identity. (The optimal N for revealing bad firms is infinity.) Thus, a potential low-quality imitator has less to gain from imitation when there are fewer creditors. This forces the relationship between $N^\star$ and the exogenous variables:

PROPOSITION 2: *The optimal number of creditors, $N^\star$, is:*

1. *Independent of the new opportunities V_{New}.*
2. *Increasing in the pre-existing firm value V_{Old}.*
3. *Decreasing in the cost of investment I.*
4. *Non-decreasing in the disputable amount X.*
5. *Increasing in π_G and decreasing in π_B.*
6. *Increasing in c_e and decreasing in c_d.*

The promised interest rate, $FV^\star/I - 1$, also increases in $N^\star$.

These comparative statics should be unsurprising to connoisseurs of signaling models. They are determined by the self-punishing mechanisms necessary to deter low-quality imitation. Part (1) holds because debt has a fixed claim on the project's net present value (NPV), so the profitability of the project is irrelevant. When V_{Old} is high, high-quality firms need to raise little debt. Thus, imitation is less attractive and $N^\star$ can be larger. Similarly, when I is very low, firms need to sell little debt. Imitation is again relatively less attractive and $N^\star$ can be larger. Moreover, as the difference $\pi_G - \pi_B$ decreases, there is less need for signaling, which implies that $N^\star$ is higher. Finally, a higher c_d implies that the number of creditors that is necessary to incur the same cost of signaling, decreases. Similarly, as c_e increases, creditors fight more and signaling is more costly, so a higher number of creditors is sufficient for separation.

When separation by choice of creditors is insufficient, entrepreneurs may have to underprice their debt, that is, pay an extra-high interest rate. Interestingly, this has a direct implication: Even though the *required* yields on highly concentrated bank debt can be lower than those on dispersed public debts (to allow for the superior ability of banks to defend their APR), banks earn excess rents (positive expected returns) from the loans they extend. This is not to purchase bank services, but "money burning" to ensure separation.[2]

PROPOSITION 3: *When firms can use either yields or creditor concentration for signaling, two choices emerge in equilibrium:*

1. *The firm offers fairly priced debt to a creditor base, concentrated or unconcentrated.*
2. *The firm offers good-deal debt to a single concentrated creditor (bank debt).*

In particular, the firm will not offer good-deal debt to public creditors.

The intuition is that signaling with creditor concentration is the more efficient signal: If firms pay a higher yield to signal their quality, they also pay the cost of signaling when they are solvent. Instead, creditor concentration inflicts pain when the firm goes bankrupt, which is more likely to happen to a low-quality firm. Only when the signal is exhausted, that is, $N = 1$—which we interpret as bank debt—would a high-quality firm pay a higher credit price to separate. High bank interest rates do not arise from credit-rationing or poor quality or the purchase of monitoring services, but instead from high quality, high uncertainty, and the need to separate from other firms! Naturally, in real life, banks probably both monitor and permit signaling.

A figure may help visualize the model. Using the numerical values from an example on our web site, Figure 1 shows the two regions for which it is optimal to signal with either N only, or with N and the debt yield r. For $\pi_B > 0.6$, at least the bad firm's (and possibly also the good firm's) project has a negative NPV, so a signaling equilibrium makes no sense. The upward sloping curve solves π_G as a function of π_B in equation (16), where $N^\star = 1$. The optimal number of creditors $N^\star$ becomes larger as π_B and π_G converge. When $r^\star > 0$, the debt yield decreases as both probabilities of default converge.

Signaling is not the only mechanism that produces interior credit-concentration choices. For example, in an agency model, if management is better kept in check by fewer creditors, then concentrated creditors have an incentive to invest more in monitoring activities even if the firm is not in distress. Management with more concentrated debt would not be ex ante better, but only become ex post better. Similarly, more concentrated creditors might make better continuation choices *after* the firm enters financial distress. Finally, it may be more expensive to market debt claims to multiple creditors than to just a few creditors.

[2] Necessarily, we would expect competitive banks to compete these rents away (e.g., through higher fixed costs). More importantly, we would expect a signaling equilibrium to allow some recovery of signaling costs: If good firms could recover signaling costs in the far-away future, after the bad firms have gone bankrupt and are not capable of recovering the cost, the signaling equilibrium can still remain.

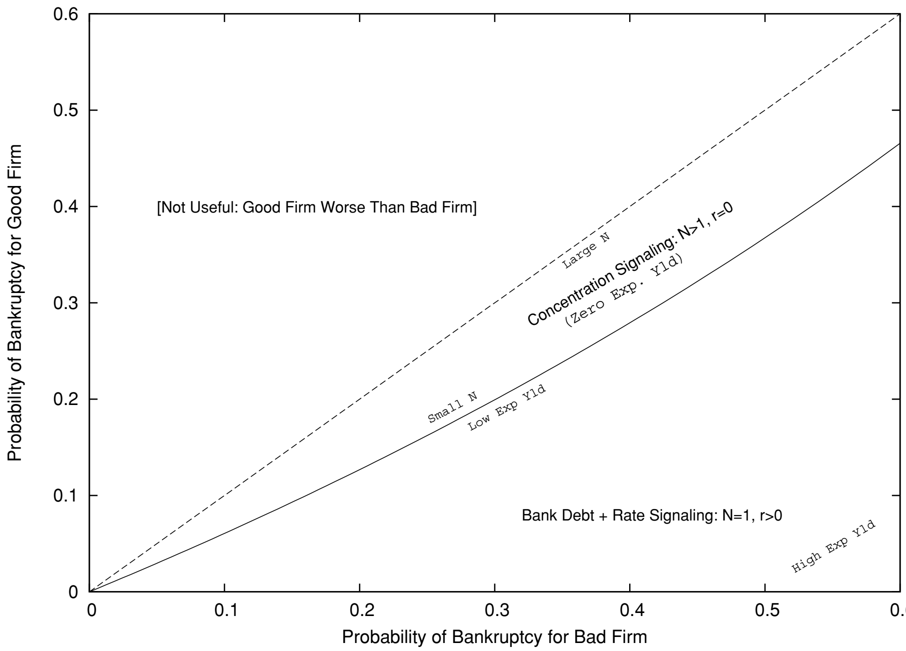

Figure 1. Signaling regions in equilibrium. Figure 1 plots the regions in which signaling by creditor concentration alone signals creditor quality and in which signaling requires not only the ultimate concentration ($N = 1$, i.e., bank debt), but also an expected interest above zero. The parameter values for this figure are as in our numerical examples on our web site: $V_{\text{Old}} = \$80$, $V_{\text{New}} = \$250$, $X = \$80$, $I = \$100$, $c_e = c_d = \$1$, and $\lambda = 1.5$. A positive interest rate is required when $\pi_g < 2.33\pi_b/4{,}010 - 1{,}680\pi_b$. If $(1 - \pi_B) \cdot V_{\text{New}} < I$, that is, when $\pi_B > 0.6$, the new project is not a positive-NPV project.

III. Implications

A. Bolton and Scharfstein (1996)

The paper most interested in the optimal concentration of creditors, and thus most similar in its objectives to our own paper, is the classic by Bolton and Scharfstein (1996). It is fair to state that their model is considerably richer than our own. In our model, the influence of creditor dispersion is more unambiguous and less subtle than in Bolton and Scharfstein (1996). By relying on the signaling model, there is no “strategic default” issue “in which the firm defaults because managers want to divert cash to themselves” (Bolton and Scharfstein (1996 p. 2)). Our model works even if management is already sufficiently intrinsically motivated to avoid financial distress and bankruptcy. In one sense, their outside option is a modeling device designed to obtain the same predictions as our contest success function: Two creditors receive less (not more) than one creditor. Bolton and Scharfstein (1996) identify the source of

Table I
Comparison of Implications

Relationship	BS 1996	BW 2004
Creditor concentration in liquidations	Helps creditors	Helps creditors
Creditor concentration in reorganizations	Undefined	Helps creditors more recovery
Concentration versus corporate termination	Less frequent	Undefined
Concentration versus promised interest rate	High or undefined	Low
Bank debt versus expected interest rate	Zero	Zero (or positive)
Concentration versus holdout (time)	Negative	Undefined
Concentration versus creditor lobbying expenses	Undefined	Higher
Concentration versus lobbying expenses of firm	Undefined	Higher
Concentration versus total lobbying expenses	Undefined	Higher
Concentration versus inefficient outcome	Ambiguous	Higher (except with signaling)
Concentration incentives for creditors ex post	Negative or ambiguous	Positive
Lawyer expenses	Maybe uncover value	Seek rents
Public debt versus known firm quality	Positive	Positive
Public debt versus unknown firm quality	Undefined	Negative

Note: BS 1996 is Bolton and Scharfstein (1996). BW 2004 is our own paper. Public debt is assumed to be equivalent to highly dispersed debt. Bank debt is assumed to be equivalent to highly concentrated debt.

creditor concentration strength. In contrast, we do not. One could therefore test whether creditor concentration still helps in situations in which the firm reorganizes (which does not occur in Bolton and Scharfstein) and there is no outside option available to exert pressure on management that forces it to treat a single creditor better than multiple creditors. The main intuition and some different empirical implications of our approach are summarized in Table I.

B. Empirical Implications

This section provides a set of ceteris paribus predictions that can be empirically examined.

Collection Activity $\Longrightarrow$ Outcome. The model's principal assumption is that more collection efforts translate into better precollection cost outcomes. However, future tests cannot merely rely on bankruptcy records, because creditors that spend zero effort on collection may not appear in the bankruptcy records. A better test would seek to identify claimants *pre*-bankruptcy, not *in*-bankruptcy.

The remaining implications derive from the team free-riding incentive:

Dispersion $\Longrightarrow$ Less Creditor Collection Activity. This follows immediately from the "team free-riding" ingredient, namely that dispersed creditors spend less on collection than concentrated creditors. Naturally, although the model assumes equal-size creditors, we would expect the actual claim sizes ("effective dispersion") to be more related to collection activity than the simple number of creditors. A test might relate a company's prebankruptcy estimates of the number of (small) claimants to the creditors' ultimate aggregate lobbying and collection expenses (e.g., their aggregate legal representation, both quality and quantity).

Dispersion $\Longrightarrow$ Less Management Resistance Activity. This implication follows from the following conflict: As creditors spend more on collection activities, management responds by spending more to resist. We know of no empirical study testing this implication.

Dispersion $\Longrightarrow$ Worse Outcome. Putting together both the preceding team free-riding implication and the preceding collection-outcome assumption suggests that dispersed creditors should fare worse than concentrated creditors. A test might relate the prebankruptcy *estimated* number of creditors claimants to the *actual* number of claims (and their collection success) that are eventually filed. Interestingly, this implication is opposite to the implication of some papers that emphasize the Grossman and Hart (1980) effect.

Concentration $\Longrightarrow$ More Social Waste. An implication of our model is that the total conflict waste is negatively related to creditor concentration. Total waste is the sum of expenses by both management (equity) and creditors. Collection activity is a purely redistributive activity in our model, but an empirical test would benefit from stripping firm value-enhancing activities from lobbying expenses. There are good alternatives that suggest the opposite implication: Creditor coalitions could be formed to avoid a creditor run on the firm and thereby enhance firm value. Like the relation between concentration and outcome, contrasting implications make this an interesting empirical test.

A second set of empirical implications of our model derives from the agency/signaling model, which embeds creditor concentration in an ex ante perspective. Better (performing) firms can accept more in-bankruptcy waste, because bankruptcy is less likely—but only to the extent that firm quality is otherwise unknown. Therefore, careful control of observable measures of firm riskiness (profitability, size, and credit quality) is important. The firm signaling perspective adds richer implications to those just described.

Unknown Firm Quality $\Longrightarrow$ More Creditor Concentration. In the signaling model, the issuer knows quality ex ante. In an agency model analog, the higher quality would arise endogenously via the firm's acceptance of debt with a "higher punishment" feature (fewer creditors). The most stark form of creditor concentration relies on only a single credit provider—typically a house bank—chosen by the borrowers needing to signal highest quality.

Known Firm Quality $\Longrightarrow$ Less Creditor Concentration. Naturally, firms that require sending no signal or that require no managerial discipline imposed by the financial markets can avoid the costs associated with higher creditor concentration.

The alternative nonsignaling mechanisms sketched above can also predict that firms with fewer creditors suffer from fewer agency conflicts, fewer bad termination choices, and lower marketing costs than firms with more creditors. Only one implication arises purely in a signaling context:

Extreme Creditor Concentration $\Longrightarrow$ Higher Expected Debt Yield. Unlike in Bolton and Scharfstein (1996), creditor concentration and promised yields are strategic substitutes for firms with unobservable quality in our model. Because signaling with creditor concentration is cheaper than signaling with high-yield debt, our model suggests that bank (concentrated) debt is associated with lower yields than public debt. Denis and Mihov (2003) report that yields are higher for public debt (8.24%) than for bank debt (7.14%).

IV. Conclusion

Our model applies best to the contrast between small trade and accounts payables creditors and organized bank credit. Our interest is akin to that of Bolton and Scharfstein (1996), although our approach can offer an intuition and set of implications that is both similar and different. Together with relatively easy measurability of the models' parameters and proxies, we hope that this will appeal to empiricists.

Appendix: Proofs

Proof of Proposition 1: Equity Value in Equilibrium: The first statement is straightforward since $W^\star(N)$ is decreasing in N. The second statement follows because $W^\star(N)$ is decreasing in c_d and increasing in c_e. To show that $E^\star$ is decreasing in c_e for $c_e > \frac{X^2}{4 \cdot c_d \cdot N^{\star 2}}$, we can express $W^\star(N)$ as

$$W^\star = c_d\left(\frac{L_d^{\star 2}}{N}\right) + c_e \cdot \frac{a \cdot d}{(b \cdot c_e + d)^2}, \tag{A1}$$

where $a \equiv 4 \cdot N^{\star 2}$, $b \equiv 4 \cdot c_d \cdot N^{\star 2}$, and $d \equiv X^2$. The first term in the previous expression is decreasing in c_e. Defining

$$Z^\star \equiv c_e \cdot \frac{a \cdot d}{(b \cdot c_e + d)^2}, \tag{A2}$$

then

$$\frac{\partial Z^*}{\partial c_e} = \frac{a \cdot d \cdot (d - b \cdot c_e)}{(b \cdot c_e + d)^3}, \tag{A3}$$

which is negative if $c_e > \frac{d}{b}$. Finally, to show that $E^\star$ is decreasing in X, note that we can express $W^\star(N)$ as

$$W^\star = c_e \cdot \frac{y+a}{(y+b)^2}. \tag{A4}$$

Proof of Proposition 2: Comparative Statics: GFI is monotonically increasing in N; we can express GFI from equation (16) as

$$\mathsf{GFI} = \left(\frac{\pi_G - \pi_B}{1-\pi_G}\right) \cdot (V_{\text{Old}} - I - X) + \left(\frac{\pi_G - \pi_B}{1-\pi_G}\right) \cdot \alpha(L_e, L_d) \cdot X$$
$$- \pi_B \cdot c_e \cdot L_e^2 - \pi_G \cdot c_d \cdot \left(\frac{1-\pi_B}{1-\pi_G}\right) \cdot \left(\frac{L_d^2}{N}\right). \tag{A5}$$

This expression is increasing in N, because $\alpha(L_e, L_d)$ is decreasing in N, L_e and L_d are decreasing in N, and $(\pi_G - \pi_B)/(1-\pi_G) < 0$.

The comparative statics are determined by the incentive compatibility constraint to prevent low-quality firms' imitation. The sign of the implicit differentiated $(\partial N^\star/\partial\cdot)$ is the opposite to the sign of $(\partial\mathsf{GFI}/\partial\cdot)$. (Using the implicit function theorem, $\partial N^\star/\partial\cdot = -\partial\mathsf{GFI}/\partial\cdot/\partial\mathsf{GFI}/\partial N$. Consequently, $\text{sign}(\partial N^\star/\partial\cdot) = -\text{sign}(\partial\mathsf{GFI}/\partial\cdot)$.)

It is straightforward to show that V_{New} is irrelevant, because

$$\frac{\partial\mathsf{GFI}}{\partial V_{\text{New}}} = 0 \Rightarrow \frac{\partial N^\star}{\partial V_{\text{New}}} = 0. \tag{A6}$$

Similarly,

$$\frac{\partial\mathsf{GFI}}{\partial V_{\text{Old}}} = \frac{\pi_G - \pi_B}{1-\pi_G} < 0 \Rightarrow \frac{\partial N^\star}{\partial V_{\text{Old}}} > 0. \tag{A7}$$

Thus, $N^\star$ increases when firms have more assets in place (V_{Old}). Statement three of the proposition holds because,

$$\frac{\partial\mathsf{GFI}}{\partial I} = \frac{\pi_B - \pi_G}{1-\pi_G} > 0 \Rightarrow \frac{\partial N^\star}{\partial I} < 0. \tag{A8}$$

Now note that, when $X = 0$, then $\mathsf{GFI} = \frac{\pi_G - \pi_B}{1-\pi_G}(V_{Old} - I) > 0$. Moreover, for all $X > 0$, in equilibrium $\mathsf{GFI}^\star(N^\star(X), X) = 0$. Suppose that in equilibrium, $\frac{\partial\mathsf{GFI}^\star}{\partial X} > 0$. Therefore, because GFI is continuous in X, and $\mathsf{GFI}(N^\star(X), 0) > 0$, there must exist $X_o < X$ such that $\mathsf{GFI}^\star(N^\star(X), X_o) = 0$. However, $\frac{\partial\mathsf{GFI}^\star}{\partial X} > 0$ and $\frac{\partial\mathsf{GFI}^\star}{\partial N} > 0$ imply $\frac{\partial N^\star}{\partial X} < 0$, and therefore $N^\star(X_o) > N^\star(X) \Rightarrow 0 = \mathsf{GFI}^\star(N^\star(X), X_o) < \mathsf{GFI}^\star(N^\star(X_o), X_o)$, which is absurd because $(N^\star(X_o), X_o)$ is an equilibrium. Therefore, it cannot be that $\frac{\partial\mathsf{GFI}^\star}{\partial X} > 0$. Therefore, $\frac{\partial\mathsf{GFI}^\star}{\partial X} \leq 0 \Rightarrow \frac{\partial N^\star}{\partial X} \geq 0$.

The proof for π_G and π_B is similar to the previous one, noting that when the difference between firm types tends to zero, we know that GFI could not be positive (N here can be finite):

$$\pi_B = \pi_G = \pi \Rightarrow \mathsf{GFI} = -\left(c_d \cdot \frac{L_d^{\star 2}}{N} + c_e \cdot L_e^{\star 2}\right) \cdot \pi \leq 0. \tag{A9}$$

Therefore, for $\mathsf{GFI}^\star = 0$ and for $\pi_B > \pi_G$, it must be that the gains to imitation decrease in the probability of bankruptcy for the good firm ($\partial\mathsf{GFI}/\partial\pi_G < 0$), and increase in the probability that the bad firm goes bankrupt ($\partial\mathsf{GFI}/\partial\pi_B > 0$). After all, GFI is a monotonic function of N for $0 \leq \pi_B < 1, 0 \leq \pi_G < 1$. Consequently,

$$\frac{\partial N^\star}{\partial \pi_G} > 0, \quad \frac{\partial N^\star}{\partial \pi_B} < 0, \quad \frac{\partial N^\star}{\partial(\pi_G - \pi_B)} > 0. \tag{A10}$$

Finally, when $c_e = 0, L_e^\star = 1$, and $L_d^\star = 0$, therefore $\mathsf{GFI}(c_e = 0) = (\frac{\pi_G - \pi_B}{1-\pi_G}) \cdot (V_{\text{Old}} - X - I) > 0 \Rightarrow \frac{\partial\mathsf{GFI}}{\partial c_e} < 0 \Rightarrow \frac{\partial N^\star}{\partial c_e} > 0$. To show $\frac{\partial N^\star}{\partial c_d} < 0$, note that $L_d^\star(1 - L_e^\star) = \frac{4 \cdot c_e^2 \cdot c_d \cdot X \cdot N^3}{(4 \cdot c_e \cdot c_d \cdot N^2 + X^2)^2}$, decreasing in c_d. Moreover, $\frac{\partial L_e}{\partial c_d} < 0$, $\frac{\partial L_d}{\partial c_d} < 0$, $\frac{\partial c_d L_d^2}{\partial c_d} < 0$, therefore $\frac{\partial\mathsf{GFI}}{\partial c_d} > 0 \Rightarrow \frac{\partial N^\star}{\partial c_d} < 0$. Finally, $\partial(\mathsf{FV}/I)/\partial N$ is positive, because $\partial\mathsf{GFI}/\partial\mathsf{FV}$ is negative.

Proof of Proposition 3: Signaling with Debt Pricing and Concentration: This appendix proves that the firm prefers to use only the number of creditors for signaling, if possible, and uses interest rate signaling only after bumping against the limit $N = 1$. We need to modify equation (10) to accommodate nonzero debt yields,

$$I \cdot (1 + r) = \pi_G \cdot \left[\alpha^\star \cdot V_{\text{Old}} + (1 - \alpha^\star) \cdot (V_{\text{Old}} - X) - c_d \cdot \left(\frac{L_d^2}{N}\right)\right] + (1 - \pi_G) \cdot \mathsf{FV}^{\mathsf{NY}}, \tag{A11}$$

where the superscript NY on FV reflects the fact that the good firm uses both N and the debt yield as signals. Hence,

$$\mathsf{FV}^{\mathsf{NY}} = \frac{I \cdot (1 + r) - \pi_G \cdot \left[V_{\text{Old}} - (1 - \alpha^\star) \cdot X - c_d \cdot \left(\frac{L_d^2}{N}\right)\right]}{1 - \pi_G}. \tag{A12}$$

Separation will occur as long as $\mathsf{GFI}^{\mathsf{NY}} = 0$, where GFI is defined in equation (16),

$$\begin{aligned}\mathsf{GFI}^{\mathsf{NY}} = \pi_B \cdot \left[(1 - \alpha^\star) \cdot X - c_e \cdot L_e^2\right] + (1 - \pi_B) \cdot \left(V_{\text{New}} + V_{\text{Old}} - \mathsf{FV}^{\mathsf{NY}}\right) \\ - [V_{\text{Old}} - I + (1 - \pi_B) \cdot V_{\text{New}}] = 0.\end{aligned} \tag{A13}$$

Substituting in $\mathsf{FV}^{\mathsf{NY}}$,

$$\begin{aligned}
\mathsf{GFI}^{\mathsf{NY}} &= \pi_B \cdot \left[(1-\alpha^\star)\cdot X - c_e \cdot L_e^2\right] \\
&\quad + (1-\pi_B)\cdot \left\{ V_{\text{New}} + V_{\text{Old}} - \frac{I\cdot(1+r)\cdot \pi_G \cdot \left[V_{\text{Old}} - (1-\alpha^\star)\cdot X - c_d \cdot \left(\dfrac{L_d^2}{N}\right)\right]}{1-\pi_G} \right\} \\
&\quad - [V_{\text{Old}} - I + (1-\pi_B)\cdot V_{\text{New}}] \\
&= \left[X\cdot(1-\alpha^\star) + I - V_{\text{Old}}\right]\cdot(\pi_B - \pi_G) - (1-\pi_B)\cdot r \cdot I \\
&\quad - \left[\pi_B \cdot (1-\pi_G)\cdot c_e \cdot L_e^2 + (1-\pi_B)\cdot \pi_G \cdot c_d \cdot \left(\frac{L_d^2}{N}\right)\right]. \qquad \text{(A14)}
\end{aligned}$$

Setting this expression to zero defines the signaling equilibrium $(N^\star, r^\star)$. Solving for $r^\star$ as a function of $N^\star$,

$$r^\star = \frac{[X\cdot(1-\alpha^\star) + I - V_{\text{Old}}]\cdot(\pi_B - \pi_G) - \left[\pi_B\cdot(1-\pi_G)\cdot c_e \cdot L_e^{\star 2} + (1-\pi_B)\cdot \pi_G \cdot c_d \cdot \left(\dfrac{L_d^{\star 2}}{N}\right)\right]}{(1-\pi_B)\cdot I}. \qquad \text{(A15)}$$

Note that $r^\star$ depends on $N^\star$ through $\alpha^\star, L_d^\star$, and $L_e^\star$. Substitute the value of $r^\star$ into $\mathsf{FV}^{\mathsf{NY}}$, to obtain

$$\mathsf{FV}^{\mathsf{NY}} = \left(\frac{I}{1-\pi_B}\right) + \pi_B \cdot (1-\alpha^\star)\cdot X - \left(\frac{\pi_B}{1-\pi_B}\right)\cdot \left(V_{\text{Old}} + c_e \cdot L_e^{\star 2}\right). \qquad \text{(A16)}$$

Finally, substitute $\mathsf{FV}^{\mathsf{NY}}$ into the expression for E to obtain

$$\begin{aligned}
E^{\mathsf{NY}} &= \pi_G \cdot \left[(1-\alpha^\star)\cdot X - c_e \cdot L_e^{\star 2}\right] + (1-\pi_G)\cdot \left(V_{\text{Old}} + V_{\text{New}} - \mathsf{FV}^{\mathsf{NY}}\right) \\
&= \left(\frac{\pi_B - \pi_G}{1-\pi_B}\right)\cdot \left[c_e \cdot L_e^{\star 2} - X\cdot(1-\alpha^\star)\right] \\
&\quad + (V_{\text{Old}} - I)\cdot \left(\frac{1-\pi_G}{1-\pi_B}\right) + (1-\pi_G)\cdot V_{\text{New}}. \qquad \text{(A17)}
\end{aligned}$$

In terms of entrepreneurial proceeds, the equilibrium $(\widehat{N}, r=0)$ dominates the equilibrium $(N^\star, r^\star \neq 0)$ defined in equation (14). This is because E^{NY} increases with $\alpha^\star$, but is independent of r. Thus, any equilibrium with both signals is dominated by an equilibrium of the type $(N, r=0)$, as long as the latter is feasible (i.e., does not run into the $N=1$ constraint).[3]

[3] The single-crossing property also assures us that the high-quality firm prefers to adhere to the equilibrium over pretending that it is a low-quality firm.

When $N^{\star} = 1$:} We now consider when N alone is not sufficient for the firms to separate (i.e., even with $N^{\star} = 1$). We now show that the firm needs to additionally increase the debt yield to induce separation. To characterize this equilibrium, let us define

$$\alpha_{N=1} \equiv L_{d,N=1} \cdot (1 - L_{e,N=1}), \tag{A18}$$

that is, the value of α when $N = 1$. In this case, the entrepreneur offers debt with face value such that

$$I \cdot (1+r) = \pi_G \cdot \left[\alpha_{N=1} \cdot V_{\text{Old}} + (1 - \alpha_{N=1}) \cdot (V_{\text{Old}} - X) - c_d \cdot \left(\frac{L^2_{d,N=1}}{N}\right)\right] + (1 - \pi_G) \cdot \mathsf{FV}^{\star\star}. \tag{A19}$$

Because $N = 1$,

$$L^{\star}_{e,N=1} = \frac{X^2}{4 \cdot c_e \cdot c_d + X^2} L^{\star}_{d,N=1} = \frac{2 \cdot c_e \cdot X}{4 \cdot c_e \cdot c_d + X^2}. \tag{A20}$$

Solving for $\mathsf{FV}^{\star\star}$,

$$\mathsf{FV}^{\star\star} = \frac{I \cdot (1 + r^{\star\star}) - \pi_G \cdot \left[\alpha_{N=1} \cdot V_{\text{Old}} + (1 - \alpha_{N=1}) \cdot (V_{\text{Old}} - X) - c_d \cdot \left(\frac{L^2_{d,N=1}}{N}\right)\right]}{1 - \pi_G}. \tag{A21}$$

Therefore, separation will occur as long as the bad firms find the gains from separation nonpositive (equal to zero),

$$\mathsf{GFI}^{\star\star} = \pi_B \cdot \left[(1 - \alpha_{N=1}) \cdot X - c_e \cdot L^{\star 2}_{e,N=1}\right] + (1 - \pi_B) \cdot \left(V_{\text{New}} + V_{\text{Old}} - \mathsf{FV}^{\star\star}\right) - [V_{\text{Old}} - I + (1 - \pi_B) \cdot V_{\text{New}}] = 0. \tag{A22}$$

The last two equations define $r^{\star\star}$ as a function of the parameters in the model, together with the condition that

$$\mathsf{GFI} = \pi_B \cdot \left[(1 - \alpha_{N=1}) \cdot X - c_e \cdot L^{\star 2}_{e,N=1}\right] + (1 - \pi_B) \cdot (V_{\text{New}} + V_{\text{Old}} - \mathsf{FV}_{r=0,N=1}) - [V_{\text{Old}} - I + (1 - \pi_B) \cdot V_{\text{New}}] > 0. \tag{A23}$$

This equation states that $N = 1$ is insufficient to separate (profits from imitation are greater than zero). That is, separation with N only is not enough, even for $N = 1$. It is also the case that signaling with $N = 1$ and $r^{\star\star}$ is preferred to signaling with r alone: From equation (17), $E(1, r^{\star\star}) > E(\infty, r)$, where $E(\infty, r)$ is the value of equity when the firm optimally signals with r alone.

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