The (Large) Effect of Return Horizon on Fund Alpha*

Hendrik Bessembinder, Arizona State University, hb@asu.edu

Michael J. Cooper, University of Utah, mike.cooper@eccles.utah.edu

Feng Zhang, Southern Methodist University, fengzhang@smu.edu

Current Draft: October 31, 2023

Abstract

Alpha depends on the return measurement horizon, particularly as the horizon becomes long. We introduce a procedure to estimate long-horizon alphas from short-horizon returns. Among those sample mutual funds with positive alphas estimated from monthly returns, nearly half have negative alpha estimates when returns are measured at the ten-year horizon. Among sample funds with positive monthly alpha estimates and monthly beta estimates that exceed one, over 70% have negative alpha estimates at the decade horizon. Alphas estimated from short-horizon returns can be uninformative or misleading regarding fund performance for both active and passive investors over longer horizons.

(*JEL* G10, G23)

* The authors thank Ivo Welch (the Editor), an anonymous referee, Stephen Brown, Mikhail Chernov, John Cochrane, Wayne Ferson, Alan Hess, Chris Hrdlicka, Paul Irvine, Haim Levy, Lars Lochstoer, Zhongjin (Gene) Lu, Seth Pruitt, Clemens Sialm, Harald Uhlig, Sunil Wahal, Ivo Welch, Morad Zekhnini, Min Zhu, and seminar participants at the Virtual Asset Management Seminar, Melbourne Asset Pricing Conference, Arizona State University, Southern Methodist University, Georgetown University, Michigan State University, the University of Wyoming, University of Utah, and the University of Georgia for valuable comments and discussions on earlier versions of this paper.

The (Large) Effect of Return Horizon on Fund Alpha

Abstract

Alpha depends on the return measurement horizon, particularly as the horizon becomes long. We introduce a procedure to estimate long-horizon alphas from short-horizon returns. Among those sample mutual funds with positive alphas estimated from monthly returns, nearly half have negative alpha estimates when returns are measured at the ten-year horizon. Among sample funds with positive monthly alpha estimates and monthly beta estimates that exceed one, over 70% have negative alpha estimates at the decade horizon. Alphas estimated from short-horizon returns can be uninformative or misleading regarding fund performance for both active and passive investors over longer horizons.

(*JEL* G10, G23)

Investors select portfolio weights and researchers test asset pricing models relying on estimates of the parameters that describe the probability distributions of returns, including means, variances, covariances, as well as "beta" and "alpha" coefficients. The literature described below has taken note that these parameters depend on the horizon over which returns are measured. However, the focus to date has been on relatively short horizons ranging from daily to annual, where the effects are modest. For example, Lewellen and Nagel (2006) acknowledge that betas depend on return measurement horizon under the heading "microstructure effects" and dismiss the effect of horizon on beta as "tiny."

We show that it is a mistake to view the effects of return measurement horizon as unimportant when considering outcomes to those who invest over long horizons. We focus in particular on the fact that alphas, both parameters and estimates thereof, differ quite notably when returns are measured over long vs. short horizons. Our application is to measures of mutual fund performance and we consider horizons of up to ten years. Of course, parallel issues arise in the assessment of any type of investment performance, including returns to hedge funds, pension funds, endowments and trusts, and individual investors. We show that alphas estimated from short-horizon returns can be uninformative or misleading regarding performance for investors with longer horizons. Importantly, the issues we highlight pertain to all who invest over long horizons, including those who periodically rebalance or otherwise trade actively, not just those who hold fixed positions over long periods.

The literature that reports on measures of investment performance mainly studies returns assessed over relatively short time horizons, often monthly. Investors who are interested in parameter estimates that pertain to the distribution of monthly returns on various securities, portfolios, and funds can consider the results of hundreds of published studies. Yet, a much smaller literature that includes Levhari and Levy (1977), Handa, Kothari and Wasley (1989), Longstaff (1989), and Lee, Wu and Wei (1990) has shown mathematically and empirically that the parameters that describe the distribution of returns not only depend on the horizon over which returns are measured, but in complex, non-linear ways.[1] This

[1] In contrast to simple returns, some parameters of the distribution of logarithmic returns (e.g. mean and variance) are proportionate to horizon if return distributions are independent and identical over time. Since the simple return

literature has largely focused on CAPM-related tests and as noted relatively short horizons that range from daily to annual, where the effects of horizon are modest.[2] We focus attention on the fact that alphas as well as betas depend on return measurement horizon and show that the effects become large when horizons are long.

Jensen (1968) introduced and explicitly motivated alpha based on the CAPM of Sharpe (1964) and Lintner (1965). The CAPM is a single-period model of unspecified length. Of course, researchers have since adapted alpha to multifactor models. More recent factor models, such as the five-factor model of Fama and French (2015), the Q-factor model of Hou, Xue, and Zhang (2015) and the four-factor model of Carhart (1997), are motivated in part based on their ability to explain aspects of the empirical distribution of monthly returns, but without explicit consideration of whether the monthly horizon is the most relevant or informative.

Importantly, our emphasis is not on forecasting (e.g. the degree to which returns measured from a short sample period are informative about returns measured over a subsequent period), learning (e.g. the Bayesian updating of parameter estimates as the sample becomes larger with the passage of time), or changes in parameters as the economy evolves (as in conditional asset pricing models). Rather, we focus directly on the effects of altering the time interval over which returns are measured, e.g. from monthly to annual to decadal. Our theoretical analysis focuses on a known stable probability distribution, while our empirical analyses in all cases rely on the identical 30-year sample. Even though each long-horizon return is simply obtained by compounding the relevant shorter-horizon returns within the same sample,

over multiple periods is obtained by the nonlinear (exponential) transformation of the sum of the log returns, the proportional-to-time property for means and variances in log returns does not carry over to simple returns.

[2] Kothari, Shanken, and Sloan (1995) estimate a positive return premium associated with CAPM betas when returns are measured at the annual horizon, but not at the monthly horizon. Handa, Kothari and Wasley (1989) show that the estimated return premium associated with firm size is sensitive to the length of return interval used to estimate beta. Gilbert, Hrdlicka, Kalodimos, and Siegel (2014) estimate alphas and betas for equity portfolios over horizons ranging from daily to quarterly, and argue that differences across horizon are explained by differences in firms' opacity, i.e., in investors difficulty in assessing the value implications of events. Boguth, Carlson, Fisher and Simutin (2016) focus on slow information diffusion as an explanation for differing equity portfolio returns for horizons ranging from daily to annual. Kamara, Korajczyk, Lou, and Sadka (2016) also focus on heterogeneous stock price reactions and assess the extent to which systematic factors earn risk premia at horizons from monthly to biannual. We focus on returns measured over longer horizons where these frictions are less important, and highlight the effect of horizon *per se*.

we show that performance measures constructed from long-horizon returns contain notably different information than measures constructed from monthly returns.

The estimation of fund alphas and betas when returns are measured over longer horizons presents substantive challenges. The available databases are too short to allow estimation of long-return-horizon alphas and betas by means of standard time series regressions. Parameter estimates obtained from short horizon returns can be used in the formulas we develop that express long-horizon parameters as a function of short-horizon parameters, but since these functions are non-linear the resulting long-horizon parameter estimates will be biased due to Jensen's inequality. We rely on simulations calibrated to the actual data on a fund-by-fund basis to quantify and correct these biases.

Having done so, we illustrate the practical importance of return horizon in measuring performance. For the full sample, 30.4% of alphas estimated for decade-horizon returns are positive, compared to 43.2% of alphas estimated from monthly returns. Among those mutual funds with a monthly return beta estimate less than one, the percentage of funds with positive decade-horizon alphas increases to 54.6%, while, strikingly, among funds with a monthly return beta estimate greater than one the percentage of funds with positive decade-return horizon alphas decreases to 14.5%.

These results imply that a given fund's risk-adjusted performance, i.e., its alpha, can be positive over short return measurement horizons and negative over long return measurement horizons (or vice versa), even when results are based on a single sample. To understand this potentially counterintuitive result, recognize that an alpha estimate is simply a mean return less the portion of that mean attributed to outcomes on "factors" (such as the overall market), i.e., on the product of estimated betas and mean factor outcomes. Since betas depend on return measurement horizon, so does the partition of a mean return between factor outcome and alpha.

But why does beta depend on return horizon? To gain intuition for this fact it is useful to consider the details of return compounding. If the expected return for the first period is positive then the investment at the beginning of the second period will be larger on average than at the beginning of the first. As a consequence, any given *percentage* return during the second period has a larger average effect on investment value as compared to the same percentage return during the first period, and deviations of individual outcomes from the mean outcome, which are the building blocks of parameters such as variances and covariances, are also larger relative to the initial investment during the second period. Compound returns computed over two periods capture this reality, while successive single-period returns do not.

The importance of this distinction depends on the average magnitude of the earlier returns, with the effects being relatively larger for high alpha securities and for high beta securities if prior market returns are large, other things equal. Consequently, market betas diverge in the cross-section from their average (of one) as the compounding horizon becomes longer. The upshot is that an investor with a short return horizon can experience a positive alpha even while a long-horizon investor experiences a negative alpha (or vice versa), because the long-horizon investor can be subject to more (or less) beta risk as compared to the short-horizon investor.

It has been suggested to us by a prominent Finance researcher that the results we report in this paper show that alpha is a "terrible statistic" for long-horizon investors, in part because it was originally introduced with reference to the mean-variance CAPM. Among other shortcomings, alpha does not consider the strong positive skewness in the distribution of long horizon returns, as emphasized by Farago and Hjalmarsson (2023) and Bessembinder, Cooper, and Zhang (2023), as well as others. Yet, many investors are interested in investment outcomes over long horizons, and until superior measures emerge will likely continue to focus on alpha as a measure of mean returns after allowing for systematic risk exposures. Our results imply that the degree to which mean returns are abnormal cannot be evaluated independent of information regarding the horizons that matter to disparate investors. Measures of mutual fund performance that are based on short-horizon returns may be uninformative or even misleading regarding fund performance for those investors who are focused on longer horizons.

Our goal in this paper is to demonstrate the importance of return measurement horizon for estimates of investment alpha. We rely on a simple single-factor market model because our focus is on the effects of the compounding of random returns over long horizons, not on the widely studied question of which benchmarks or factor models are most appropriate. Our results suggest the desirability of reporting alpha estimates for returns measured over a variety of horizons, but do not answer the intriguing question of which return measurement horizon is most relevant to investors. Some relevant evidence might be obtained by assessing the return measurement horizon that best explains relations between mutual fund performance and fund flows originating with various types of investors. Additional evidence might be provided by assessing the return measurement horizon for which various asset pricing models perform best.

1. Investment Horizon and the Role of Rebalancing

The familiarity that stems from the many empirical studies conducted in monthly returns might lead to a natural tendency to think of the monthly horizon as the "correct" one. Yet, we know of no theory with the implication that the CAPM or other factor models should hold at the monthly horizon in particular, or that would support the reasoning that the one-month horizon is necessarily the most relevant to disparate investors.

For longer term investors to focus only on the distribution of returns over the upcoming month would be *myopic*.[3] Yet, Samuelson (1969) shows that rational long-horizon investors do, under certain assumptions, behave myopically.[4] Specifically, he shows that if returns are independent and identically distributed (iid) over time, then an investor who maximizes the expectation of a power utility function rebalances each period to the same portfolio weights that are optimal for a single-period investor. That is,

[3] Relatively little research provides direct empirical evidence regarding investment horizons. Among the available studies, Ameriks and Zeldes (2004) report that nearly half of the participants in a sample of defined contribution retirement plans made no changes to their allocations or investments over a ten-year period. A 2020 report issued by the Vanguard Group (Vanguard, 2020) indicates that nearly half of the accounts associated with affluent households made no trades at all during each calendar year from 2015 to 2019, and that among those accounts that did trade, only about eight to ten percent of existing positions were turned over. Smith (2015), using demographic data, estimates that the investment horizon of the marginal individual investor exceeds fifteen years.

[4] While the word "myopic" does not appear in Samuelson's study, the literature (e.g. Campbell and Viceira, 1999) has adopted it to describe outcomes in Samuelson's setting.

under these assumptions, the information contained in compound multi-period returns is of no interest to an investor who already knows the parameters of returns over the shorter horizon at which they rebalance.

It is, however, an important point of perspective that Samuelson's myopic investor prescription *cannot* apply to all investors or to the market as a whole. Since relative security prices change each period, the weight of each security in the market portfolio changes as well. The market as a whole is, but for the effects of issuers' primary transactions, a buy-and-hold portfolio, not a rebalanced portfolio.[5] If some investors sell assets that have outperformed and purchase those that have underperformed to return to prior weights, then other investors must trade in the opposite direction. The upshot is that, while it can be rational under specific assumptions regarding objectives (power utility) and the distribution of returns (iid) for some investors to follow myopic investment rules guided by single-period parameters, the parameters of compound long-horizon returns will be relevant to other investors and to those who invest in the market as a whole.[6]

Our analysis of return horizon and alpha is relevant to both buy-and-hold and active investors. In fact, the expressions we develop apply directly to a portfolio that is rebalanced each period. Consider the following example. Assume that monthly returns are iid, that the risk-free interest rate is zero, and that the mean continuously compounded market return is 10% per year with a continuously compounded standard deviation of 20% per year. Consider two securities: the first has an instantaneous market beta equal to 0.9 while the second has an instantaneous market beta of 1.4, and each has an instantaneous alpha equal to zero. Consider also an initially equal-weighted portfolio of these two securities, which has

---

[5] To avoid this issue would require the counterfactual assumption that firms issue or repurchase shares every period to offset relative price changes.

[6] While the theory of myopic portfolio rebalancing was influential, a large literature has studied long-horizon investors and asset pricing in settings where myopic behavior is not optimal. Merton (1973) allows for economic state variables that affect parameters of the investment opportunity set, and shows how investors' desire to hedge against changes in such state variables affects equilibrium pricing. Campbell and Viceira (1999) and Barberis (2000), among others, consider the effect of allowing for time varying risk premia, and Campbell and Viceira (2002) describe many of the related studies. Cochrane (2022) and Cochrane (2014) emphasize the importance to long-horizon investors of the stream of inflation-adjusted cash flows produced by their portfolios. Such investors are concerned with changes in the long run dividend stream itself, not in the discount rate variations that Cochrane (2011) shows are the main determinants of changes in market values, and would not necessarily rebalance in response to changes in market values induced by changes in discount rates.

zero instantaneous alpha and a continuous beta of 1.15 (the average of the component alphas and betas). Applying the expressions in Section 4 of this paper reveals that, if there is no additional trading (that is, the portfolio is buy-and-hold), the decade-horizon portfolio beta is 1.54 and the decade-horizon alpha is -38.39% (or -4.73% annualized). On the other hand, if the portfolio is actively rebalanced to maintain equal weights, the decade horizon beta is 1.38 and the decade horizon alpha is -21.26% (or -2.36% annualized).

That is, while rebalancing alters the effects of compounding, the decadal portfolio beta is greater than the instantaneous beta and the decadal portfolio alpha is negative even though the instantaneous alpha is zero, *with or without* rebalancing. Further, the effects at the decade horizon are economically large. Parallel conclusions apply to other active trading strategies. The issues highlighted in this paper arise from the compounding of successive random returns and are relevant to any investment strategy that involves risky returns realized across multiple periods. In contrast, short return horizon parameters are the sole object of interest only for those investors who are myopic (as in Samuelson, 1969), or for those who invest for a single short period before permanently exiting the market.

2. Return Measurement Horizon and Alpha

Jensen (1968) introduced alpha as a measure of mutual fund performance, motivating it as a "direct application" of the asset pricing model now broadly referred to as the CAPM. The CAPM is a single-period model, but the length of the period is unspecified. In practice, investment and decision horizons can differ across investors, a fact not explicitly considered by the CAPM. Nevertheless, investors are interested in assessing fund performance after allowing for exposure to systematic risk, and researchers are interested in testing asset pricing models. Alpha estimates are central to each exercise.

Of course, researchers have adapted the concept of alpha to multi-factor models. Our intent here is to focus attention on the fact that measures of investment performance depend on return horizon in the simplest possible settings, so we follow Pastor and Stambaugh (2012) in focusing on the single-market-factor model. Despite the single factor model's simplicity, there is evidence it is of substantial relevance to investors. Berk and van Binsbergen (2016) and Barber, Huang, and Odean (2016) present evidence that the single factor model better explains investor flows into and out of mutual funds as compared to multi-factor models (though this conclusion has been challenged, e.g., by Ben-David, Li, Rossi, and Song (2022)).

To assess how mutual fund alphas depend on the horizon over which returns are measured, we must accommodate the fact, previously noted by Levhari and Levy (1977), Handa, Kothari and Wasley (1989), and Longstaff (1989) among others, that beta coefficients depend on the horizon over which returns are measured. We highlight that alphas also depend on the return measurement horizon, and in complex ways. Apart from Levy and Levy (2011), this fact does not appear to have been emphasized in the literature.[7] It is an important point of perspective that alphas and betas *as parameters* vary as a function of return measurement horizon, even in the absence of estimation challenges.

We first demonstrate the theoretical relation between short-return-horizon and long-return-horizon alphas. We then describe our empirical methods to estimate long-return-horizon alphas and betas for the funds in the sample. Finally, we describe the empirical evidence regarding long-return-horizon alphas for US equity mutual funds.

2.1 Return Horizon, Beta and Alpha

In this section, we demonstrate relations between short-return-horizon and long-return-horizon alphas and betas in the simplest possible setting: the short horizon is instantaneous, there is only a single relevant factor (the market return), and returns are iid over time.[8] Let *i* denote an individual fund or asset and *m* denote the market, and let μ_i and μ_m denote their respective mean instantaneous returns, stated on a per-period (e.g. monthly) basis. We assume for expositional simplicity that the risk-free interest rate is zero. Let σ_m^2 denote the variance of instantaneous market returns and let β_i denote asset *i*'s instantaneous market beta. Then the instantaneous alpha is

[7] Levy and Levy (2011) note that if the CAPM holds (i.e. implies zero alphas) for a horizon longer than that used to measure returns, then positive alpha estimates are induced for small firms, due to their high betas. They do not, however, develop an expression for true alpha as a function of return horizon.

[8] We thank John Cochrane for specific modeling suggestions employed in this analysis.

$$\alpha_i = \mu_i - \beta_i \mu_m. \tag{1}$$

Let a superscript L denote long-horizon (e.g. decade) returns, which are instantaneous returns compounded over the relevant period. Let σ_m^{2L} denote the variance of long-return-horizon market returns, σ_{im}^{L} the covariance of between long-horizon asset *i* returns and long-horizon market returns, and $\beta_i^L = \frac{\sigma_{im}^L}{\sigma_m^{2L}}$ denote asset *i*'s long-return-horizon market beta. The alpha for long-horizon returns is:

$$\alpha_i^L = \mu_i^L - \beta_i^L \mu_m^L. \tag{2}$$

Expressions (1) and (2) are identical definitions of alpha, differing only in the horizon over which returns are measured.

If returns are iid, then compounding over T periods gives $\mu_i^L = e^{\mu_i T} - 1$ and $\mu_m^L = e^{\mu_m T} - 1$. Substituting into (2), the long-return-horizon alpha can be stated as:

$$\alpha_i^L = e^{\mu_i T} - 1 - \beta_i^L (e^{\mu_m T} - 1). \tag{3}$$

If $\mu_m = 0$ or $\beta_i = \beta_i^L = 0$ then, using expression (1), the long-return-horizon alpha is simply $\alpha_i^L = e^{\alpha_i T} - 1$. That is, in these cases the long-return-horizon alpha is the compounded equivalent of the instantaneous alpha. More broadly this simple relation does not hold, and the long-horizon alpha differs from this benchmark as a function of beta and the mean market return, μ_m.

We show in Appendix A that the long-horizon market return variance, σ_m^{2L}, and the covariance between the long-horizon market return and the long-horizon return to asset *i*, σ_{im}^L, are:

$$\sigma_m^{2L} = (e^{\sigma_m^2 T} - 1)(e^{2\mu_m T}) \text{ and} \tag{4}$$

$$\sigma_{im}^L = (e^{\beta_i \sigma_m^2 T} - 1)(e^{[\alpha_i + (\beta_i + 1)\mu_m]T}). \tag{5}$$

The variance of a given asset's return increases with the return measurement horizon, T.[9] The long-horizon covariance between a pair of asset returns (in this case asset *i* and the market) depends on

[9] The point that the variance of compound returns increases with investment horizon has been underappreciated at times, potentially attributable to a focus on the variance of arithmetic mean returns or the imprecise use of the phrase "time diversification." Samuelson (1969, page 239) sought to put this confusion to rest when he referred to the "mistaken notion that multiplying the same kind of risk leads to cancelation rather than augmentation of risk."

the return measurement horizon but need not grow at the same rate as the variance of either asset. Combining (4) and (5), the long-horizon beta is:

$$\beta_i^L = \frac{(e^{\beta_i \sigma_m^2 T}-1)(e^{[\alpha_i+(\beta_i+1)\mu_m]T})}{(e^{\sigma_m^2 T}-1)(e^{2\mu_m T})} \,. \qquad (6)$$

It can be verified that (6) implies that instantaneous and long-horizon betas are equal if α_i = 0 and $\beta_i = 1$, or if $\beta_i = 0$.[10]

To illustrate the implications of expression (6), Panel A of Figure 1 displays long-horizon (one year, five year, and ten year) asset *i* betas that are implied by various combinations of instantaneous betas and alphas.[11] As noted, the long-horizon beta is equal to the instantaneous beta when the short-horizon beta is one and the instantaneous alpha is zero. Long-horizon betas increase in instantaneous betas (given a positive mean market excess return), but the effect is non-linear, being greatest when short-horizon betas are large. This implies an asymmetric impact, whereby increases in the return measurement horizon have the greatest effect on betas, and by extension on alphas, when short-horizon betas are large. The absolute divergence between instantaneous and long horizon betas increases with the return measurement horizon. Note that the long-horizon betas also increase in short-horizon alphas. For example, when instantaneous alpha is zero an instantaneous beta equal to one implies long horizon betas also equal to one, while a positive (negative) instantaneous alpha implies long horizon betas that exceed (are less than) one (and more so at longer horizons).

Combining expressions (3) and (6), long-return-horizon alpha depends on short-return-horizon (instantaneous) parameters according to:

$$\alpha_i^L = \overbrace{e^{(\alpha_i+\beta_i\mu_m)T}-1}^{expected\ return} - \overbrace{\left[\frac{(e^{\beta_i \sigma_m^2 T}-1)(e^{[\alpha_i+(\beta_i+1)\mu_m]T})}{(e^{\sigma_m^2 T}-1)(e^{2\mu_m T})}\right]}^{beta} \overbrace{(e^{\mu_m T}-1)}^{market\ premium} \,. \qquad (7)$$

---

[10] A limitation of this analysis is that the asset return and the market return are simply specified as correlated variables, without explicitly considering that the asset is itself a component of the market. However, this simplification is not of practical importance as long as the weighting on the asset in question is small.

[11] The illustration relies also on $\mu_m = .09\ and\ \sigma_m = .19$, which are reasonable in light of historical annual outcomes for the U.S. market.

Panel B of Figure 1 displays long-return-horizon alphas implied by expression (7) for various combinations of instantaneous alphas and betas. To facilitate comparisons across horizons, the Figure displays alphas that are annualized. The most notable results that can be observed on Panel B of Figure 1 are that zero short-return-horizon alpha does not generally imply zero long-return-horizon alpha, and that deviations from zero are greater for longer return measurement horizons. Long- and short-return-horizon alphas are approximately equal (when each is stated on an annualized basis) if the short-return-horizon beta is one, but not otherwise. For assets with instantaneous betas less than one the long-return-horizon alpha is greater than the short-return-horizon alpha, while for assets with instantaneous betas greater than one the long-horizon alpha is less than the short-horizon alpha. That is, investors who are concerned with outcomes measured over short horizons will experience different alphas than investors who are concerned with outcomes over long horizons, even if returns are iid and in the absence of estimation issues.

This analysis therefore implies that mutual fund performance cannot be evaluated independent of the question of which return horizon is most relevant to investors. Estimated long-return-horizon alphas are more likely to be negative if short-return-horizon betas are greater than one, while estimated long-return-horizon alphas are less likely to be negative if short-return-horizon betas are less than one, other things equal. However, as discussed below, this outcome depends on the positive market return premium; a negative average market return would reverse these effects. Further, the functions displayed in Figure 1 are non-linear. That is, the effect of return horizon on alpha is asymmetric, and is most pronounced when short run betas are large.

Expressions (6) and (7) apply to investments with iid returns. The period-by-period returns to portfolios with fixed positions, i.e., buy-and-hold portfolios, are not iid, because weights change over time as a function of prior performance. However, expressions (6) and (7) can be applied to individual securities, and relying on standard portfolio theory results, the long return horizon alpha and beta of a buy-and-hold portfolio can be obtained as the average (using initial weights) of the security-specific long-horizon alphas and betas. On the other hand, returns to portfolios that are rebalanced each period to maintain constant weights are iid if the component returns are iid. As a consequence, expressions (6) and (7) apply directly for such rebalanced portfolios.

To gain intuition for why both alpha and beta depend on the return horizon it is useful to consider the details of return compounding. Assume an initial investment of $1. If the return for the first period is positive, then the investment at the beginning of the second period is larger than $1. As a consequence, any given percentage return during the second period has a larger effect on investment value as compared to the same percentage return during the first period, and deviations of individual outcomes from the mean, which are the building blocks of parameters such as variances and covariances, are also larger (relative to the initial $1investment) during the second period. Compound percentage returns computed over two periods capture this reality, while successive single period *percentage* returns do not.

The discussion in the prior paragraph is predicated on a positive return during the earlier period. If the first period return were negative instead, then returns would be of less consequence during the second period than the first. If mean returns are positive, then the second periods will be of greater importance on average. Other things equal this effect will be stronger for firms with positive alphas or with larger betas (assuming a positive market premium). However, in a given sample the mean return can be negative, in which case the effects are reversed. Of course, expressions (6) and (7) are implemented in sample data, and the relation between short and long horizon alphas and betas for any given fund or asset will depend on estimated alphas, betas, and mean market returns during the relevant sample period.

2.2 The Modified LL Method of Estimating Long-horizon Alphas and Betas

Short-horizon (e.g. monthly) betas are typically estimated by time series regressions, where the sample size is the number of months for which the requisite data is available. Here, we seek to estimate betas for longer return horizons. On average, funds are included in our sample for only 133 months (about 11 years), so time series regression methods cannot be implemented for most funds if the return measurement horizon is long.

Expressions (1) to (6) are stated in terms of instantaneous parameters. Since mutual fund return data is observable only over discrete horizons, we rely on a version of expression (6) that applies when returns are measured at the monthly horizon rather than the instantaneous horizon. Letting $\beta_i, \sigma_m^2, u_i \; and \; u_m$ now denote monthly rather than instantaneous parameters, Lehavi and Levi (1977) show that long horizon beta can be expressed based on monthly parameters as:

$$\beta_i^L = \frac{\left(\beta_i \sigma_m^2 + (1+\mu_i)(1+\mu_m)\right)^T - (1+\mu_i)^T (1+\mu_m)^T}{\left(\sigma_m^2 + (1+\mu_m)^2\right)^T - (1+\mu_m)^{2T}}. \qquad (6a)$$

The challenge lies in estimating long horizon betas. If these are in hand, long horizon alphas are estimated simply based on expression (2) and sample estimates of expected returns measured at the relevant horizon. For example, the alpha estimate for returns measured at the decade horizon is the arithmetic mean decade-horizon fund return less the product of the decade-horizon beta estimate and the arithmetic mean of decade horizon market returns over the same sample interval.

To estimate long horizon betas, we introduce the procedure described below, which relies on a combination of theory and simulations. We anticipate that future researchers may refine this estimation procedure, in particular by relaxing our reliance on the assumption that returns are iid. In the meantime, the results obtained here illustrate the importance of return measurement horizon to the assessment of fund performance and to the estimation of alphas. We estimate annual betas for all funds with at least twelve monthly observations, five-year betas for all funds with at least sixty monthly observations, and decadal betas for all funds with at least 120 monthly observations.

We rely on the following sequence of empirical steps.

Step 1: Obtain monthly alpha and beta estimates. We begin by estimating single factor alpha ($\hat{\alpha}_i$) and beta ($\hat{\beta}_i$) for each sample fund using time series regressions of monthly excess fund returns on monthly excess SPY returns. Following Welch (2021), we winsorize fund returns at -2 times and 4 times the same-month SPY return prior to estimating these regressions.

Step 2: Obtain estimates of the other parameters contained in expression (6a). We next record fund-specific estimates of the mean excess fund monthly return as well as the mean and variance of the excess market monthly return during the same sample months. Since some of these estimates are obtained from short samples, we winsorize these estimates at the 10th and 90th percentiles.

Step 3: Obtain estimates of long-horizon betas using (6a). We employ the sample estimates of monthly horizon parameters in expression (6a) to obtain corresponding estimates of long-horizon betas at the annual, 5-year, and decade horizons, which we denote as $\hat{\beta}_i^L$ (i.e., $\hat{\beta}_i^{1y}$, $\hat{\beta}_i^{5y}$, and $\hat{\beta}_i^{10y}$). Note, though that since expression (6a) is non-linear in its parameters, the resulting $\hat{\beta}_i^L$ estimates are not only noisy, but are biased due to Jensen's inequality. Further, since the expression relies on several parameter estimates characterized by varying degrees of estimation error, no obvious prediction as to the direction of the bias arises. The remaining steps describe how we quantify and adjust for such bias.

Step 4: Create a simulated fund to match each sample fund and assign parameters to it. For each fund in our sample, we create a matching simulated fund and assign to it a true monthly beta, β_i, as a random draw from a normal distribution with mean equal to the sample fund's empirically estimated monthly beta and variance equal to the cross-sectional sample variance of monthly fund beta estimates obtained in Step 1. We also assign to each fund a true monthly alpha, α_i, as a random draw from a normal distribution with mean equal to the matched sample fund's alpha estimate and variance equal to the cross-sectional variance of sample monthly alphas across all funds obtained in Step 1. Further, we assign μ_m and σ_m^2 parameters to each simulated fund that match the sample estimates for each fund in Step 2. Having done so, we use expression (6a) and the assigned short-term parameters to compute the actual long-horizon (1-year, 5-year, and 10-year) beta for each simulated fund, denoted β_i^L (i.e., β_i^{1y}, β_i^{5y}, and β_i^{10y}), that corresponds to the true short-horizon beta also assigned, β_i.

Step 5: Simulate sample returns for each fund and estimate the simulated fund's monthly beta using the simulated data and standard time-series regressions. We then create simulated sample returns for each fund. If the sample for the actual fund includes N monthly returns, we generate for the matched simulated fund N monthly excess market returns, R_{mt} as random draws from a normal distribution with mean and variance equal to the matching fund sample estimates of such, and create N monthly excess simulated fund returns as $R_{it} = \alpha_i + \beta_i * R_{mt} + e_{it}$, where each e_{it} is a random draw from a zero-mean normal distribution with variance equal the sample residual volatility for the sample fund. Having generated a N-month return sample for each simulated fund, we obtain an estimated monthly beta for each simulated fund by standard time-series regression methods, after again implementing the winsorization method recommended by Welch (2021).

Step 6: Convert the monthly beta estimate from the simulated monthly sample to a corresponding long-horizon beta estimate. We take the monthly beta estimate obtained in the simulated data for each fund in Step 5 and convert it to a corresponding long-horizon (1-year, 5-year, and 10-year) estimate (denoted as $\hat{\beta}_i^{LS}$ (i.e., $\hat{\beta}_i^{1y,S}$, $\hat{\beta}_i^{5y,S}$, and $\hat{\beta}_i^{10y,S}$), where the *S* in the superscript denotes a simulation-based estimate) using expression (6a) and associated parameter estimates from the simulated sample. That is, $\hat{\beta}_i^{LS}$ is estimated for each fund from the simulated data using the same procedure by which $\hat{\beta}_i^{L}$ was estimated from the sample data in Step 3.

Step 7: Repeat, to obtain a bootstrap distribution. We repeat Steps 4 to 6 1,000 times, to obtain for each simulated fund a distribution of 1,000 true long-horizon betas (i.e., β_i^L in Step 4) as well as 1,000 estimates of long-horizon betas (i.e., $\hat{\beta}_i^{LS}$ in Step 6) that are obtained by employing short-horizon parameter estimates in expression (6a).

Step 8: Assess the nature of the bias in estimates of long run beta that are obtained by using short run beta estimates and other parameter estimates in expression (6a). As noted, the estimates of long-run beta obtained by substituting short-horizon parameter estimates in expression (6a) are likely to be biased in complex ways. We seek to quantify the nature of such bias. To do so, we estimate fund-specific regressions of the form $\beta_i^L = a_i + b_i * \hat{\beta}_i^{LS} + u_i$ for each of the three investment horizons (i.e., 1 year, 5 years, and 10 years) across the 1,000 simulation outcomes in Step 7.[12] If the $\hat{\beta}_i^{LS}$ were unbiased estimates of β_i^L (an outcome we do not anticipate) these regressions would produce intercepts indistinguishable from zero and slope coefficients indistinguishable from one. If not, the estimated intercept and slope are informative regarding the nature of the bias.

Table 1 provides summary statistics regarding the resulting distribution of regression coefficient estimates across the simulated funds, when the simulations are applied based on annual, five year, and decade return horizons. It can be observed that intercepts are positive for virtually all simulated funds (the 5th percentile is positive at all three horizons), and average 0.39 in annual returns, and 0.27 in both five-year and in decadal returns. Slope coefficients are virtually all less than one (the 95th percentile is less than one at all three horizons) and average 0.62 in annual returns, 0.75 in five-year returns, and 0.78 in decadal returns. These results verify that employing monthly parameter estimates in expression (6a) leads to biased estimates of long-horizon betas. They also suggest a solution, which we implement.

Step 9: Adjust the biased estimates of long horizon beta obtained from the actual sample using the information gleaned from the simulations. Our final estimate of the long-horizon beta for each sample fund is $\hat{\beta}_i^{LL} = \hat{a}_i + \hat{b}_i\hat{\beta}_i^L,$ where $\hat{a}_i$ and $\hat{b}_i$ are the estimated fund *i* regression coefficients based on simulated data in Step 8, and $\hat{\beta}_i^L$ is the long-horizon beta estimate obtained when the monthly beta estimate and other monthly parameter estimates were employed in expression (6a), as described at Step 3. We obtain $\hat{\beta}_i^{LL}$ for the 1-year, 5-year, and 10-year investment horizons (i.e., $\hat{\beta}_i^{LL,1y}$, $\hat{\beta}_i^{LL,5y}$, and $\hat{\beta}_i^{LL,10y}$) for each fund. Since expression (6a) is attributable to Levhari and Levy (1977), we refer to this procedure as the modified LL method, and denote the estimate itself with the superscript LL. Note that the final estimate includes both the estimate obtained directly from the sample data and the information obtained from the bootstrap simulation regarding the nature of the bias in the sample estimate.

[12] Here also, we winsorize the dependent and independent variables at the 10th and 90th percentiles to mitigate the influence of outliers.

Step 10: Compute each fund's 1-year, 5-year, and 10-year alpha estimate, and convert each to a monthly equivalent to facilitate comparison. We compile each fund's N-month (N = 12, 60, or 120) returns on a rolling basis, and compute the arithmetic mean, denoted F_N^{Avg}, as well as the corresponding mean N-month rolling returns to the risk-free asset and SPY over the same months, denoted as RF_N^{Avg} and SPY_N^{Avg}, respectively. The fund's 10-year alpha estimate (expressed as a monthly rate) is $\left[F_{120}^{Avg} - RF_{120}^{Avg} - \hat{\beta}_i^{LL,10y}\left(SPY_{120}^{Avg} - RF_{120}^{Avg}\right)\right]/120$, where $\hat{\beta}_i^{LL,10y}$ is the funds' 10-year beta obtained in Step 9.[13] Note that 10-year alpha is only available for funds with at least 120 monthly returns. Similarly, we compute the fund's 5-year alpha for all funds with at least 60 monthly returns (expressed in monthly rate) by replacing $\hat{\beta}_i^{LL,10y}$ with $\hat{\beta}_i^{LL,5y}$ and replacing 120 with 60 in the formula. The fund's 1-year alpha is computed in the corresponding manner.

3. Data and a Validation of the Modified LL Method

We study a broad sample of nearly 8,000 U.S. equity mutual funds during the 1991 to 2020 period. This sample is also employed by Bessembinder, Cooper, and Zhang (2023), who study the aggregate wealth effects of mutual fund investing and describe the distribution of long-horizon mutual fund returns. While specific point estimates would differ if we considered alternative samples, e.g. non-domestic, balanced, or levered funds, the central point we emphasize, that alphas depend on the horizon over which returns are measured, is not sample-specific. Data are obtained from the CRSP survivorship bias free Mutual Fund Database. We begin at 1991, as data regarding fund total net assets (TNA), which we use to aggregate fund returns across share classes, are not consistently available for earlier periods. We study domestic equity funds while excluding ETFs, target date funds, hedged funds, and leveraged

[13] We restate long-horizon alphas as monthly equivalents by dividing by the number of months in the sample. The more natural alternative, to focus on the N^{th} root (where N is the number of months in the sample) of one plus the long-horizon alpha, is precluded for some funds because the estimated long-horizon alpha is less than -100%. Note that the alternative of focusing on the N^{th} root after winsorizing alpha estimates that are less than -100% would lead to extreme negative estimates for the (mainly) high-beta funds involved. For example, if a fund with a ten-year life had a winsorized alpha of -99%, the implied monthly alpha would be -3.77% per month.

funds. Specific fund filters are described in Appendix B. Prior studies (e.g., Elton, Gruber, and Blake, 2001) have documented the presence of errors in the CRSP mutual fund data. We identify and correct specific potentially influential errors, and omit a small number of funds with apparent data errors that we are not able to correct or verify from alternative sources, as also described in Appendix B. We also exclude funds that have fewer than twelve months of non-missing return data.

Table 2 presents summary statistics regarding the sample, which contains 7,883 domestic equity mutual funds. The sample includes 1,048,111 fund/months. Mean TNA is $1.177 billion. However, the TNA distribution is strongly positively skewed, reflecting the presence of very large funds, and the sample median TNA is $149 million. Figure 2 displays the number of funds contained in the sample and total TNA for sample funds on an annual basis. The number of domestic equity mutual funds increased rapidly from about 1,000 in 1991 to over 3,400 in 2002, remaining relatively constant until 2007, before expanding to approximately 4,300 in 2008. Sample funds' aggregate TNA not only rose rapidly in the early years of the sample period, from about $300 billion in 1991 to approximately $2.8 trillion in 2000, but continued to increase thereafter, to approximately $9.5 trillion in 2020.

To assess performance at longer horizons we compound the monthly returns. Since the returns include any dividends or other cash distributions, we implicitly assume that distributions are reinvested in fund shares.[14] As Pastor and Stambaugh (2012) note, investors cannot directly capture the overall market return, since transaction costs would be incurred at the times of dividends, stock repurchases, or new equity issues. While Pastor and Stambaugh address this issue by deducting a constant fifteen basis points per year from the CRSP value-weighted return, we instead focus on SPY returns, which are net of the

[14] Fama (1972) notes that the assumption that dividends and other distributions are reinvested is desirable when measuring performance over intervals that are longer than the elapsed time between such distributions, because of the implicit assumption that funds invested at the beginning of the sample remain invested throughout. However, he also notes that the approach is "less pure" than some alternatives, because it assumes a reinvestment policy "not followed in the (mutual fund) portfolio."

fund's fees and expenses and could in principle have been captured by investors.[15] Table 2 shows that the pooled mean of matched-month SPY returns is 0.84%.

3.1 A Check on the Modified LL Procedure

The modified LL procedure adjusts for the biases that are revealed by the simulations. However, the simulation as well as expression (6a) itself rely on the simplifying assumption that returns are iid. We therefore provide additional evidence regarding the validity of the procedure. As noted, it is not feasible to estimate betas for long return horizons using standard time series regression methods, due to an insufficient sample size. However, it is viable to obtain estimates of annual horizon betas using time series methods that, while potentially noisy, are unbiased under standard assumptions. In light of the fact that these regressions are estimated from small sample sizes, we employ median regression rather than typical OLS techniques to mitigate the effect of outliers.

In Table 3 we report data regarding annual-horizon beta estimates obtained using time series median regressions (again implementing the winsorization recommendation of Welch, 2021) and those obtained based on the modified LL procedure described above. Results pertain to all funds with at least 120 monthly returns, i.e. where the time series regression includes at least ten 12-month observations.

The results on Table 3 show that the distribution of beta estimates is quite similar across the two methods. Across all 3,768 funds with at least ten annual return observations, the mean annual beta is 1.02 whether estimated by time series methods or by the modified LL method. The correlation coefficient for the annual beta estimates obtained across the two methods is 0.66, which seems reasonable considering that each method provides noisy estimates. Mean annual horizon beta estimates remain similar when the sample is broken into funds with an estimated monthly beta greater versus less than one. For the 2,272 funds with a monthly beta estimate greater than one the mean annual beta estimate is 1.13 when estimated by time series methods and by the modified LL method. For the 1,496 funds with a monthly beta estimate less than one the mean annual beta estimate is 0.84 when estimated by time series regressions

[15] The SPY ETF started trading in January of 1993. For 1991 and 1992, we rely on the return on the Vanguard S&P500 index fund (ticker symbol VFINX) instead.

compared to 0.86 when estimated by the modified LL method. It can be observed that annual beta estimates obtained by the modified LL method are considerably less volatile as compared to annual beta estimates obtained by time series regressions. On balance we view the data reported in Table 3 as supporting the conclusion that the modified LL method provides reasonable beta estimates.

4. Long-Horizon Alpha and Beta Estimates

We implement the modified LL approach described in Section 2.2 to estimate alphas and betas for sample funds at the one-year, five-year and ten-year horizons.

4.1 Empirical Estimates of Long-Horizon Beta

In Table 4 we report on estimates of short-return-horizon (monthly) betas obtained by standard time series regressions and long-return-horizon (annual, five-year, and decadal) betas obtained by the modified LL approach. Here, and in subsequent tables where our emphasis is on comparisons of parameters across return measurement horizons, we focus on the 3,768 sample funds with at least 120 monthly observations, i.e., those funds where we are able to estimate decade horizon parameters, to ensure that differences across return measurement horizons are attributable to horizon per se, not to the use of differing funds at different horizons.

The mean beta estimated in monthly returns across the 3,768 funds (Panel A) is 1.03, while the mean beta estimated by the modified LL method is 1.02 at the annual horizon, 1.03 at the five-year and 1.06 at the decade horizons. We also report results for subsamples delineated by whether the monthly alpha estimate is positive or negative and whether the monthly beta estimate is greater than or less than one. These estimates demonstrate the implications discussed earlier. Mean beta estimates increase with return horizon for funds with large monthly betas and positive monthly alphas (Panel B), from 1.14 at the annual return horizon to 1.51 at the decade horizon. The effect of horizon is attenuated for funds with negative monthly alpha estimates, even if they have large positive monthly beta estimates (Panel C). The mean beta estimate for these funds is 1.12 at the annual horizon, 1.08 at the five-year horizon, and 1.05 at the ten-year horizon. For funds with small monthly beta estimates and positive monthly alpha estimates

(Panel D), estimated betas increase moderately with return measurement horizon, from a mean of 0.86 at the annual horizon to 0.88 at the five-year horizon and to 0.93 at the decade horizon. The smallest long-return-horizon beta estimates are obtained for funds with small monthly beta estimates and negative monthly alpha estimates (Panel E), where the average beta estimates declines from 0.86 at the annual horizon to 0.69 at the decade horizon.

The estimates reported in Table 4 verify that the relation between long-horizon and short-horizon betas depends on magnitudes of both short return horizon beta and alpha estimates. The effect of alpha estimates is empirically more important, as demonstrated by the outcomes on Panels C and D, where betas change in the direction predicted by the effect of the sign of the short horizon alphas rather than in the direction predicted by the magnitude of the short horizon betas.

4.2 Empirical Estimates of Long-Return-Horizon Alpha

Table 5 describes the distributions of fund alpha estimates when returns are measured at various horizons. Panel A contains results pertaining to all sample funds with at least 120 monthly returns. Each alpha estimate, regardless of return horizon, is restated as a monthly equivalent to make estimates directly comparable. While the hypothesis that longer horizon mean alphas are equal to monthly mean alphas can be rejected, mean alphas for the full sample do not differ greatly across return measurement horizons. The cross-fund mean alpha estimate is -0.04% in monthly returns, -0.06% in annual returns, -0.13% in five-year returns and -0.18% in decadal returns.[16]

Empirical estimates of long-horizon alphas are affected by random sampling noise and by changes in true alphas as a function of horizon. The implication of expression (7) that the effect of return horizon on alpha estimates depends on both short horizon alpha and short horizon beta can help to distinguish the effects of changes in true alpha as a function of horizon versus the effects of randomness. In particular, the theory implies that alphas will tend to decrease with return horizon if short-return-

[16] Berk and van Binsbergen (2015) argue that foreign funds, which are not included in our sample, perform better than domestic funds, and report monthly-horizon alphas for their broader sample that do not differ significantly from zero when using returns to an array of Vanguard index funds as benchmarks.

measurement-horizon betas are large. This implication is strongly illustrated by the estimates reported on Panels B and C of Table 5.

Panel B pertains to funds where short horizon beta estimates exceed one and short horizon alpha estimates are positive. For this sample, average alpha estimates decrease from 0.16% at the monthly horizon to 0.14% at the annual horizon, -0.05% at the five-year horizon, and remarkably, to -0.20% at the decade horizon.   Panel C pertains to funds where short horizon beta estimates are large and short horizon alpha estimates are negative. Alpha estimates decrease with horizon for this sample as well, from an average of -0.22% at the monthly horizon to -0.42% at the decade horizon.

Expression (7) also implies that alphas will tend to increase with return horizon if short horizon betas are small. The estimates on Panels D and E of Table 5 illustrate this implication as well, but less strongly. Specifically, the average alpha estimate for funds with small monthly betas and negative monthly alphas grows from -0.14% at the monthly horizon to -0.10% at the decade horizon while the mean estimated alpha for funds with small monthly betas and positive monthly alphas grows from 0.16% at the monthly horizon to 0.22% at the decade horizon.

To provide a sense of the overall variation in alpha estimates as a function of return-measurement horizon, we report in Panel F average pairwise correlations between monthly, 1-year, 5-year and 10-year horizon alphas. Focusing on all 3,768 funds with at least 120 monthly returns, the correlations of longer-horizon with monthly alphas decrease from the 1-year to the 10-year horizons, with the smallest correlation of 0.53 being observed between monthly and 10-year alpha estimates. The correlation decreases are more pronounced for the subsample of funds with short horizon beta estimates greater than one. Specifically, the correlation between 1-month and decade-horizon alpha estimates for funds with estimated monthly SPY betas greater than one is 0.46 versus a correlation of 0.75 between 1-month and 10-year alpha estimates for funds with estimated monthly SPY betas less than one.

The change in alpha estimates and the decrease in correlations between short and long-horizon alphas are suggestive of important differences in the economic information conveyed by alpha estimates across return measurement horizons. Therefore, we next assess the extent to which the signs of alpha estimates are altered by the horizon over which returns are measured. Since mutual fund alpha is often interpreted as informative regarding management skill it is particularly useful to know how often the same dataset can imply positive alpha estimates if returns are measured at one horizon and negative alpha estimates if returns are measured at an alternative horizon.

For the full sample of 3,768 funds with at least 120 monthly returns, the results on Panel G of Table 5 reveal a reduction in the percentage of funds with positive alpha estimates, from 43.2% at the monthly horizon to 30.4% at the decade horizon. However, the effects are asymmetric. Among those funds with positive monthly alpha estimates, 52.6% have positive decade horizon alpha estimates, or equivalently, 47.4% have negative alpha estimates. In contrast, among those funds with negative monthly alpha estimates, a smaller percentage (13.5%) have a decade-horizon alpha estimate with the opposite, i.e. positive, sign. These results suggest that long-horizon investors might consider avoiding funds with negative monthly alphas because such funds are likely to have negative long-horizon alphas. We display in Figure 3 scatter plots of decade-horizon versus monthly-horizon alpha estimates.[17] Fund outcomes displayed in the lower right quadrant and upper left quadrants of Figure 3 are those where the sign of the alpha estimate differs across short versus long return measurement horizons. For example, the lower right quadrant of the "All funds" Panel illustrates the 47.4% of funds with positive monthly alpha estimates and negative decade-horizon alpha estimates.

As anticipated based on expression (7), the divergence in alpha estimates is more notable when focusing on funds with monthly beta estimates greater than one. Panel G of Table 5 shows that among those funds with a monthly beta estimate greater than one and a monthly alpha estimate that is negative, relatively few divergences arise; only 5.0% have a positive alpha estimate based on decadal returns. In contrast, among the funds with a monthly beta estimate greater than one and a monthly return alpha estimate that is positive, only 28.9% have a positive alpha or, equivalently, nearly three quarters (71.1%)

[17] To minimize the impact of outliers, Figure 3 display only those outcomes where the absolute monthly alpha estimate is less than one percent per month.

have a negative alpha estimate based on decade-horizon returns. The scatter plot for this group of funds is illustrated in Figure 3, in the Panel for "Funds with monthly SPY beta >1", in the lower right quadrant of the figure. Thus, for most high beta funds with positive monthly alpha estimates increasing the return measurement horizon results in a negative alpha estimate, even within the same sample. These results also reflect the fact that the effect of return horizon on alpha estimates is asymmetric, being stronger for funds with larger short-horizon beta estimates.

Finally, we report in Panel H of Table 5 information regarding the magnitudes of divergences between short-horizon and long-horizon alpha estimates for sample funds. Large divergences become more common at longer return measurement horizons. In the full sample of 3,768 funds with at least 120 monthly returns, the difference in alpha estimates as compared to those obtained in monthly returns exceeds five percent per year for 0.7% of funds when returns are measured at the annual horizon and for 18.3% of funds when returns are measured at the decade horizon. The differences exceed two percent per year for 4.3% of funds when returns are measured at the annual horizon and for 48.9% of funds when returns are measured at the decade horizon.

The asymmetry predicted by our analysis is also observable in Panel H. Focusing on the subsample of funds with monthly return beta estimates that exceed one and with negative alpha estimates, the percentage of funds where the difference in alpha estimates as compared to the monthly horizon exceeds five percent per year is 1.5% at the annual horizon and 20.8% at the decade horizon, while corresponding results for the subsample with negative monthly alpha estimates and monthly beta estimates that are less than one are 0.1% at the annual horizon and 0.9% at the decade horizon.

On balance, the results reported on Table 5 verify that the information regarding mutual fund performance conveyed by studying compound long-horizon returns differs from that conveyed by monthly returns. Divergences in alpha estimates across long vs. short horizon returns are particularly pertinent for those funds with large estimates of short horizon betas.

4.2.1 Outcomes for Fund Families

The results described to this point either pertain to all funds with sufficient return observations in the sample or to subsamples delineated by the magnitude of short horizon alpha and beta estimates. We next assess outcomes by mutual fund family, for the twenty-five families with the largest combined TNA as of the end of our sample period. Our intent is to assess whether altering the horizon over which returns are measured differentially affects conclusions regarding fund performance across families.

To do so, we implement the procedures described in the prior section for each fund within a family, and then compile average alphas across funds within each family, for returns measured at the one-month, five-year, and ten-year horizons. Outcomes, in each case restated as the monthly equivalent, are reported on Table 6 and show that average alpha estimates for these larger and more successful fund families are less negative than for the full sample. Specifically, the cross-fund mean monthly alpha at the five-year horizon is -0.08% for these large funds, compared to -0.13% for the full sample (Table 5, Panel A).

Changing the horizon over which returns are measured has meaningful impacts on fund average alphas, which are -0.01%, -0.08%, and -0.11% at the one-month, five-year, and ten-year return measurement horizons, respectively. The decrease of 0.10% per month or 1.20% per year when alphas are estimated at the 10-year horizon rather than the one-month horizon is economically substantive, being comparable to average expense ratios. The effect of return measurement horizon differs across fund families, with the spread between 10-year and monthly alphas ranging between -0.26% and 0.24% per month. For the fund family listed in row 7 of Table 6, whose average monthly beta is 1.01, the alpha estimated from monthly returns is (barely) positive, equal to 0.005%, while that estimated from decade-horizon returns is -0.221% per month. The sign of the average monthly alpha differs from the sign of the average decade-horizon alpha for twelve of the twenty-five fund families, and for sixteen of the fund families the absolute difference between the mean monthly-horizon and decade-horizon alphas exceeds ten basis points per month. The upshot is that changes in the horizon over which returns are measured has meaningful and differential effects on measured performance across mutual fund families.

4.2.2 Robustness Tests

In the Internet Appendix we present two sets of robustness tests. The first considers the fact that many investors' rebalancing periods will be less than their investment horizons. To obtain some information regarding the relevance of rebalancing periods we report on estimated alphas and betas for mutual funds that are grouped based on their trading activity, reasoning that funds with higher turnover also effectively have shorter rebalancing periods. Table A1 in the Internet Appendix shows that the effect of return measurement horizon is consistent across turnover terciles.

Linnainmaa (2013) observes that cross-sectional averages of fund-specific alpha estimates yield biased estimates of average fund manager skill, due to the endogenous relation between performance and fund lives. Our focus is not on manager skill per se, but on the distinct issue of how the return measurement horizon affects parameter estimates. Nevertheless, we provide evidence regarding the effect of return measurement horizon for estimated alphas and beta on mutual fund portfolios. Linnainmaa (2013) notes that since returns to such portfolios can be computed for every month of the sample, they are not affected by the biases that he identifies. The results presented in Table A2 of the Internet Appendix show that the effect of return horizon on alpha estimates is broadly similar for mutual fund portfolios as for individual mutual funds.

5. Conclusions

The literature that studies funds' return performance (including mutual funds, hedge funds, pension funds, etc.) is vast, but most of the evidence is based on returns measured over short, most often monthly, horizons. Investment horizons differ across investors and can stretch to decades. While many investors periodically rebalance their portfolios, the parameters of return distributions (means, medians, standard deviations, covariances, skewness, etc.) vary with return horizon in complex and non-linear ways, with or without periodic portfolio rebalancing. We know of no compelling reason to believe that parameters estimated from monthly returns are necessarily the most relevant to investors with disparate investment and decision horizons.

In this paper, we focus attention on the effects of measuring returns over various horizons for estimates of investors mean return after allowing for factor outcomes, i.e. alpha estimates. Importantly, our emphasis is not on forecasting, learning, or time-varying parameters, but simply on the effects of how returns are measured within a given sample. We study U.S. equity mutual funds for the 1991 to 2020 period. Investors are generally concerned with the systematic risk to which they are exposed, and want to know whether the expected return on their position compensates for that risk. In short, they are concerned with alphas and betas. We show theoretically and empirically that the sign of the short-horizon (e.g. monthly) alpha does not necessarily reveal the sign of the longer horizon alpha, and that the relation depends on the magnitude of both short-return-horizon betas and alphas. While these effects are relatively modest over shorter intervals, e.g. daily vs. monthly, they become large when horizons are long. Further, horizon effects are asymmetric. In particular, alphas will tend to increase for funds with smaller short-return-horizon betas and will tend to decrease for funds with larger short-horizon betas as the return measurement interval increases, and more so when short-horizon beta estimates are larger. Long run betas, and by implication, long run alphas are also affected by magnitudes of short run alphas.

The biggest practical hurdle to measuring investment performance over longer return measurement horizons arises from the fact that betas for longer return measurement horizons cannot be estimated by means of standard time series regressions, due to an insufficient number of time series observations. We obtain estimates of long-horizon betas using the theoretical relation between short and long return measurement horizon parameters, and guided by the outcomes of simulations. While the estimates we obtain appear reasonable, they do rely on simplifying assumptions (including that returns are iid over time) that are violated in the data, and we anticipate that future researchers may well be able to refine these methods. Relying on these beta estimates, we show alpha estimates vary meaningfully depending on return measurement horizon.

The interpretation of these results is intrinsically related to the evaluation of asset pricing models. Jensen (1968) introduced alpha as a measure of mutual fund performance, noting (page 390) that "the measure of portfolio performance summarized below is derived from a direct application of the theoretical results of the capital asset pricing models derived independently by Sharpe, Lintner and Treynor." More broadly, a central implication of linear factor-based asset pricing models is that alphas estimated with respect to the model's factors should not differ significantly from zero. However, linear factor models generally apply at a single horizon. The Capital Asset Pricing Model (CAPM) of Sharpe (1964) and Lintner (1965) is explicitly a single period model, though the length of the period is left unspecified. Levhari and Levy (1977) demonstrate that tests of the CAPM are biased if researchers implement tests using returns measured over the wrong horizon. More recently, Chernov, Lochstoer, and Lundeby (2020) observe that a linear factor model that is valid (i.e. generates zero alphas) at a single period horizon does not generally extend (as a linear model in compound factor outcomes) to multi-period horizons.[18]

Our results suggest that the degree to which returns are abnormal cannot be evaluated independent of the question of which return measurement horizon is most relevant to various investors. Further, measures of mutual fund performance that are based on short-horizon returns may be uninformative or even misleading as to whether funds deliver abnormal returns after allowing for beta risk over the longer horizons that may be relevant to many investors.

Our results that beta and alpha depend on the horizon over which returns are measured suggest a number of possible extensions. Alphas and betas depend on return measurement horizon because of the effects of compounding. In particular, a given percentage return during a specified time interval is more (less) influential if it was preceded by positive (negative) returns that increased (decreased) the investment base prior to the start of the interval. We follow Levhari and Levy (1977), Longstaff (1989),

[18] In the case of the CAPM, Cochrane (2005, page 152) observes that the time t "stochastic discount factor" can be stated as $m_t = a + b(1+R_{mt})$, where R_{mt} is the time t return on the market portfolio. The T-period stochastic discount factor is therefore $m_t^T = \prod_{t=1}^{T}(\mathrm{a} + \mathrm{b}(1 + \mathrm{Rmt}))$, which contains interaction terms and cannot be stated as a linear function of the compound market return $\prod_{t=1}^{T}(1 + R_{mt}) - 1$ alone.

and Kothari, Shanken, and Sloan (1995) in modeling the effect of return measurement horizon on beta while focusing on total returns that include both cash distribution and capital gain components. An implicit assumption is that multiperiod returns include the effects of reinvesting cash disbursements. However, mutual fund investors may or may not in aggregate reinvest dividends or other net cash disbursements, implying that the effect of horizon on beta will be muted or amplified depending on the rate of net cash disbursements via mutual fund flows. It would be interesting to refine our analysis to allow for this consideration and assess if the resulting changes in long-horizon alpha and beta estimates meaningfully affect long-horizon performance measures. Similarly, any growth in the investment base that simply reflect the effects of inflation need not imply greater changes in investor utility because of given percentage returns, so it would be of interest to assess if a focus on real rather than nominal returns has a noteworthy effect on inferences regarding investment performance measured over various horizons. Further, while our central conclusion that alphas depend on return measurement horizon is likely to be robust, point estimates would be altered if the sample were broadened, e.g. to include international, target date, and levered funds, or narrowed to focus on specific fund categories. Outcomes will also vary if multi-factor or style-specific benchmarks are employed.

Perhaps the most intriguing questions are related to the assessment of which return measurement horizon is most relevant to investors. Some relevant evidence might be obtained by assessing the return measurement horizon that best explains relations between mutual fund performance and fund flows originating with various types of investors. Additional evidence might be provided by assessing the return measurement horizon for which various asset pricing models best perform.

Appendix A: Instantaneous and Long-Horizon Alphas and Betas

In this appendix, we derive the relationship between instantaneous alpha and long-horizon alpha in a continuous time CAPM. Let the market price P^m and asset i's idiosyncratic return V^ϵ follows Brownian motions:

$$\frac{dP^m}{P^m} = \mu_m dt + \sigma_m dZ^m$$

$$\frac{dV^\epsilon}{V^\epsilon} = \sigma_\epsilon dZ^\epsilon.$$

Applying Ito's lemma yields:

$$d\log P^m = \left(\mu_m - \frac{\sigma_m^2}{2}\right)dt + \sigma_m dZ^m$$

$$d\log V^\epsilon = -\frac{\sigma_\epsilon^2}{2}dt + \sigma_\epsilon dZ^\epsilon.$$

Long-horizon market return and return variance are as follows:

$$\log P_T^m - \log P_0^m = \left(\mu_m - \frac{\sigma_m^2}{2}\right)T + \sigma_m \int_0^T dZ^m$$

$$E\left(\frac{P_T^m}{P_0^m}\right) = E\left[e^{\left(\mu_m - \frac{1}{2}\sigma_m^2\right)T + \sigma_m \int_0^T dZ^m}\right] = e^{\mu_m T}$$

$$Var\left(\frac{P_T^m}{P_0^m}\right) = E\left[\left(\frac{P_T^m}{P_0^m}\right)^2\right] - E\left[\left(\frac{P_T^m}{P_0^m}\right)\right]^2 = E\left[e^{\left(2\mu_m - \sigma_m^2\right)T + 2\sigma_m \int_0^T dZ^m}\right] - e^{2\mu_m T}$$

$$= e^{\left(2\mu_m + \sigma_m^2\right)T} - e^{2\mu_m T} = \left(e^{\sigma_m^2 T} - 1\right)e^{2\mu_m T}$$

The asset's instantaneous return follows:

$$\frac{dP_i}{P_i} = \alpha_i dt + \beta_i \frac{dP^m}{P^m} + \frac{dV^\epsilon}{V^\epsilon} = \alpha_i dt + \beta_i(\mu_m dt + \sigma_m dZ^m) + \sigma_\epsilon dZ^\epsilon$$

$$= (\alpha_i + \beta_i \mu_m)dt + \beta_i \sigma_m dZ^m + \sigma_\epsilon dZ^\epsilon$$

Applying Ito's lemma to the last equation yields:

$$d\log P_i = \left(\alpha_i + \beta_i \mu_m - \frac{\beta_i^2 \sigma_m^2}{2} - \frac{\sigma_\epsilon^2}{2}\right)dt + \beta_i \sigma_m dZ^m + \sigma_\epsilon dZ^\epsilon$$

The asset's long-horizon return and return variance are as follows:

$$\log P_T^i - \log P_0^i = \left(\alpha_i + \beta_i\mu_m - \frac{\beta_i^2\sigma_m^2}{2} - \frac{\sigma_\epsilon^2}{2}\right)T + \beta_i\sigma_m\int_0^T dZ^m + \sigma_\epsilon\int_0^T dZ^{\epsilon\frac{dV^\epsilon}{V^\epsilon}}$$

$$E\left(\frac{P_T^i}{P_0^i}\right) = E\left[e^{\left(\alpha_i+\beta_i\mu_m-\frac{\beta_i^2\sigma_m^2}{2}-\frac{\sigma_\epsilon^2}{2}\right)T+\beta_i\sigma_m\int_0^T dZ^m+\sigma_{\epsilon\int_0^T dZ^\epsilon}}\right]$$

$$= e^{\left(\alpha_i+\beta_i\mu_m-\frac{\beta_i^2\sigma_m^2}{2}-\frac{\sigma_\epsilon^2}{2}\right)T+\left(\frac{\beta_i^2\sigma_m^2}{2}+\frac{\sigma_\epsilon^2}{2}\right)T} = e^{(\alpha_i+\beta_i\mu_m)T}$$

$$Var\left(\frac{P_T^i}{P_0^i}\right) = E\left[\left(\frac{P_T^i}{P_0^i}\right)^2\right] - E\left[\left(\frac{P_T^i}{P_0^i}\right)\right]^2$$

$$= E\left[e^{2\left(\alpha_i+\beta_i\mu_m-\frac{\beta_i^2\sigma_m^2}{2}-\frac{\sigma_\epsilon^2}{2}\right)T+2\sigma_m\int_0^T dZ^m+2\sigma_\epsilon\int_0^T dZ^\epsilon}\right] - e^{2(\alpha_i+\beta_i\mu_m)T}$$

$$= e^{2\left(\alpha_i+\beta_i\mu_m-\frac{\beta_i^2\sigma_m^2}{2}-\frac{\sigma_\epsilon^2}{2}\right)T+2\beta_i^2\sigma_m^2T+2\sigma_\epsilon^2T} - e^{2(\alpha_i+\beta_i\mu_m)T}$$

$$= \left(e^{(\beta_i^2\sigma_m^2+\sigma_\epsilon^2)T} - 1\right)e^{2(\alpha_i+\beta\mu_m)T}$$

$$Cov\left(\frac{P_T^i}{P_0^i}, \frac{P_T^m}{P_0^m}\right) = E\left[\left(\frac{P_T^i}{P_0^i}\frac{P_T^m}{P_0^m}\right)\right] - E\left[\frac{P_T^i}{P_0^i}\right]E\left[\frac{P_T^m}{P_0^m}\right]$$

$$= E\left[e^{\left(\alpha_i+\beta_i\mu_m-\frac{\beta_i^2\sigma_m^2}{2}-\frac{\sigma_\epsilon^2}{2}\right)T+\beta_i\sigma_m\int_0^T dZ^m+\sigma_\epsilon\int_0^T dZ^\epsilon}e^{\left(\mu_m-\frac{\sigma_m^2}{2}\right)T+\sigma_m\int_0^T dZ^m}\right] - e^{(\alpha_i+\beta_i\mu_m)T}e^{\mu_mT}$$

$$= E\left[e^{\left[\alpha_i+(\beta_i+1)\mu_m-\frac{1}{2}(\beta_i^2\sigma_m^2+\sigma_m^2+\sigma_\epsilon^2)\right]T+(\beta_i+1)\sigma_m\int_0^T dZ^m+\sigma_\epsilon\int_0^T dZ^\epsilon}\right] - e^{[\alpha_i+(\beta_i+1)\mu_m]T}$$

$$= e^{\left[\alpha_i+(\beta_i+1)\mu_m-\frac{1}{2}(\beta_i^2\sigma_m^2+\sigma_m^2+\sigma_\epsilon^2)\right]T+\frac{1}{2}(\beta_i+1)^2\sigma_m^2T+\frac{1}{2}\sigma_\epsilon^2T} - e^{[\alpha_i+(\beta_i+1)\mu_m]T}$$

$$= \left(e^{\beta_i\sigma_m^2T} - 1\right)e^{[\alpha_i+(\beta_i+1)\mu_m]T}$$

The asset's long-horizon beta is:

$$\beta_i^L = \frac{Cov\left(\frac{P_T^i}{P_0^i}, \frac{P_T^m}{P_0^m}\right)}{Var\left(\frac{P_T^m}{P_0^m}\right)} = \frac{\left(e^{\beta_i \sigma_m^2 T} - 1\right) e^{[\alpha_i + (\beta_i + 1)\mu_m]T}}{\left(e^{\sigma_m^2 T} - 1\right) e^{2\mu_m T}}$$

The relationship between long-horizon alpha and instantaneous alpha is:

$$E\left(\frac{P_T^i}{P_0^i}\right) - 1 = \alpha_i^L + \beta_i^L \left[E\left(\frac{P_T^m}{P_0^m}\right) - 1\right]$$

$$e^{(\alpha_i + \beta_i \mu_m)T} - 1 = \alpha_i^L + \frac{\left(e^{\beta_i \sigma_m^2 T} - 1\right) e^{[\alpha_i + (\beta_i + 1)\mu_m]T}}{\left(e^{\sigma_m^2 T} - 1\right) e^{2\mu_m T}} (e^{\mu_m T} - 1)$$

$$\alpha_i^L = e^{(\alpha_i + \beta_i \mu_m)T} - 1 - \frac{e^{[\alpha_i + (\beta_i + 1)\mu_m]T} \left(e^{\beta_i \sigma_m^2 T} - 1\right)}{\left(e^{\sigma_m^2 T} - 1\right) e^{2\mu_m T}} (e^{\mu_m T} - 1)$$

The relationship between long-horizon alpha and instantaneous alpha is non-linear and complicated. Long-horizon and instantaneous alphas could have different signs. They have the same sign when instantaneous beta is one. When $\beta_i = 1$, $\alpha_i^L = e^{\alpha_i T} - 1$ and has the same sign as α_i.

Appendix B: Sample Construction and Data Filters

We obtain data for the 1991 to 2020 period from the CRSP survivorship bias free Mutual Fund Database. We begin at 1991, as data regarding fund total net assets (TNA), which we use to aggregate fund returns across share classes, is not consistently available for earlier periods. We rely on the CRSP share class group number (*crsp_cl_grp*) in the fund names file. For funds without a CRSP share class group number, we identify share classes of the same fund based on fund names. When funds have multiple share classes CRSP fund names contain "/" or ";". The part of the fund name after the last "/" or ";" refers to the sub share class, while the prior part refers to the main fund name. For example, the fund named "MainStay Funds: MainStay Small Cap Growth Fund; Class A Shares" is Class A of the MainStay Small Cap Growth Fund; the fund named "Alliance Strategic Balanced Fund/A" is Class A of the Alliance Strategic Balanced Fund.

We study domestic equity funds (CRSP fund style code starting with "ED"), while excluding exchange traded funds, exchange traded notes (those with CRSP *et_flag* equal to "F" or "N"), funds that take short positions (CRSP fund style "EDYS"), commodity funds (CRSP fund style "EDSC") and real estate funds (CRSP fund style "EDSR"). We exclude target date funds, since these hold substantial non-equity positions. To exclude target date funds and college savings funds we remove all funds with names that contain a four-digit number between 1990 and 2050 and the word "target", except that we do not exclude funds with "Russell 2000" or "Russell2000" in their names.

We further exclude hedged funds (CRSP fund style of "EDYH" and Lipper fund style code of "LSE"), market neutral funds (CRSP fund style of "EDYH" and Lipper fund style code of "EMN") and absolute return funds (CRSP fund style of "EDYH" and Lipper fund style code of "ABR"). We also screen some funds within the CRSP style code starting with "ED", but with names that are inconsistent with this categorization. Specifically, we exclude a fund with "VIX" in its name, funds with "Long/Short", "Long-Short", and "OTC/Short" in their names, funds whose name includes "ETF" or

“ETN”, leveraged funds with “1.25x”, “1.5x”, “2x”, “2.5x”, “3x”, and “4x” in their names, and one fixed income fund with “Government Portfolio” in its name.

Prior studies (e.g., Elton, Gruber, and Blake, 2001) have documented the presence of errors in the CRSP mutual fund data. We mitigate the effect of potentially influential errors by comparing large reported returns to those contained in the Morningstar Mutual Fund database, or if Morningstar data is not available to the returns implied by percentage changes in CRSP-reported NAV or TNA. Specifically, we identify 836 extreme fund returns based on a deviation from the same-month CRSP value-weighted market return of 30% or more. With the help of Professor Shuaiyu Chen, we are able to match 633 of these to monthly return data in the Morningstar mutual fund database. For 524 of these cases, the CRSP and Morningstar returns differ by less than 1% and we retain the CRSP return. For the remaining 109 cases we retain the Morningstar return, which in every instance is less extreme that the CRSP return. For the 203 instances that cannot be matched to Morningstar, we focus on the percentage change in the CRSP reported net-asset-value (NAV) as well as the TNA. We retain observations where the reported return deviates from both the NAV and TNA-implied returns by less than 30%. For 75 observations from 53 funds the deviation exceeds 30%, and we delete the associated funds from the sample. Finally, we exclude funds that have fewer than twelve months of non-missing return data. The sample employed here is also used by Bessembinder, Cooper, and Zhang (2023).

References

Ameriks, John, and Stephen Zeldes, 2004, How do household portfolio shares vary with age?. Available at www0.gsb.columbia.edu/mygsb/faculty/research/pubfiles/16/Ameriks_Zeldes_age_Sept_2004d.pdf

Barber, Brad, Xing Huang, and Terrance Odean, 2016, Which factors matter to investors? Evidence from mutual fund flows, *Review of Financial Studies* 29, 2600-2642.

Ben-David, Itzhak, Jiacui Li, Andrea Rossi, and Yang Song, 2022. What do mutual fund investors really care about? *Review of Financial Studies* 35, 1723–1774.

Barberis, Nicholas, 2000, Investing for the long run when returns are predictable, *Journal of Finance* 60, 225-264.

Bessembinder, Hendrik, Michael Cooper, and Feng Zhang, 2023, Mutual fund performance at long horizons, *Journal of Financial Economics* 147, 132-158.

Berk, Jonathan, and Jules H. van Binsbergen, 2016, Assessing asset pricing models using revealed preference, *Journal of Financial Economics* 119, 1-23.

Boguth, Oliver, Murray Carlson, Adlai Fisher, and Mikhail Simutin, 2016, Horizon effects in average returns: The role of slow information diffusion, *Review of Financial Studies* 29, 2241-81.

Campbell, John, and Luis Viceira, 1999, Consumption and portfolio decisions when expected returns are time varying, *Quarterly Journal of Economics* 114, 433-495.

Campbell, John and Luis Viceira, 2002, *Strategic Asset Allocation: Portfolio Choice for Long-term Investors*, Oxford University Press.

Carhart, Mark, 1997, On persistence in mutual fund performance, *Journal of Finance* 52, 57-87.

Chernov, Mikhail, Lars Lochstoer, and Stig Lundeby, 2022, Conditional dynamics and the multi-horizon risk-return tradeoff, *Review of Financial Studies* 35, 1310-1347.

Cremers, K.J. Martijn, Jon A. Fulkerson, and Timothy B. Riley, 2019, Challenging the conventional wisdom on active management: A review of the past 20 years of academic literature on actively managed mutual funds, *Financial Analysts Journal* 75, 8-35.

Cremers, Martijn and Ankur Parkeek, 2016, Patient capital outperformance: The investment skill of high active share managers who trade infrequently, *Journal of Financial Economics* 122, 288-306.

Cochrane, John, 2005, *Asset Pricing*, Princeton University Press.

Cochrane, John. 2022, Portfolios for long-term investors, *Review of Finance* 26, 1-42.

Cochrane, John, 2014, A mean-variance benchmark for intertemporal portfolio theory. *Journal of Finance* 69, 1-49.

Cochrane, John, 2011, Presidential address: Discount rates, *Journal of Finance* 66, 1047-1108.

Elton, Edwin J., Martin J. Gruber, and Christopher R. Blake, 2001, A first look at the accuracy of the CRSP mutual fund database and a comparison of the CRSP and Morningstar mutual fund databases, *Journal of Finance* 56, 2415–2430.

Fama, Eugene, 1972, Components of investment performance, *Journal of Finance* 27, 551-567.

Fama, Eugene, and Kenneth French, 2015, A five-factor asset pricing model, *Journal of Financial Economics* 116, 1–22.

Farago, A., Hjalmarsson, E., 2023, Compound returns are positively skewed, *Review of Finance* 27, 495-538.

Gilbert, Thomas, Christopher Hrdlicka, Jonathan Kalodimos, and Stephan Siegel, 2014, Daily data is bad for beta: Opacity and frequency-dependent betas, *Review of Asset Pricing Studies* 4, 78-117.

Handa, Puneet, Kothari, S. P. and Wasley, Charles, 1989, The relation between the return interval and betas: Implications for the size effect, *Journal of Financial Economics* 23, 79-100,

Jensen, Michael, 1968, The performance of mutual funds in the period 1945-1967, *Journal of Finance* 23, 389-416.

Kamara, Avraham, Robert Korajczyk, Xiaoxia Lou, and Ronnie Sadka, 2016, Horizon pricing, *Journal of Financial and Quantitative Analysis* 51, 1769-1793.

Kothari, S.P., Jay Shanken, and Richard Sloan, 1995, Another look at the cross-section of expected stock returns, *Journal of Finance* 50, 185-224.

Lee, Cheng, Chunchi Wu, and K.C. John Wei, 1990, The heterogeneous investment horizon and the capital asset pricing model, *Journal of Financial and Quantitative Analysis* 25, 361-376.

Levhari, David, and Haim Levy, 1977, The capital asset pricing model and the investment horizon, *Review of Economics and Statistics* 54, 92–104.

Levy, Haim, and Moshe Levy, 2011, The small firm effect: A financial mirage?, *Journal of Portfolio Management* 37, 129-139.

Lewellen, J. and S. Nagel, 2006, The conditional CAPM does not explain asset-pricing anomalies, *Journal of Financial Economics* 82, 289–314.

Linnainmaa, Juhani, 2013, Reverse survivorship bias, *Journal of Finance* 68, 789-813.

Lintner, John, 1965, Security prices, risk, and maximal gains from diversification, *Journal of Finance* 20, 587-616.

Longstaff, Frances, 1989, Temporal aggregation and the continuous-time Capital Asset Pricing Model, *Journal of Finance* 44, 871-887.

Merton, Robert, 1973, An intertemporal asset pricing model, *Econometrica* 61, 867-887.

P̌astor, Lubos and Robert Stambaugh, 2012, On the size of the active management industry, *Journal of Political Economy* 120, 740–781.

Samuelson, Paul, 1969, Lifetime portfolio selection by dynamic stochastic programming, *The Review of Economics and Statistics* 51, 239-246.

Sharpe, William, 1964, Capital asset prices: A theory of market equilibrium under conditions of risk, *Journal of Finance* 19, 425-442.

Smith, Kathleen, 2015, The investment horizon and asset pricing models, *Australian Journal of Management*, 40, 277-294.

Vasicek, Oldrich A., 1973, A note on using cross-sectional information in Bayesian estimation of security beta, *Journal of Finance*, 28, 1233–1239.

Vanguard Group, 2020, How America invests: A look at the personal investing behavior of Vanguard's retail households with assets of $500,000 or more. Available at https://advisors.vanguard.com/iwe/pdf/FASHAIFR.pdf.

Welch, Ivo, 2021, Simply better market betas. Available at https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3371240

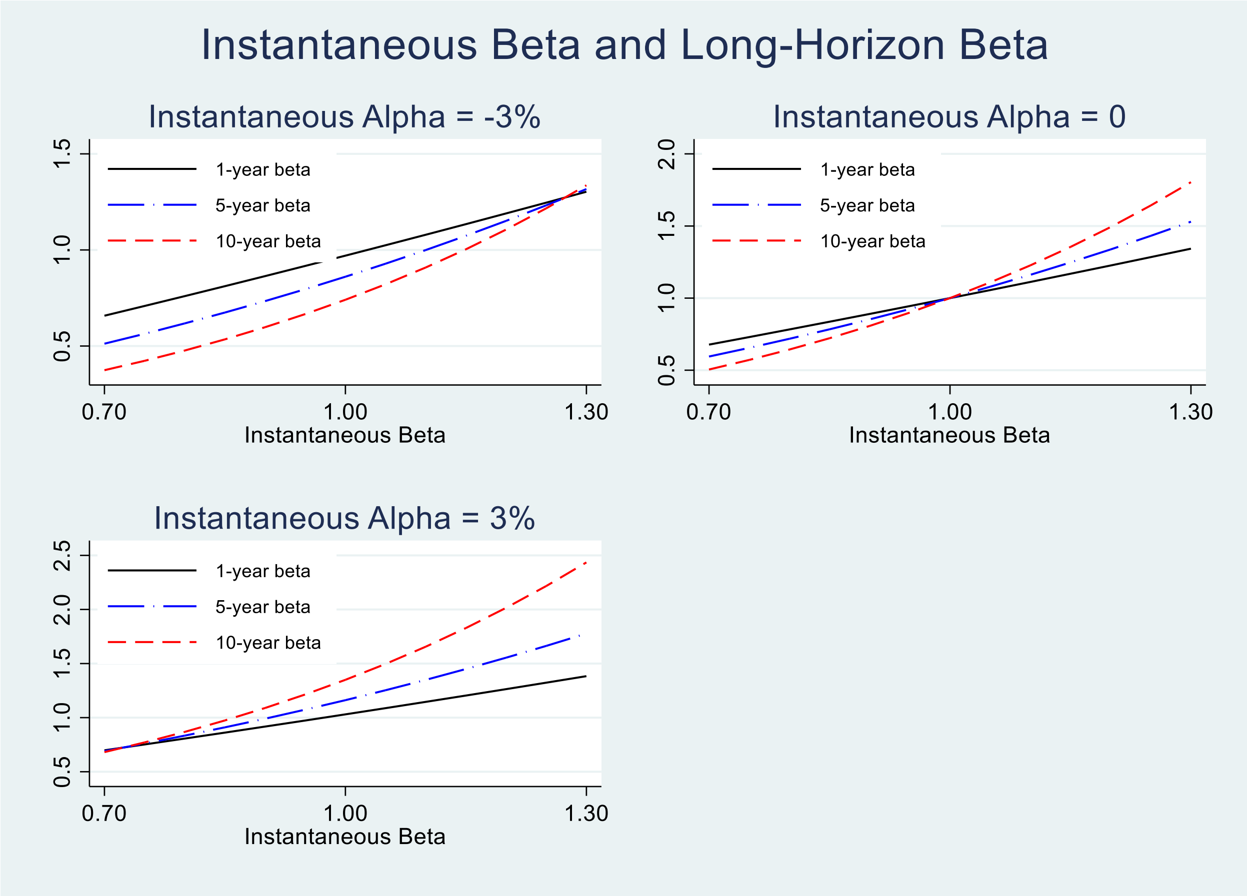

Figure 1, Panel A
Relation between instantaneous beta and long-horizon beta.
This figure displays long-horizon (1, 5, and 10 year) betas implied by text equation (6), for a variety of possible instantaneous betas and alphas. The computations incorporate a mean instantaneous market return of 9% per year and a standard deviation of the instantaneous market return of 19% per year. The figure reveals that long-horizon beta depends on not only the instantaneous beta but also the instantaneous alpha, and that the relationship between long-horizon and short-horizon beta is non-linear. When instantaneous alpha is zero, long-horizon beta is greater (smaller) than short-horizon beta if short-horizon beta is above (below) one.

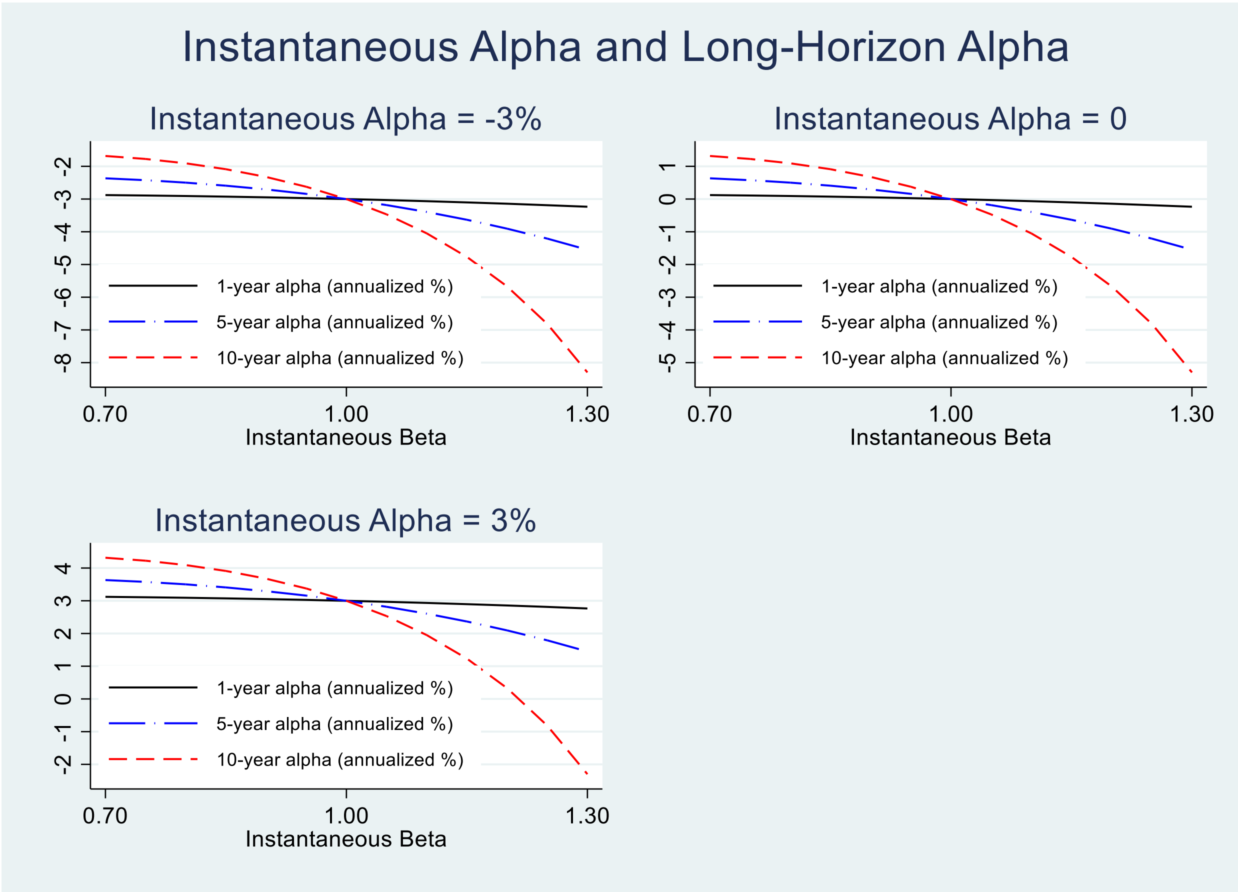

Figure 1, Panel B
Relation between instantaneous beta and long-horizon alpha
This figure displays long-horizon (1, 5, and 10 year) annualized alphas implied by text equation (6), for a variety of possible instantaneous betas and alphas. The computations incorporate a mean instantaneous market return of 9% per year and a standard deviation of the instantaneous market return of 19% per year. The figure reveals that long-horizon and short-horizon alphas differ considerably and may have different signs. Short-horizon alphas could be misinformative or even misleading about funds' long-run performance.

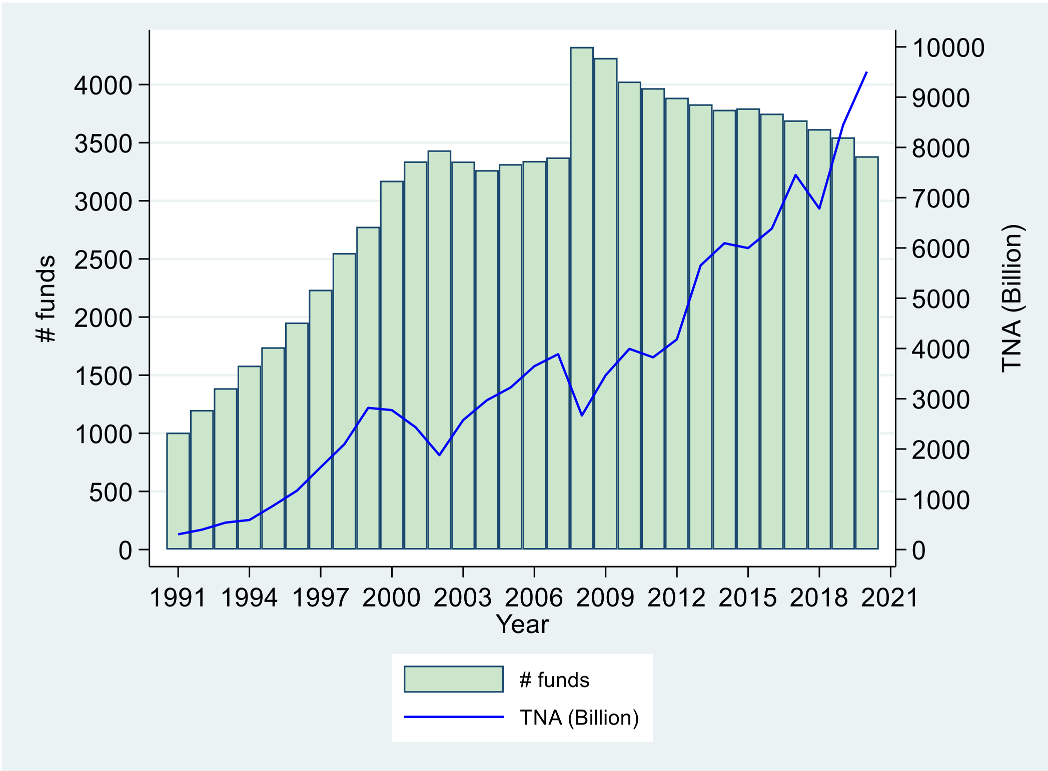

Figure 2
Number of funds and aggregate TNA, by year
This figure plots the annual number of sample equity funds (left axis) and the aggregate TNA in $Billion (right axis) in each year from 1991 to 2020.

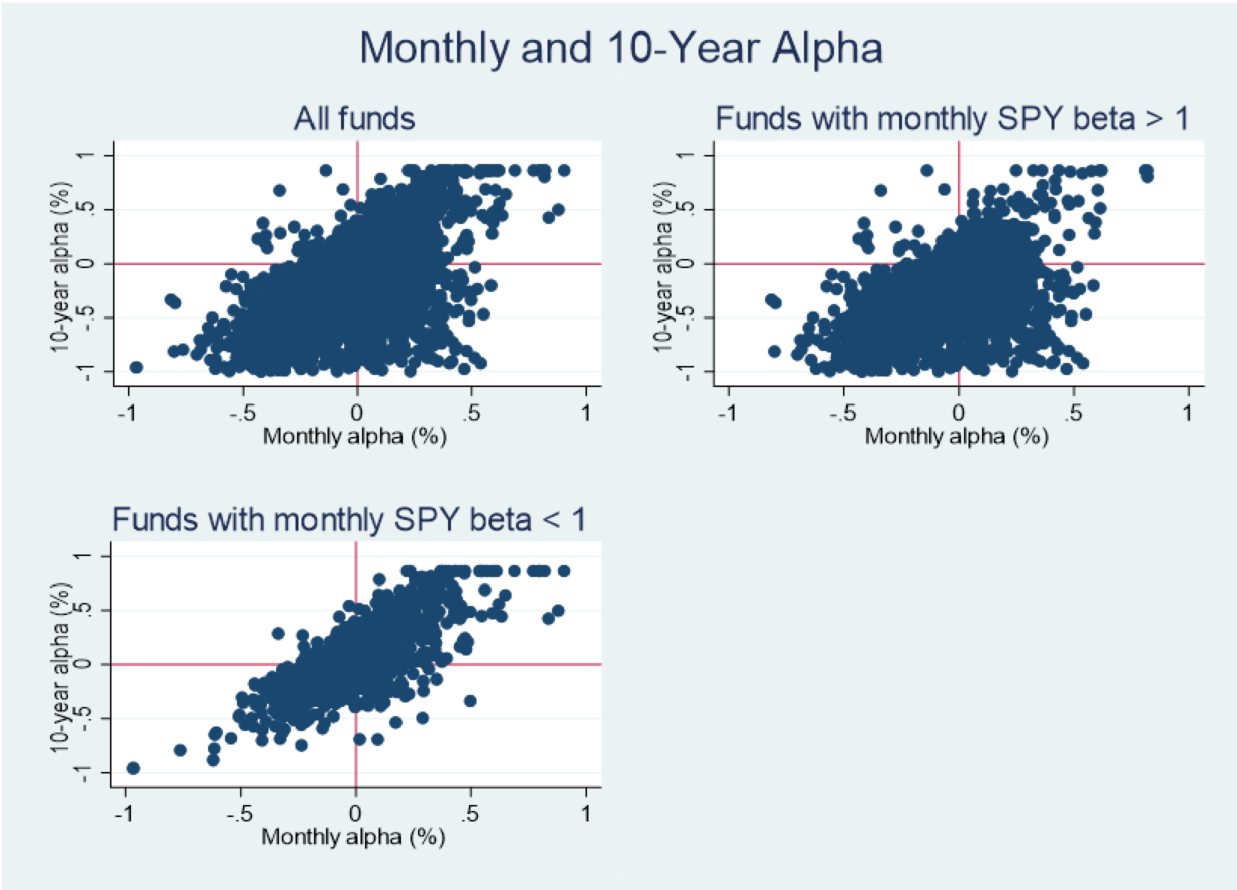

Figure 3
Plots of fund monthly alphas and 10-year alphas
This figure displays plots of fund monthly and 10-year alphas for All funds, Funds with monthly SPY beta >1 and Funds with monthly SPY beta <1. In these figures, to minimize the visual impact of outliers, we plot the funds with abs(monthly alpha) < 1% and abs(10-year alpha) < 1%/month. This figure reveals that 10-year alphas differ significantly from monthly alphas. For funds with positive monthly alpha and monthly beta above one, the majority (71.1%) of them have negative l0-year alpha. For funds with negative monthly alpha and monthly beta below one, 28.7% of them have positive l0-year alpha.

Table 1: Estimating relations between the true long-horizon beta and the sample long-horizon beta

This table presents summary statistics for regression results of the true long-horizon beta on the long-horizon beta computed from an observed sample using bootstrap simulations. In each simulation, we generate a true monthly beta for each fund and compute the corresponding true N-month beta (β_i^L) using Equation (6a) in the text. We then generate a random sample of N-month fund excess returns and SPY excess returns using the true monthly beta and other parameters detailed in Section 2.2. Lastly, we estimate the fund's monthly beta in this randomly generated sample and compute the corresponding N-month beta ($\hat{\beta}_i^{LS}$) estimate using Equation (6a) in the text and other parameters estimated from this random sample. We repeat the simulation 1,000 times for each fund and then estimate the following regression: $\beta_i^L = a + b * \hat{\beta}_i^{LS} + u$. We consider three long-horizon investment horizons for each fund: 1-year, 5-year, and 10-years. This table shows that long-horizon betas based on Equation (6a) and monthly beta estimate are likely biased, and it is necessary to correct the bias using the relationship between true and estimated long-horizon beta based on bootstrap simulations.

Variable	N	mean	sd	p5	p25	p50	p75	p95
				1-year beta				
a_hat	7883	0.394	0.264	0.067	0.161	0.350	0.575	0.903
b_hat	7883	0.623	0.221	0.252	0.440	0.640	0.827	0.923
R-squared	7883	0.529	0.243	0.154	0.318	0.526	0.755	0.881
				5-year beta				
a_hat	5578	0.273	0.231	0.034	0.086	0.200	0.397	0.762
b_hat	5578	0.748	0.179	0.396	0.634	0.791	0.903	0.954
R-squared	5578	0.670	0.207	0.278	0.526	0.712	0.852	0.923
				10-year beta				
a_hat	3768	0.266	0.236	0.038	0.086	0.191	0.368	0.766
b_hat	3768	0.778	0.150	0.473	0.687	0.813	0.904	0.952
R-squared	3768	0.707	0.176	0.364	0.589	0.740	0.858	0.920

Table 2: Summary statistics of fund return, expense ratio and TNA

This table reports summary statistics of fund expense ratios and TNA at the fund-month level, as well as monthly fund returns and monthly returns to the CRSP value-weighted market portfolio and the SPDR S&P 500 ETF. Our sample includes 7,883 U.S. equity mutual funds from 1991 to 2020.

Variable	# unique funds	# fund-months	Mean	Median	Std. dev.	Skewness
Fund return (%), monthly	7,883	1,048,111	0.776	1.158	5.419	-0.425
Market return (%), monthly	7,883	1,048,111	0.882	1.380	4.496	-0.626
SPY return (%), monthly	7,883	1,048,111	0.835	1.328	4.332	-0.616
Fees (%), monthly	7,883	1,048,111	0.095	0.094	0.049	1.583
TNA ($B), monthly	7,883	1,048,111	1.177	0.149	7.703	42.553

Table 3: Comparing annual betas estimated using the modified LL approach to annual beta estimated based on time-series return regressions.

For those mutual funds with at least 120 monthly returns, we estimate the 1-year (i.e., 12-month) beta against the S&P 500 ETF (SPY) by estimating time series median regressions of excess 12-month fund returns on excess 12-month SPY returns and by the modified Levhari and Levy (LL) approach described in Section 2.2. This table presents summary statistics of the monthly beta, the annual beta estimated from the modified LL approach, and the annual beta estimated from the conventional time series regressions. Following Welch (2021), we winsorize fund returns at -2 times and 4 times the contemporaneous SPY returns when estimating monthly and annual betas using time-series regressions. This table shows that 1-year betas estimated from our modified LL method are close to 1-year betas estimated from time-series regressions. The results help validate our modified LL method.

Variable	N	Mean	Median	Std. dev.	Skewness
All funds					
Monthly beta	3768	1.025	1.027	0.202	0.184
1-year beta estimated from modified LL method	3768	1.020	1.028	0.173	-0.632
1-year beta estimated from actual returns	3768	1.016	0.998	0.300	1.566
Funds with monthly beta against SPY > 1					
Monthly beta	2272	1.133	1.096	0.149	3.706
1-year beta estimated from modified LL method	2272	1.125	1.104	0.101	0.745
1-year beta estimated from actual returns	2272	1.132	1.080	0.288	3.069
Funds with monthly beta against SPY < 1					
Monthly beta	1496	0.861	0.912	0.157	-2.899
1-year beta estimated from modified LL method	1496	0.862	0.906	0.134	-1.370
1-year beta estimated from actual returns	1496	0.840	0.878	0.223	-1.766

Table 4: Long-horizon beta versus short-horizon beta

We compute each fund's monthly beta by regressing excess monthly fund return on excess return to the SPDR S&P 500 ETF (SPY) and compute its long-horizon beta against SPY over three return horizons (1 year, 5 years, and 10 years) using the modified Levhari and Levy (LL) approach described in Section 2.2. This table compares the monthly versus long-horizon fund betas for all funds with at least 120 monthly returns and for four sub-groups of funds depending on whether their monthly SPY beta is above 1 and whether their monthly SPY alpha is positive. Following Welch (2021), we winsorize fund returns at -2 times and 4 times the contemporaneous SPY returns when estimating monthly betas using time-series regressions. This table shows that long-horizon beta is greater (smaller) than monthly beta for funds with positive (negative) monthly alpha.

Beta	N	Mean	Median	Std. dev.	Skewness
Panel A: All funds					
Monthly beta	3768	1.025	1.027	0.202	0.184
1-year beta	3768	1.020	1.028	0.173	-0.632
5-year beta	3768	1.027	1.014	0.251	0.029
10-year beta	3768	1.061	1.003	0.395	0.988
Panel B: Funds with monthly SPY beta > 1 & monthly SPY alpha > 0					
Monthly beta	903	1.116	1.088	0.114	3.885
1-year beta	903	1.139	1.119	0.094	0.861
5-year beta	903	1.281	1.249	0.176	0.921
10-year beta	903	1.509	1.441	0.347	1.312
Panel C: Funds with monthly SPY beta > 1 & monthly SPY alpha < 0					
Monthly beta	1369	1.144	1.102	0.167	3.431
1-year beta	1369	1.116	1.090	0.104	0.751
5-year beta	1369	1.078	1.049	0.157	1.459
10-year beta	1369	1.046	1.008	0.257	2.560
Panel D: Funds with monthly SPY beta < 1 & monthly SPY alpha > 0					
Monthly beta	723	0.846	0.898	0.157	-1.648
1-year beta	723	0.861	0.902	0.142	-1.256
5-year beta	723	0.882	0.923	0.200	-0.981
10-year beta	723	0.928	0.944	0.285	-0.139
Panel E: Funds with monthly SPY beta < 1 & monthly SPY alpha < 0					
Monthly beta	773	0.875	0.927	0.156	-4.161
1-year beta	773	0.862	0.908	0.127	-1.503
5-year beta	773	0.775	0.817	0.161	-1.175
10-year beta	773	0.688	0.721	0.191	-0.759

Table 5: Long-horizon alpha versus short-horizon alpha and beta

We compute each fund's monthly beta by regressing excess monthly fund return on excess return to the SPDR S&P 500 ETF (SPY) and compute its long-horizon beta against SPY over three investment horizons (1 year, 5 years, and 10 years) using the modified Levhari and Levy (LL) approach detailed in Section 2.2. Lastly, we compute each fund's long-horizon alpha using its long-horizon returns and the long-horizon beta. Panels A-E report the monthly and long-horizon fund alphas for all funds with at least 120 monthly returns and for four sub-groups of funds depending on whether their monthly SPY beta is above 1 and whether their monthly SPY alpha is positive. Each mean estimate differs significantly from zero, and each long horizon estimates differs significantly from the corresponding monthly estimates, each at the .01 level. Panel F reports the pairwise correlation coefficients between the alphas at different investment horizons for all funds with at least 120 monthly returns and subsamples. Panel G compares the sign of the alpha at different investment horizons for all funds with at least 120 monthly returns and subsamples. Panel H presents the fraction of funds whose long-horizon and short-horizons differ by 1%, 2%, and 5% per year, respectively, for all funds with at least 120 monthly returns and subsamples. This tables shows that long-horizon alphas are significantly smaller than monthly alphas, and the spread between monthly and long-horizon alphas depends on the fund's monthly beta and alpha. Overall, monthly alphas based on conventional time-series regressions are misinformative or even misleading about funds' long-horizon performance.

Panels A-E: Summary statistics of long-horizon alpha and short-horizon alpha against SPY

	N	Mean	Median	Std. dev.	Skewness
Panel A: All funds					
Monthly alpha (%)	3768	-0.042	-0.028	0.263	-2.350
1-year alpha (%, monthly rate)	3768	-0.057	-0.043	0.258	-1.012
5-year alpha (%, monthly rate)	3768	-0.125	-0.103	0.293	-0.601
10-year alpha (%, monthly rate)	3768	-0.178	-0.129	0.412	-0.572
Panel B: Funds with monthly SPY beta SPY > 1 & monthly SPY alpha > 0					
Monthly alpha (%)	903	0.160	0.128	0.139	1.668
1-year alpha (%, monthly rate)	903	0.136	0.098	0.178	2.060
5-year alpha (%, monthly rate)	903	-0.051	-0.053	0.254	-0.022
10-year alpha (%, monthly rate)	903	-0.196	-0.154	0.418	-0.520
Panel C: Funds with monthly SPY beta SPY > 1 & monthly SPY alpha < 0					
Monthly alpha (%)	1369	-0.223	-0.162	0.265	-5.146
1-year alpha (%, monthly rate)	1369	-0.236	-0.184	0.239	-2.993
5-year alpha (%, monthly rate)	1369	-0.304	-0.253	0.273	-1.382
10-year alpha (%, monthly rate)	1369	-0.421	-0.340	0.374	-0.835
Panel D: Funds with monthly SPY beta SPY < 1 & monthly SPY alpha > 0					
Monthly alpha (%)	723	0.155	0.108	0.146	1.841
1-year alpha (%, monthly rate)	723	0.138	0.090	0.153	1.763
5-year alpha (%, monthly rate)	723	0.138	0.092	0.219	0.467
10-year alpha (%, monthly rate)	723	0.222	0.191	0.281	-0.119
Panel E: Funds with monthly SPY beta SPY < 1 & monthly SPY alpha < 0					
Monthly alpha (%)	773	-0.141	-0.107	0.130	-2.782
1-year alpha (%, monthly rate)	773	-0.146	-0.115	0.144	-3.345
5-year alpha (%, monthly rate)	773	-0.141	-0.113	0.196	-1.962
10-year alpha (%, monthly rate)	773	-0.101	-0.076	0.203	-1.402

Panel F: Correlations between long- and short-horizon fund alphas

	Monthly alpha	1-year alpha	5-year alpha	10-year alpha
	All funds (N = 3768)			
Monthly alpha	1.000			
1-year alpha	0.942	1.000		
5-year alpha	0.688	0.762	1.000	
10-year alpha	0.531	0.592	0.869	1.000
	Funds with monthly beta against SPY > 1 (N = 2272)			
Monthly alpha	1.000			
1-year alpha	0.934	1.000		
5-year alpha	0.655	0.731	1.000	
10-year alpha	0.459	0.527	0.848	1.000
	Funds with monthly beta against SPY < 1 (N = 1496)			
Monthly alpha	1.000			
1-year alpha	0.961	1.000		
5-year alpha	0.783	0.850	1.000	
10-year alpha	0.754	0.795	0.886	1.000

Panel G: Sign of fund alpha at long- and short-horizon

	All funds	Funds with monthly SPY alpha > 0	Funds with monthly SPY alpha < 0
	All funds (N = 3768)		
Fraction, monthly alpha > 0	0.432	1.000	0.000
Fraction, 1-year alpha > 0	0.391	0.862	0.033
Fraction, 5-year alpha > 0	0.298	0.563	0.096
Fraction, 10-year alpha > 0	0.304	0.526	0.135
	Funds with monthly beta against SPY > 1 (N = 2272)		
Fraction, monthly alpha > 0	0.397	1.000	0.000
Fraction, 1-year alpha > 0	0.346	0.823	0.031
Fraction, 5-year alpha > 0	0.184	0.388	0.049
Fraction, 10-year alpha > 0	0.145	0.289	0.050
	Funds with monthly beta against SPY < 1 (N = 1496)		
Fraction, monthly alpha > 0	0.483	1.000	0.000
Fraction, 1-year alpha > 0	0.460	0.911	0.038
Fraction, 5-year alpha > 0	0.471	0.781	0.180
Fraction, 10-year alpha > 0	0.546	0.823	0.287

Panel H: Fraction of funds whose long-horizon and short-horizon alphas differ significantly

		Fraction of funds whose long run and short run alphas differ by at least		
Investment horizon	N	1% / year	2% / year	5% / year
All funds				
1 year	3768	0.189	0.043	0.007
5 years	3768	0.578	0.338	0.073
10 years	3768	0.693	0.489	0.183
Funds with monthly SPY beta > 1 & monthly SPY alpha > 0				
1 year	903	0.292	0.063	0.008
5 years	903	0.784	0.599	0.164
10 years	903	0.839	0.703	0.385
Funds with monthly SPY beta > 1 & monthly SPY alpha < 0				
1 year	1369	0.231	0.053	0.015
5 years	1369	0.565	0.326	0.065
10 years	1369	0.692	0.508	0.208
Funds with monthly SPY beta < 1 & monthly SPY alpha > 0				
1 year	723	0.122	0.026	0.000
5 years	723	0.488	0.228	0.036
10 years	723	0.698	0.452	0.069
Funds with monthly SPY beta < 1 & monthly SPY alpha < 0				
1 year	773	0.058	0.016	0.001
5 years	773	0.442	0.155	0.016
10 years	773	0.519	0.237	0.009

Table 6: The largest 25 fund families by TNA, sorted by their average 10-year fund alpha

For each of the 7,883 U.S. equity funds in our sample, we estimate its monthly beta by regressing excess monthly fund return on excess return to the SPDR S&P 500 ETF (SPY) and estimate its long-horizon beta against the SPY over five- and ten-year horizons using the modified Levhari and Levy (LL) approach detailed in Section 2.2. We then estimate each fund's long horizon alpha based on fund and SPY mean returns at the indicated horizon together with the long horizon beta estimate. Reported are average monthly beta and monthly, 5-year, and 10-year alphas across funds with at least 120 monthly returns within a fund family. Results pertain to the 25 funds families with the largest aggregate TNA at the end of 2020 and are sorted from lowest to highest average 10-year alpha estimate. This table reveals that long-horizon alphas are considerably smaller than monthly alphas for the large, successful fund families.

	Beta,	Alpha (Monthly Equivalent; %)			5-year alpha minus	10-year alpha minus
Row	Monthly	Monthly	5-year	10-year	Monthly	Monthly
1	1.027	-0.187	-0.281	-0.437	-0.094	-0.250
2	1.015	-0.143	-0.246	-0.401	-0.103	-0.258
3	0.976	-0.102	-0.227	-0.348	-0.125	-0.246
4	1.056	-0.157	-0.205	-0.306	-0.048	-0.149
5	0.886	-0.146	-0.207	-0.259	-0.062	-0.113
6	1.043	-0.070	-0.144	-0.234	-0.074	-0.164
7	1.008	0.005	-0.062	-0.221	-0.067	-0.226
8	0.991	-0.004	-0.082	-0.145	-0.078	-0.142
9	1.053	0.024	-0.067	-0.137	-0.091	-0.160
10	1.038	-0.010	-0.046	-0.104	-0.037	-0.095
11	1.009	-0.011	-0.119	-0.102	-0.108	-0.091
12	0.971	0.034	-0.063	-0.099	-0.097	-0.133
13	1.053	0.006	-0.090	-0.098	-0.095	-0.104
14	1.039	0.031	-0.099	-0.096	-0.130	-0.127
15	1.050	0.030	-0.068	-0.092	-0.098	-0.122
16	1.038	0.008	-0.053	-0.079	-0.061	-0.087
17	0.973	0.026	-0.071	-0.073	-0.097	-0.099
18	1.084	0.012	-0.037	-0.060	-0.049	-0.073
19	1.028	0.012	-0.067	-0.060	-0.079	-0.071
20	1.000	0.031	-0.056	-0.032	-0.086	-0.062
21	1.028	-0.003	-0.031	-0.026	-0.028	-0.023
22	0.993	0.103	0.031	-0.007	-0.072	-0.110
23	1.038	0.142	0.048	0.060	-0.094	-0.082
24	0.892	0.075	0.054	0.211	-0.021	0.136
25	0.991	0.103	0.102	0.342	-0.001	0.239
Mean	1.011	-0.008	-0.083	-0.112	-0.076	-0.104
Median	1.027	0.008	-0.067	-0.098	-0.079	-0.110

Internet Appendix to:

Fund Alphas Depend on the Return Horizon

October 31, 2023

Table A1: Fund turnover ratio and fund beta/alpha

We compute each fund's monthly beta by regressing excess monthly fund return on excess return to the SPDR S&P 500 ETF (SPY) and compute its long-horizon beta against SPY over three investment horizons (1 year, 5 years, and 10 years) using the modified Levhari and Levy (LL) approach detailed in Section 2.2. Lastly, we compute each fund's long-horizon alpha using its long-horizon returns and the long-horizon beta. We also compute each fund's turnover ratio as its median quarterly turnover ratio reported in CRSP, and sort the funds into four groups depending on whether the fund's monthly SPY beta is above 1 and whether its monthly SPY alpha is positive. For each group of funds, we divide them into terciles based on their turnover ratio. The table reports the average turnover ratio, the average monthly, annual, 5-year, and decade betas, and the average monthly, annual, 5-year, and decade alphas for each tercile of funds with at least 120 monthly returns. This table shows that investment horizon has significant effects on fund alpha estimates regardless of the fund's turnover rate.

Tercile, turnover ratio	N	Average turnover	Average SPY beta				Average SPY alpha (%, monthly rate)			
			Monthly	1-year	5-year	10-year	Monthly	1-year	5-year	10-year
Funds with monthly SPY beta > 1 & SPY alpha > 0										
1	248	0.259	1.106	1.122	1.252	1.455	0.152	0.131	0.025	-0.058
2	249	0.666	1.114	1.139	1.273	1.487	0.150	0.127	-0.030	-0.130
3	242	1.471	1.150	1.171	1.321	1.568	0.156	0.160	-0.036	-0.163
All	739	0.793	1.123	1.144	1.282	1.503	0.153	0.139	-0.014	-0.117
Funds with monthly SPY beta > 1 & SPY alpha < 0										
1	340	0.261	1.107	1.089	1.054	1.018	-0.169	-0.181	-0.231	-0.299
2	329	0.677	1.122	1.108	1.074	1.043	-0.182	-0.195	-0.270	-0.341
3	332	1.772	1.204	1.147	1.114	1.095	-0.256	-0.234	-0.334	-0.450
All	1001	0.899	1.144	1.115	1.080	1.052	-0.202	-0.203	-0.278	-0.363
Funds with monthly SPY beta < 1 & SPY alpha > 0										
1	218	0.185	0.837	0.853	0.862	0.888	0.139	0.118	0.136	0.257
2	208	0.457	0.860	0.874	0.902	0.957	0.155	0.144	0.139	0.232
3	213	1.155	0.878	0.890	0.929	0.997	0.172	0.153	0.141	0.205
All	639	0.597	0.858	0.872	0.897	0.947	0.155	0.138	0.139	0.232
Funds with monthly SPY beta < 1 & SPY alpha < 0										
1	204	0.190	0.892	0.880	0.808	0.733	-0.112	-0.112	-0.106	-0.065
2	202	0.491	0.894	0.879	0.798	0.715	-0.140	-0.147	-0.161	-0.119
3	195	1.756	0.871	0.870	0.784	0.702	-0.158	-0.171	-0.187	-0.151
All	601	0.799	0.886	0.876	0.797	0.717	-0.136	-0.143	-0.151	-0.111

Table A2: Long-horizon alpha versus short-horizon alpha for the portfolio of mutual funds

We form a portfolio of domestic equity mutual funds and compute its monthly beta by regressing excess monthly (equal-weighted or value-weighted) portfolio return on excess return to the SPDR S&P 500 ETF (SPY) and compute its long-horizon beta against SPY over three investment horizons (1 year, 5 years, and 10 years) using the modified Levhari and Levy (LL) approach detailed in Section 2.2. Lastly, we compute the portfolio's long-horizon alpha using its long-horizon returns and the long-horizon beta. This table compares the monthly versus long-horizon alphas for the portfolio of all funds and for four sub-groups of funds depending on whether their monthly SPY beta is above 1 and whether their monthly SPY alpha is positive. Panel A presents the results based on equal-weighted portfolio returns; Panel B presents the results based on value-weighted portfolio returns. This table shows that investment horizon has significant effects on alpha estimates for portfolio of mutual funds, alleviating the concern that the effects are driven by endogenous fund survival.

Panel A: Monthly and long-horizon beta/alpha for the equal-weighted portfolio of mutual funds

All funds (N = 7883)			
Monthly beta	1.014	Monthly alpha (%)	-0.073
1-year beta	1.013	1-year alpha (%, monthly rate)	-0.105
5-year beta	0.984	5-year alpha (%, monthly rate)	-0.189
10-year beta	0.948	10-year alpha (%, monthly rate)	-0.105
Funds with monthly SPY beta > 1 & SPY alpha > 0 (N = 1535)			
Monthly beta	1.133	Monthly alpha (%)	0.155
1-year beta	1.172	1-year alpha (%, monthly rate)	0.100
5-year beta	1.314	5-year alpha (%, monthly rate)	-0.088
10-year beta	1.545	10-year alpha (%, monthly rate)	-0.054
Funds with monthly SPY beta > 1 & SPY alpha < 0 (N = 2911)			
Monthly beta	1.156	Monthly alpha (%)	-0.253
1-year beta	1.144	1-year alpha (%, monthly rate)	-0.284
5-year beta	1.085	5-year alpha (%, monthly rate)	-0.393
10-year beta	1.015	10-year alpha (%, monthly rate)	-0.405
Funds with monthly SPY beta < 1 & SPY alpha > 0 (N = 1310)			
Monthly beta	0.831	Monthly alpha (%)	0.134
1-year beta	0.829	1-year alpha (%, monthly rate)	0.121
5-year beta	0.828	5-year alpha (%, monthly rate)	0.102
10-year beta	0.834	10-year alpha (%, monthly rate)	0.307
Funds with monthly SPY beta < 1 & SPY alpha < 0 (N = 2127)			
Monthly beta	0.854	Monthly alpha (%)	-0.186
1-year beta	0.825	1-year alpha (%, monthly rate)	-0.191
5-year beta	0.712	5-year alpha (%, monthly rate)	-0.214
10-year beta	0.590	10-year alpha (%, monthly rate)	-0.137

Panel B: Monthly and long-horizon beta/alpha for the value-weighted portfolio of mutual funds

All funds (N = 7883)			
Monthly beta	1.008	Monthly alpha (%)	-0.055
1-year beta	1.000	1-year alpha (%, monthly rate)	-0.071
5-year beta	0.982	5-year alpha (%, monthly rate)	-0.154
10-year beta	0.955	10-year alpha (%, monthly rate)	-0.118
Funds with monthly SPY beta > 1 & SPY alpha > 0 (N = 1535)			
Monthly beta	1.100	Monthly alpha (%)	0.002
1-year beta	1.115	1-year alpha (%, monthly rate)	-0.035
5-year beta	1.167	5-year alpha (%, monthly rate)	-0.191
10-year beta	1.209	10-year alpha (%, monthly rate)	-0.186
Funds with monthly SPY beta > 1 & SPY alpha < 0 (N = 2911)			
Monthly beta	1.091	Monthly alpha (%)	-0.167
1-year beta	1.076	1-year alpha (%, monthly rate)	-0.185
5-year beta	1.034	5-year alpha (%, monthly rate)	-0.281
10-year beta	0.985	10-year alpha (%, monthly rate)	-0.304
Funds with monthly SPY beta < 1 & SPY alpha > 0 (N = 1310)			
Monthly beta	0.875	Monthly alpha (%)	0.056
1-year beta	0.863	1-year alpha (%, monthly rate)	0.054
5-year beta	0.856	5-year alpha (%, monthly rate)	0.029
10-year beta	0.831	10-year alpha (%, monthly rate)	0.174
Funds with monthly SPY beta < 1 & SPY alpha < 0 (N = 2127)			
Monthly beta	0.883	Monthly alpha (%)	-0.127
1-year beta	0.861	1-year alpha (%, monthly rate)	-0.127
5-year beta	0.773	5-year alpha (%, monthly rate)	-0.146
10-year beta	0.677	10-year alpha (%, monthly rate)	-0.083