Simply Better Market Betas around the Globe*

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Abstract

This paper compares the forecasting and hedging performance of 11 market beta estimators across 53 international stock markets in 6 geographical regions. The Welch (2022) age-decayed slope-winsorized beta estimator produces the highest R-squareds in predicting future realized OLS betas in 45 markets and is always within the top 3 performers. It also forecasts future realizations of its competitors well and performs the best when hedging market risk exposures in 42 markets. On the practical side, it significantly outperforms commonly available market betas on Bloomberg, Yahoo! Finance, and Google Finance. Market participants can greatly improve their beta estimations in both developed and emerging markets with this easy-to-implement slope-winsorized beta estimator.

Keywords: Market Beta Estimation, International Stock Markets

JEL: C58, G11, G12, G14, G15, G17

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This paper evaluates the predictive and hedging performance of 11 market beta estimators in 53 markets around the globe. Candidates participating in the horse race include the standard daily OLS beta (bols), the Vasicek (1973) Bayesian beta, the Levi and Welch (2017) double-shrinkage beta (blw), the Blume (1971) adjusted beta, the Dimson (1979) synchronicity-adjusted beta, the Frazzini and Pedersen (2014) hybrid beta, the Welch (2022) slope-winsorized beta (bsw), and its age-decayed variant (bswa). Most of the betas above have been tested using US stock data, but the external validity of these beta estimators is unknown. Good empirical performance in a single market does not guarantee equal success in another market. By running a horse race in 53 international markets across 6 geographical regions, my paper contributes to the literature on estimating market betas and international finance, offering a practical guide for choosing beta estimators across developed, emerging, and frontier markets. In brief, my empirical results show good external validity of bswa. It is the best estimator of market beta yet.

I consider three criteria to compare the beta estimator's predictive performances. First, I run pooled predictive regressions of future realized daily OLS betas (Andersen et al., 2006) on beta estimators and compare their R-squared coefficients. For 12 months-ahead predictions, the bswa estimator outperforms other beta estimators in predicting future realized bols in 45 markets and consistently ranks in the top 3 across 53 markets. Moreover, bswa also performs well in forecasting bols over longer horizons and the future realizations of other beta estimates. Second, I investigate which beta estimator offers the smallest root mean squared error (RMSE) in forecasting future realized daily OLS betas. This test is equivalent to the regression RMSE by restricting the intercept and the slope coefficients to be 0 and 1 in the first test, respectively. The Levi and Welch (2017) double-shrinkage method shines in this test, as blw ranks first in producing the lowest RMSEs in 33 markets, whereas bswa ranks second with the smallest RMSEs in 19 markets. Based on the Borup et al. (2023) sequential elimination rule, bswa and blw remain in the best sets of beta predictors 27 and 41 times according to their RMSEs, respectively. Third, if the future realizations of bols proxy for stocks' exposures towards market risk, a less biased beta estimator should provide better hedging performance. Daily stock returns hedged by bswa estimates in the previous months have the lowest standard deviations in 42 markets, outperforming the hedging performance of blw except for 11 out of 53 markets. Market-hedged stock portfolios with bswa, bsw, or blw have similar standard deviations. Overall, the optimal hedging ratios under slope-winsorized betas (bsw, bswa) and double shrinkage betas (blw) are more effective in reducing portfolio variances than the standard OLS betas.

Most users do not estimate market betas with data by themselves. Instead, they usually obtain individual stocks' beta estimates from popular sources such as Bloomberg, Yahoo! Finance, and Google Finance. For instance, when a user searches a stock ticker on the Bloomberg terminal, it reports the Blume (1971) adjusted OLS weekly beta (**Beta vs SPX** for US stocks) estimated from the previous 103 weeks of price returns on a company's equity profile page. In common practice, users select the sampling period and data frequency from the terminal's historical beta page to obtain raw and Blume (1971) adjusted beta estimates for securities. Unlike the customizable Bloomberg service, Yahoo! Finance and Google Finance only provide the latest beta estimates for the past 60 months. In particular, the Yahoo! Finance website reports **Beta (5Y Monthly)** based on the price returns of the stock and the main market index over the past 60 months (OLS monthly betas). Beta estimates acquired by the **GoogleFinance("ticker", "beta")** function on the Google Sheets application are equivalent to those available on Yahoo! Finance. With the past 60 months of price returns, raw Bloomberg betas are practically identical to those available on Yahoo! Finance and Google Finance. Although these platforms are convenient, I replicate their 5-year monthly OLS betas (bmols) and the 5-year monthly Bloomberg adjusted betas (bmblm) with international data and show that they fare poorly in predicting future realized daily OLS betas (bols).[1]

Remarkably, even when market professionals estimate market betas by themselves, some sophisticated beta estimation methods proposed in the literature do more harm than good. In particular, the Dimson (1979) synchronicity adjustment and the Frazzini and Pedersen (2014) hybrid approach often produce beta estimates with unstable predictive performances that are inferior to bols over time, echoing Han (2022); Novy-Marx and Velikov (2022); Welch (2022). Simply put, bfp and bdim are not suitable for forecasting

[1] Figure A7 in the supplemental online material provides a Python program for replicating beta estimates from Yahoo! Finance and Google Finance. For stocks in the US, Bloomberg, Yahoo! Finance, and Google Finance treat the S&P 500 index as the market factor, and they estimate betas with price returns. In contrast, I estimate bmols and bmblm with excess returns derived from CRSP's holding period returns (dividend included) for individual stocks and the value-weighted market portfolio. Therefore, there are negligible differences between my beta estimates and Bloomberg betas. Following Bloomberg's definitions, I also replicate raw and adjusted weekly OLS betas with Friday to Friday returns. These weekly betas typically perform slightly better than their monthly variants but are inferior to betas estimated from daily data.

betas. Furthermore, betas estimated from monthly data generally underperform their counterparts estimated from daily data. Practitioners should not use these beta estimators in making business decisions.

Beyond the 11 beta estimators considered in the main article, the additional analysis in the supplemental online material reveals that my findings are robust to a broader set of sophisticated beta estimators that require stricter data requirements and more computing power. The bswa estimator generally outperforms betas estimated from different sampling windows, data frequencies, shrinkage methods, and estimation procedures. Relaxing the data requirements, I extend my analysis to 26 additional international markets. Here, bswa has the highest R-squared values 16 times and belongs to the best sets of beta predictors based on RMSEs 20 times. More importantly, bswa performs best not only in mature markets such as the US, Japan, the UK, France, and Canada but also up-and-coming markets such as China, India, and South Korea. In other words, bswa, on average, performs better than other beta estimators in over 80% of global equity by market value.

This article therefore recommends using the Welch (2022) age-decayed slope-winsorized bswa estimator with daily stock returns also for international markets. On the one hand, bswa is simple to implement with merely daily stock returns and it performs well across different market types. It has the same data requirement of 1-year daily observations as bols and accounts for all available data with exponential decay only up to the point of estimation. On the other hand, although the Levi and Welch (2017) blw estimator is on par with bswa in many markets, blw users must define their prior shrinkage weights and targets for stocks of different sizes. The empirical performance of blw thus depends relatively more on its users' familiarity and subjective judgments towards a particular market. Furthermore, the performance of double-shrinkage betas may benefit from the hindsight of knowing the full sample average beta for certain groups of stocks. On the contrary, bswa users only need to think that reasonable beta estimates should fall between -2 and $+4$. They also do not need to know about the size of their target stocks relative to the entire stock market. Hence, bswa is a superbly easy-to-compute tool for estimating risk in markets unfamiliar to business managers or investors.

My evaluation of beta estimators focuses on their ability to predict market risk exposures and hedging. It is however useful to discuss briefly some capital budgeting applications. Some practitioners apply the CAPM with a market beta to measure the cost of capital. However, even in the CAPM, a firm's capital cost is jointly determined by its market beta and the equity risk premium, leading to complications with their estimations and time-varying parameters. Estimating the cost of capital with the CAPM also ignores other sources of systematic risk premium (Merton, 1973; Ross, 1976). For example, in a US court case on company valuation,[2] the valuation expert for the petitioners put forward a Bloomberg historical beta of 1.32, while his counterpart for the respondent advocated the Bloomberg adjusted beta of 1.17. Neither is the "best" practice any longer.[3]

The remainder of this paper proceeds as follows. Section 1 recaps the methodologies for estimating market betas. Section 2 describes the sample selection and data requirements. Section 3 discusses the empirical performance of different beta estimators in forecasting future realized betas and hedging. Section 4 concludes my findings.

1 Beta estimators

This section summarizes the estimation procedures of 11 beta estimators selected in the horse races. Unless otherwise specified, estimators using daily returns employ rolling windows of 252 daily returns with at least 100 data points per estimation and a minimum of 5 trading days during the month of the estimation. For estimators using monthly returns, there must be 60 observations per estimation.

bols: The daily least-squares beta, $\widehat{\text{bols}}_{i,t}$, is estimated from $r_{i,d} = \alpha_i + \text{bols}_{i,t} \cdot r_{m,d} + \varepsilon_{i,d}$ using rolling windows of 252 daily excess returns prior to the end of month t.

bvck: The Vasicek (1973) Bayesian shrinkage beta estimator takes the functional form of $\widehat{\text{bvck}}_{i,t} = \frac{\overline{\sigma^2_{\widehat{\text{bols}}_t}}}{\overline{\sigma^2_{\widehat{\text{bols}}_t}} + \text{se}^2_{\widehat{\text{bols}}_{i,t}}} \cdot \widehat{\text{bols}}_{i,t} + \frac{\text{se}^2_{\widehat{\text{bols}}_{i,t}}}{\overline{\sigma^2_{\widehat{\text{bols}}_t}} + \text{se}^2_{\widehat{\text{bols}}_{i,t}}} \cdot \overline{\widehat{\text{bols}}_t}$, where $\text{se}_{\widehat{\text{bols}}_{i,t}}$ is the standard error of stock i's bols estimate. $\overline{\widehat{\text{bols}}_t}$ and $\overline{\sigma_{\widehat{\text{bols}}_t}}$ are the cross-sectional average and standard deviation of every stock's bols estimated in month t, respectively.

[2]Global GT LP v. Golden Telecom, Inc., C.A. No. 3698-VCS (Del. Ch. Apr. 23, 2010).

[3]The judge decided on an alternative beta estimate, ruling out the suggestions from both parties. The judge's beta assigned 2/3 weight to the Bloomberg historical beta of 1.32 and 1/3 weight to the industry average beta of 1.24, resulting in a beta of 1.29. The court also rejected the MSCI Barra's proprietary predictive beta. In addition, there were disagreements on the equity risk premium estimate and the judge agreed with the 6% proposed by the expert for the petitioners. Therefore, the court may not recognize the raw historical beta or the Bloomberg adjustment as the standard "best" practice in valuation lawsuits. Legal and valuation professionals should acquaint themselves with the latest industry and academic valuation literature to defend their beta and equity risk premium estimates.

bdim: The Dimson (1979) aggregated coefficients method adjusts for biases in estimating betas of infrequently traded stocks. It first estimates the contemporaneous beta, the lagged beta, and the lead beta from $r_{i,d} = \alpha_i + \text{bcont}_{i,t} \cdot r_{m,d} + \text{blag}_{i,t} \cdot r_{m,d-1} + \text{blead}_{i,t} \cdot r_{m,d+1} + \varepsilon_{i,d}$ using prior daily returns until month t. The synchronicity-adjusted beta is $\widehat{\text{bdim}}_{i,t} = \widehat{\text{bcont}}_{i,t} + \widehat{\text{blag}}_{i,t} + \widehat{\text{blead}}_{i,t}$.

blw: Levi and Welch (2017) advocate using the double-shrink beta estimator (blw), $\widehat{\text{blw}}_{i,t} = 0.75 \cdot \widehat{\text{bvck}}_{i,t} + 0.25 \cdot \text{target}_i$, to estimate market betas. The shrinkage target of an individual stock depends on its market cap.[4]

bsw: Welch (2022) proposes the slope-winsorized beta estimator (bsw), which bounds daily stock excess returns by $\text{rsw}_{i,d} \in (-2 \cdot r_{m,d}, 4 \cdot r_{m,d})$. In effect, extreme positive (negative) values in observed daily excess returns on day d are set to the maximum (minimum) of the multiples of daily market excess return, $-2 \cdot r_{m,d}$ and $4 \cdot r_{m,d}$, on day d. The resulting beta estimate is $\widehat{\text{bsw}}_{i,t} = \frac{\widehat{\text{cov}}(\text{rsw}_{i,d}, r_{m,d})}{\widehat{\text{var}}(r_{m,d})}$.

bswa: Welch (2022) shows that imposing age decay on daily observations further enhances the predictive performance of bsw. The weight of the nth daily observation is $w_n = (1 + \frac{2}{252})^n$ in the weighted least squares (WLS) estimation. Effectively, the age-decayed slope-winsorized beta estimator (bswa) captures time variations in beta by over-weighting recent returns and under-weighting old ones.

bfp: Frazzini and Pedersen (2014) use rolling 1-year windows of 1-day log returns for estimating volatilities and rolling 5-year windows of overlapping 3-day log returns for estimating correlations. The hybrid beta estimate is then shrunk towards 1 by $\widehat{\text{bfp}}_{i,t} = 0.6 \cdot \widehat{\rho^{\text{3-day}}_{i,m,t}} \cdot \frac{\widehat{\sigma^{\text{1-day}}_{i,t}}}{\widehat{\sigma^{\text{1-day}}_{m,t}}} + 0.4 \cdot 1$.

bmols: The monthly OLS beta variant is obtained by running rolling regressions with the previous 60 months of excess returns at the end of month t.

bmvck: The monthly Vasicek (1973) beta uses monthly returns to estimate bmols and cross-sectional moments of bmols across stocks.

[4] As in Levi and Welch (2017), I use $\text{target}_i = 1$ for non-micro stocks while $\text{target}_i = 0.75$ and $\text{target}_i = 0.5$ for micro and nano stocks, respectively. In line with Jensen et al. (2022), I use percentile cutoffs of NYSE stocks' market capitalization to classify all stocks. For international markets, non-micro stocks have market equities above the 20th percentile of NYSE stocks' market capitalization, micro stocks' market equities are between the 20th and 1st percentile, and nano stocks' market equities are less than the 1st percentile. The blw estimator cannot apply to stocks missing market equities.

bmblm: Due to mean-reverting tendencies in monthly OLS beta estimates over time, Blume (1971) suggests adjusting the current betas by the regression coefficients estimated from regressing the current beta estimates on past beta estimates. The adjusted monthly OLS beta follows $\widehat{\text{bmblm}}_{i,t} = \frac{2}{3} \cdot \widehat{\text{bmols}}_{i,t} + \frac{1}{3} \cdot 1$.

bmlw: The monthly double-shrink beta is $\widehat{\text{bmlw}}_{i,t} = 0.75 \cdot \widehat{\text{bmvck}}_{i,t} + 0.25 \cdot \text{target}_i$ with the same market cap dependent targets for bmvck as in blw.

2 Data

2.1 Sample description

I gather stock returns, index returns, and firm sizes from the CRSP and the Compustat North America databases for US and Canadian ordinary stocks and the Compustat Global database for international ordinary stocks. For each market, the market factor is the value-weighted excess returns of all existing stocks in that market during the estimation period. To ease comparison, I convert local currency returns in global markets into US dollar (USD) returns by their respective daily and monthly foreign exchange rates on Compustat. These USD returns are then subtracted by the US treasury bill rates to obtain excess returns. This study focuses on estimating stock betas in the local market. Because exchange rate fluctuations between the USD and the local currency affect both local stock returns and local market portfolio returns equally, beta estimates in USD and local currency are highly similar. Following Jensen, Kelly, and Pedersen (2022), there must be only one monthly observation per stock. I only consider primary listed common stocks on their respective main exchanges. I further remove extreme stock daily returns outside the range of $\pm 99\%$ to mitigate data errors.[5]

The initial data set contains stock market data from 79 markets around the globe. About 33% of them have limited observations. Hence, I further restrict the sample to markets having at least 50 listed companies with 5 years of price history and a market index with at least 5 constituents. The resulting sample consists of **53 markets** across 6 geographical regions.[6]

[5]These extreme outliers are rare in the sample. They are likely entry errors resulting from omitted corporate events such as stock splits and reverse stock splits. This filter has little effect on beta estimates but mitigates distortions in beta-hedged returns.

[6]In the supplemental online material, I provide additional findings for 16 markets that do not meet

Africa (4 markets): Morocco, Nigeria, Tunisia, and South Africa.

Asia Pacific (17 markets): Australia, Bangladesh, China, Hong Kong, Indonesia, India, Japan, South Korea, Sri Lanka, Malaysia, New Zealand, Pakistan, Philippines, Singapore, Thailand, Taiwan, and Vietnam.

Europe (19 markets): Austria, Belgium, Switzerland, Germany, Denmark, Spain, Finland, France, United Kingdom, Greece, Italy, Netherlands, Norway, Poland, Portugal, Romania, Russia, Sweden, and Turkey.

Middle East (7 markets): United Arab Emirates, Egypt, Israel, Jordan, Kuwait, Oman, and Saudi Arabia.

North America (3 markets): Canada, Mexico, and United States.

South America (3 markets): Argentina, Brazil, and Chile.

The sample has a diverse mix of MSCI market classifications, including 22 developed markets, 19 emerging markets, 10 frontier markets, and 2 standalone markets. Table 1 reports their ISO 3-digit country codes, MSCI market classifications, stock counts, numbers of monthly beta observations, and sampling periods by regions.[7]

[Insert Table 1 here: **Sample Summary**]

For most developed markets, data are available in the early 1990s on Compustat Global, whereas the USA sample dates back to the 1930s on CRSP.[8] Europe has the highest number of developed markets, followed by the Asia Pacific and North America. Emerging, frontier, and standalone markets tend to have shorter sampling periods. Across developed markets, the US, Japan, and the UK have the highest numbers of individual equity issuers. For emerging markets, large Asian markets, including China, India, and Korea, have over 2,000 unique firms, reflecting their rapidly growing economies and financial sectors.

my data requirements. My initial sample of 79 markets excludes 14 out of 93 markets covered in Jensen, Kelly, and Pedersen (2022), such as Iran, Uganda, and Zimbabwe, because there is insufficient data in these markets to estimate all 11 beta estimators for the horse race.

[7]The market classification follows the MSCI's 2022 Global Market Accessibility Review published on June 9, 2022. MSCI monitors market conditions regularly, and market reclassification can occur. A summary of a report is available at https://www.msci.com/our-solutions/indexes/market-classification.

[8]This study includes all available stocks in the US, while Welch (2022) focuses on the performance of beta estimators among the largest 1,000, 2,000, or 3,000 US stocks.

2.2 Summary statistics

Table 2 gives the pooled summary statistics of 11 beta estimates by 6 geographical regions. There are three salient features: First, betas estimated with rolling windows of 252 days (bols, bvck, blw, bdim, and bsw) or age-decayed expanding windows (bswa) have lower pooled means than betas estimated with 5-year rolling windows of daily data (bfp) or monthly data (bmols, bmvck, bmblm, and bmlw). The value-weighted market portfolio of all stocks has a market beta of 1. However, thinly traded stocks (Dimson, 1979) and small stocks (Levi and Welch, 2017) often have beta estimates below 1, explaining why some of the sample equal-weighted beta averages are less than 1. Second, betas estimated from daily data tend to have larger pooled dispersion than estimates from monthly data. Third, shrinkage adjustment and slope-winsorization reduce the pooled dispersion in beta estimates relative to their OLS counterparts. The bdim estimates have the highest pooled variations in all 6 regions.

[Insert Table 2 here: **Summary Statistics of Beta Estimates by Geographical Regions**]

Betas estimated using the same estimation windows have similar summary statistics. To better understand their differences, the supplemental online material Table A2 also presents the pairwise root mean squared differences across all 11 beta estimators by geographical regions. Using bols as the benchmark, its differences with bvck, blw, and bsw are in the moderate range of 0.043 to 0.215 across regions. These differences are small because bvck and blw are adjusted bols towards fixed targets while bsw equals bols if stock returns never exceed the slope winsorization limits. Because of the expanding sampling windows with exponential decay, bswa differs slightly from 1-year block-sampled bsw with distances between 0.067 and 0.102. In contrast, bdim, bfp, or estimators that use 5-year block sampling of monthly data produce beta estimates that are more distinct from bols, with distances ranging from 0.167 to 0.593. Across regions, the North America has the largest average distance between bols and other beta estimators. Stock markets with more listed companies tend to exhibit greater heterogeneity among beta estimates.

3 Results

3.1 Predicting 12 months-ahead bols

In line with Welch (2022), I evaluate the forecasting performance of beta estimator based on both its ability to predict the future realized bols and its ability to predict the future realizations of other beta estimators. The pooled predictive regression takes the form of

$$\mathrm{b}_{i,t} = \gamma_a + \gamma_b \cdot \widehat{\mathrm{b}}_{i,t-l} + \epsilon_{i,t}. \tag{1}$$

In particular, $\mathrm{b}_{i,t}$ are the ex-post predicted betas, $\widehat{\mathrm{b}}_{i,t-l}$ are the ex-ante predictor betas, and l is the prediction horizon. If $\widehat{\mathrm{b}}_{i,t-l}$ predict $\mathrm{b}_{i,t}$ perfectly, the pooled estimate of $\widehat{\gamma_b}$ equals 1 while $\widehat{\gamma_a}$ equals 0. When evaluating the forecasting performances of beta estimators, the first benchmark is to consider the R-squared from the pooled regression, $\overline{R}^2 = \left[\hat{\mathrm{Corr}}(\mathrm{b}_{n,y}, \hat{\mathrm{b}}_{n,y-l})\right]^2$, which is equivalent to the squared value of the estimated correlation between $\mathrm{b}_{i,t}$ and $\widehat{\mathrm{b}}_{i,t-l}$. The R-squared benchmark evaluates the goodness-of-fit of the predictive regression. Welch (2022) suggests that biases in beta estimates do not affect the R-squared measure. Moreover, Jegadeesh et al. (2019) show that the average correlation between estimated betas and true betas is approximately the square root of the correlation between betas estimated in adjacent months. A higher R-squared from predicting the OLS proxy of true beta with a beta estimator thus implies a higher explanatory power of the beta estimator on the true unknowable beta. For the univariate predictive regression, the R-squared is the same as the square of the correlation between $\mathrm{b}_{i,t}$ and $\widehat{\mathrm{b}}_{i,t-l}$. Therefore, I use Fisher's Z-test for the difference of correlations to examine whether the R-squared of a regression is significantly different from the highest R-squared beta observed in each international market.

However, if one wants to find a beta estimator for hedging the systematic risk exposure as proxied by the future realized OLS beta, biases in beta estimators can potentially lead to over(under)-hedging. Therefore, the second benchmark is to compare the root mean square errors (RMSEs) across different beta estimators

$$\mathrm{RMSE} = \sqrt{\frac{1}{N \cdot T} \sum_n \sum_t (\mathrm{b}_{n,t} - \hat{\mathrm{b}}_{n,t-l})^2}. \tag{2}$$

This RMSE metric directly compares a beta estimator and the future realized OLS beta, which is identical to the restricted regression RMSE by imposing $\gamma_a = 0$ and $\gamma_b = 1$ to Equation 1. To test whether the RMSEs of different beta estimators are significantly different from one another, I apply the Borup et al. (2023) algorithm to find the best set of beta predictor(s) by eliminating the inferior beta estimator(s) sequentially. Specifically, the algorithm uses a multivariate version of the Giacomini and White (2006) (GW) test to compare multiple RMSEs from different predictors. Given a set of predictors, the multivariate GW test eliminates one predictor with the worst RMSE from the set at a time until the null hypothesis of equal predictive ability across multiple predictors can no longer be rejected. Like the Hansen et al. (2011) model confidence set (MCS) framework, the surviving predictors have statistically indistinguishable forecasting abilities when more than one predictor remains in the best set.

Table 3 reports estimates of $\hat{\gamma}_a$, $\hat{\gamma}_b$, and $\overline{R}^2$ from Equation 1, and the RMSEs of Equation 2 with $\text{bols}_{i,t}$ in month t as the dependent variable and betas estimated in month $t-12$ as the independent variable in 6 geographical regions. Each column reports the regression results of a beta estimator. To improve readability, the highest $\overline{R}^2$ and the smallest RMSEs are in bold, indicating the best beta estimator for each region.[9]

[Insert Table 3 here: **Predicting 12 Months-Ahead Realized bols by Geographical Regions**]

The bswa estimator yields the highest $\overline{R}^2$ in 5 out of 6 geographical regions. Although blw has a higher R-squared than bswa in Africa, they are statistically different only at the 10% significance level. Based on the RMSE metric, bswa performs best in the Asia Pacific, the Middle East, and the North America, while blw beats bswa in the remaining 3 regions. Estimates of $\hat{\gamma}_b$ are usually below 1.0 in pooled predictive regressions, revealing that beta estimates often overshoot the future realized bols. That said, the top 3 performing beta estimators, bswa, blw, and bsw, tend to have $\hat{\gamma}_b$ closer to 1 than other estimators in most cases. While the R-squared coefficients measure how correlated the beta estimate and the future realized bols are, they do not tell us whether the beta estimates are biased. For example, if a beta estimate is exactly one-half of its future self, the $\overline{R}^2$ from the predictive regression is 1, implying a perfect positive correlation.[10] However, such a beta estimate

[9] I use Newey and West (1987) standard errors with 12 lags to correct for serial correlations and heteroskedasticity in the error terms of overlapping monthly observations. For robustness, I also compute Cameron et al. (2011) standard errors clustered by firms and years. These standard errors for $\hat{\gamma}_a$ and $\hat{\gamma}_b$ are typically between 0.01 and 0.05. To save space, I only report them in the supplemental online material's market-level tables.

[10] Let $\frac{1}{2}\text{b}_{n,t} = \hat{\text{b}}_{n,t-l}$, their correlation is $\text{Corr}(\text{b}_{n,t}, \hat{\text{b}}_{n,t-l}) = \text{Corr}(\text{b}_{n,t}, \frac{1}{2}\text{b}_{n,t}) = \sqrt{R^2} = 1$.

will also vastly underestimate the market risk and the optimal hedging ratio.

The market-level analysis involves running 583 pooled predictive regressions for 11 beta predictors in 53 markets. Table 4 shows the rankings of beta estimators by their predictive regression R-squareds and direct comparison RMSEs across 53 markets. For each pair of rankings, the $\overline{R}^2$ ranking (in *italics*) is on the left and the RMSE ranking is on the right. In 46 markets, bswa ranks first in terms of $\overline{R}^2$. It is also consistently among the top 3 performers. Around the world, bswa performs the best in predicting 1 year-ahead bols, followed by blw and bsw. Moreover, the great predictive performance of bswa is universal across developed, emerging, and frontier markets. Looking at $\overline{R}^2$, blw, bsw, and bswa are the top 3 performers in the African continent, outperforming other estimators by clear margins. While blw generally performs better than bswa in African frontier markets, bswa has the highest R-squared in emerging ZAF. In the Asia Pacific region, bswa ranks first in 15 out of 17 markets. The two exceptions are bols in LKA and blw in VNM, with bswa being second in both markets. As for Europe, bswa dominates other beta estimators in 17 out of 19 markets except for POL and ROU. The performance gap between blw and bswa in POL is negligible, but bswa is a little behind bmlw and blw in ROU. Across the Middle East, bswa has the highest $\overline{R}^2$ in 6 out of 7 markets and is a close second after blw in JOR. For North America and South America, bswa is the clear winner under the R-squared metric in all 7 markets. Generally speaking, betas estimated from daily observations are superior to those estimated from monthly observations in predicting future bols realized 12 months later. The only exception is ROU, with bmlw being marginally better than blw and bswa.[11]

[Insert Table 4 here: **Beta Rankings of Predicting 12 Months-Ahead Realized bols in 53 Markets**]

As for RMSEs, bswa is consistently among the top 3 performers across 53 markets. Nonetheless, blw performs better than bswa for being first in 33 markets. The bswa estimator comes second as the top performer in 20 markets, while the bsw estimator remains in the third position. By sequentially eliminating the beta with the highest RMSE in a market, the Borup et al. (2023) algorithm indicates that bswa and blw belong to the best sets of beta predictors in 27 and 41 markets, respectively. Simply put, blw performs better than bswa in terms of the RMSE metric, but the differences between them are usually small.

[11]For detailed results, please refer to Table A3 of the supplemental online material.

3.2 Relative predictive performance of betas over time

The pooled predictive regressions do not account for time-varying predictive performance in beta estimators. Following Welch (2022), I run year-by-year (December to December) predictive regressions in predicting 1 year-ahead future realized bols to obtain the time series of time-varying regression RMSEs for different beta estimators in each annual cross-section. This approach is similar to the Fama and Macbeth (1973) cross-sectional regression, which accounts for time fixed effects (Petersen, 2012).

Figure A2 of the supplemental online material plots the year-by-year relative performance of beta estimators in 53 international markets. In most markets, bswa outperforms other estimators most of the time. Whenever the difference between the predictive regression RMSEs of a beta estimator and the bols benchmark is positive, that beta estimator outperforms the bols estimator, and vice versa. All beta estimators exhibit variations in predictive performance over time. There are two notable patterns across regions and periods. Firstly, the predictive abilities of bswa, blw, bsw, and bvck tend to move in tandem, and they usually outperform bols. Except for Africa, bswa almost always dominates others in predicting 1 year-ahead future realized bols. The good performance of bswa also holds across developed, emerging, and frontier markets. Secondly, bdim and bfp often underperform relative to bols and have relatively unstable predictive performance, suggesting that they are unsuitable for forecasting bols.

3.3 Predicting bols over longer horizons

After establishing the superiority of bswa in predicting bols over 1 year, it is intriguing to investigate whether bswa also performs well in forecasting bols over longer horizons. The long-term predictability of bols reduces the rebalancing frequency for hedging long-term market risk exposures, which in turn lowers the cost of risk management.

Similar to Welch (2022), this subsection looks at estimates from Equation 1 with non-overlapping year-end betas (annual frequency, December to December) for 3 distinct predictive horizons: 1 year, 3 years, and 5 years. The left-hand side of Table 5 reports the R-squared values of these non-overlapping long-term pooled predictive regressions by geographical regions. To address whether bmols can better predict itself over longer horizons than betas estimated from daily observations, the last column (5yM) shows the result for predicting bmols over predictive horizons of 5 years.[12]

Except for the Africa region, bswa gives the highest $\overline{R}^2$ in predicting multiyear future realized bols, especially for 1y. The left-hand side of Table A4 of the supplemental online material shows the multiyear predictive performance of betas in 53 markets. In about half of the markets, bswa is the best predictor in predicting bols across all chosen predictive horizons for bols and bmols. For predictive horizons beyond 1 year, the double shrinkage blw estimator performs better than bswa with small margins in the remaining half of markets. The $\overline{R}^2$ of bswa and blw are often statistically indistinguishable. Overall, the predictive performance of beta estimators declines with the forecasting horizons.

[Insert Table 5 here: **Multiyear Predictive Performance of Betas by Geographical Regions, Annual**]

The right-hand side of Table 5 reports the RMSEs for multiyear beta forecasts. The bswa estimator produces the lowest RMSEs for 1y in the Asia Pacific, the Middle East, and the North America, while blw ranks first in the remaining 3 regions. For longer predictive horizons, blw outperforms bswa. Betas estimated from monthly data are usually subpar, and the commonly used bmols estimator consistently underperforms in terms of $\overline{R}^2$ and RMSE. As for market-level analysis, bswa is on par with blw for 1y, but blw broadly yields the lowest RMSEs for predictive horizons beyond 1y, outperforming bswa in most cases.[13] Moreover, blw has an upper edge over other estimators among African markets under the R-squared and the RMSE benchmarks, highlighting the practical value of having a good prior for betas in predicting market betas in frontier markets.

3.4 Predicting betas other than bols

So far, bswa performs well in forecasting the future realized bols in most markets over different predictive horizons. As Welch (2022) discussed, the common goal of different beta estimators is to uncover the unobservable true beta from noisy stock return data. Therefore, a good beta estimator should also perform well in predicting its future self as well as the future realizations of other betas. For each beta estimator in month t, I

[12]As bmols is estimated from 5 years of monthly data, the predictive horizon must also be at least 5 years to avoid overlapping data in the predictor and the predicted. For example, if I forecast bmols in year 6 (predicted) with bmols in year 5 (predictor), both variables share 4 years of overlapping data.

[13]For example, bswa and blw are members of the Borup et al. (2023) best predictor sets for 1y in 39 and 38 markets, respectively. For 3y, these figures become 7 for bswa and 46 for blw. For details, please refer to the right-hand side of Table A4 of the supplemental online material.

estimate Equation 1 with different combinations of beta predictors in month $t-12$.[14]

[Insert Figure 1 here: **Betas Rankings of Predicting Future Realized Betas in 53 Markets**]

For ease of comparison, the Panel A of Figure 1 summarizes the R-squared rankings of beta estimators in predicting bswa, bsw, blw, bvck, and bdim. Out of the 5 beta estimators, bswa has the best overall performance in predicting future realizations of itself and other beta estimators, except for blw. Likewise, the 1-year block-sampled bsw estimator performs well in predicting other beta estimators and often beats blw in predicting bswa, bsw, and bdim. In addition, the Panel B of Figure 1 shows the respective RMSE rankings. Based on the RMSE metric, blw generally ranks first in predicting its future self and future realizations of other betas, while bswa is usually the close second. Besides bswa and blw, the bvck estimator also performs well in predicting its future self. The bvck estimator shrinks bols towards its cross-sectional average, resulting in persistent bvck estimates over time. Similarly, the blw estimator shrinks bvck estimates towards market cap-specific targets. This persistency gives blw an advantage over other beta estimators in predicting the future realized blw.

To summarize, bswa is consistently among the top 3 in predictive performance across beta estimators and markets. If another beta is preferred over bols in estimating the true beta, bswa usually offers the best prediction of the future realization of that estimator.

3.5 Hedging performance

A beta estimator that performs well in predictive regressions may not provide the optimal hedging ratio, because biases in beta estimates can undermine hedging performance. Similarly, Mincer and Zarnowitz (1969) show that the mean square error (MSE) is influenced by bias, inefficiency, and noise. The MSE of a biased predictor can be smaller than that of an unbiased one. Therefore, I conduct back-tests on beta-hedged strategies that compare the hedging performance of beta estimators. For each beta estimator, I compute the hedged excess daily return as $r_{i,d} - \hat{b}_{i,m-1} r_{m,d}$, which is the daily excess return of stock i subtracted by the product of the previous month's beta estimate and the daily excess return of the market portfolio. I consider hedged daily returns because bols is optimized

[14] I lag predictor betas estimated with 5 years of data (bfp, bmols, bmvck, bmblm, and bmlw) by 60 months to avoid the mechanical correlations between predicted and predictor due to overlapping estimation periods.

to minimize the sum of squared daily residuals, the metric of hedging out-of-sample daily returns aligns closely with the in-sample R-squared and out-of-sample RMSE metrics. In practice, this procedure is equivalent to revising a stock's hedging ratio towards its exposure to market risk with the latest beta estimate at the end of each month.[15]

I repeat the procedure to get hedged stock returns of different beta estimates and compare their hedging performance. Ideally, a perfect hedge should have zero excess return and no variance. Therefore, the beta estimator with the lowest variance in hedged excess stock returns has the best hedging performance. I also use the chi-square test to determine whether the variance of each hedging strategy is statistically different from the one that yields the lowest variance within a market. As shown in Table 6, the bswa-hedged strategies have the lowest standard deviations in 5 out of 6 geographical regions. The blw-hedge has the lowest standard deviation in Africa, but it is not significantly different from the hedging strategies of bswa and bsw.[16]

[Insert Table 6 here: **Hedging Performance of Betas by Geographical Regions**]

For individual markets, bswa-hedged returns perform the best in 42 markets, trailing by blw only in 11 markets.[17] Overall, hedging results are similar for bswa, bsw, and blw, whereas the bols-hedge produces significantly higher standard deviations in 49 out of 53 markets. In addition to the 11 beta estimators, mret represents a naive hedging strategy with a fixed beta of 1. Across regions, bmols perform worse than not estimating the beta, except for Africa. Similarly, the naive hedging strategy produces smaller standard deviations than bmols-hedged returns in 34 markets. Therefore, hedging daily stock returns with betas estimated from the conventional 60-month windows is undesirable.

According to Levi and Welch (2017), an individual with better prior knowledge about beta can do better in predicting beta. To unleash the full potential of blw, the risk manager should choose shrinkage targets tailored to the attributes of the stock and the market. Therefore, the performance of blw may depend on the risk manager's market-specific knowledge and experience. My back-tests suggest the performance gaps between

[15]Unlike the R-squared and RMSE metrics, these tests do not involve comparison of betas estimated at sequential points in time. Therefore, it is no longer necessary to lag beta predictors by 12 months to avoid overlapping estimation periods. For the US market, Welch (2022) uses annual compounded returns to evaluate hedging performance. I focus on daily returns to avoid issues arising from non-synchronous trading between a stock and its market index in international markets.

[16]In the North American region, the chi-square tests of equal standard deviations are all rejected due to massive stock-day observations in the US. The standard deviations of beta-hedged daily returns with bols, bvck, bsw, bswa, and blw are economically similar.

[17]Table A5 of the supplemental online material reports the market-level hedging performance of betas.

bswa/bsw-hedged and blw-hedged portfolios are usually small. Hence, slope-winsorized betas provide practical and straightforward risk management solutions without additional managerial inputs and data on firm characteristics.

3.6 Thinly traded stocks

Although betas estimated from daily data typically have better predictive performance, they are susceptible to errors-in-variables biases induced by missing daily returns in infrequently traded stocks (Scholes and Williams, 1977; Dimson, 1979). In emerging and frontier markets, the problem of non-synchronous stock trading can potentially be more severe than in their developed counterparts. Given the positive association between stocks' betas and their trading volumes, the conventional bols estimator can severely underestimate the market betas of thinly traded stocks.

For each market, I first sort stocks into quintiles according to their average daily dollar trading volume (DVol) over 126 trading days before each year-end. I then run annual (December to December) pooled predictive regressions to compare the abilities of bdim, bols, and bswa in predicting 1 year-ahead realized bols or bdim within each market's DVol quintiles. I also compute the cross-sectional means and standard deviations of bdim, bols, and bswa within each quintile year by year before averaging their time series by quintiles.

Consistent with Welch (2022), all 3 betas are lower among infrequently traded stocks for all markets. Across quintiles, bdim usually produces larger coefficients and higher variations in beta estimates than bols and bswa. The slope winsorization technique removes the effects of outliers. As a result, bswa has the lowest variations. In most markets, bswa has the best predictive performance across quintiles, while bdim performs the worst.[18] For stocks with above median DVol, bdim has poor predictive performance even where non-synchronous trading is no longer a concern. Although the bdim estimator aims to reduce biases in estimating market betas of illiquid stocks, the empirical evidence shows bdim cannot predict its future self better than other beta estimators.

For low trading stocks in the first and second DVol quintiles, bswa has the best performance of predicting 1 year-ahead realized bdim in about three-quarters of the sampled markets. Occasionally, even bols performs better at predicting 1 year-ahead bdim for illiquid than bdim stocks for some markets. As an investor interested in the future bdim,

[18]Please refer to Tables A6, A7, and Figure A3 of the supplemental online material.

the past bdim should never be used.

3.7 Additional beta estimators

In the supplemental online material, Table A8 and Figure A5 present the robustness test results by incorporating 5 additional beta estimators covered in Hollstein (2020). It also describes the estimation procedures and data requirements of these additional estimators. The predictive performance of bswa remains strong. It ranks first by $\overline{R}^2$ in 36 out of 53 markets and is usually among the top 3. Moreover, bswa and blw are members of the best predictor sets according to the RMSE metric (Borup et al., 2023) in 33 and 39 markets, respectively. By default, the Bloomberg terminal provides the Blume (1971) adjusted betas estimated from the past 2 years of weekly data (bbbg) and their values before adjustments (braw). Table A9 summarizes the R-squared and RMSE rankings of bbbg and braw compared with the 11 beta estimators.[19] Across 53 markets, these two Bloomberg betas never rank within the top 3 for R-squared, and the bbbg estimator only ranks within the top 3 for RMSE 6 times. Generally, bswa and blw outperform bbbg, except for 3 occasions where bbbg and bswa are in the best predictor sets. Due to data limitations, this study does not consider beta estimators that utilize high-frequency data (Aït-Sahalia et al., 2020), firm fundamentals data (Cosemans et al., 2015), and stock options data (Buss and Vilkov, 2012). Only requiring daily returns, bswa is an easy-to-implement and robust beta estimator.

3.8 Additional markets

Given the data filters in Section 2, 26 international markets are excluded from the main analysis, in which bswa achieves the highest $\overline{R}^2$ in 16 of them and always ranks in the top 4. Regarding the RMSE metric, bswa and blw are the best predictors in 20 and 22 markets, respectively. Moreover, bswa-hedged daily returns achieve the lowest standard deviations in 19 of these 26 markets. These additional results are available in Tables A10 and A11 of the supplemental online material.

[19] For brevity, the table only shows the rankings of bswa, blw, bbbg, and braw out of 13 beta estimators.

4 Conclusion

In a horse race of 11 market beta estimators across 53 international markets, the Welch (2022) age-decayed slope-winsorized beta estimator (bswa) excels in predicting future realized OLS betas as well as hedging market risk exposures around the world. Compared with bswa, the next-best Levi and Welch (2017) double-shrinkage beta estimator (blw) generally produces smaller RMSEs and has similar hedging performances. However, it requires pre-specified shrinkage targets for different stock sizes. Hence, the usefulness of blw ultimately hinges on the user's prior knowledge of the relation between stocks' market capitalization and betas in a particular market. Choosing appropriate shrinkage targets for different markets may require many subjective judgments. Unlike other shrinkage estimators, bswa merely imposes that reasonable beta estimates should be between -2 and $+4$. Moreover, its minimum data requirement is identical to the commonly used OLS beta estimated from 252 days of daily returns in the literature. Because bswa offers a trivially simple adjustment correction that works well across developed and emerging markets, researchers and practitioners should use bswa to estimate the market risk of individual stocks in international markets, too.

Given the heterogeneity across international markets, it was not a foregone conclusion that estimators performing well in one market could guarantee equal success in another market. Only tautologies require no empirical tests. Therefore, the external validity of an estimator must be tested through empirical investigation. In some sense, one could not have anticipated the near universality of the evidence and could even find it surprising. Simply put, the slope-winsorized market beta is not only simple to use but also the dominant estimator of the market beta.

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Table 1: Sample Summary

Region	ISO3	Name	Class	Stock	Month	Sample
Africa	MAR	Morocco	FM	51	3,066	2004/07–2021/12
	NGA	Nigeria	FM	95	8,056	2006/05–2021/12
	TUN	Tunisia	FM	59	4,825	2004/11–2021/12
	ZAF	South Africa	EM	364	35,729	1991/12–2021/12
Asia Pacific	AUS	Australia	DM	1,348	97,190	1991/10–2021/12
	BGD	Bangladesh	FM	258	16,054	2008/04–2020/03
	CHN	China	EM	2,263	162,305	1998/03–2021/12
	HKG	Hong Kong	DM	1,645	143,472	1991/12–2021/12
	IDN	Indonesia	EM	344	27,250	1997/02–2021/12
	IND	India	EM	2,484	211,025	1994/08–2021/12
	JPN	Japan	DM	4,509	768,433	1991/12–2021/12
	KOR	South Korea	EM	2,016	218,081	1992/01–2021/12
	LKA	Sri Lanka	FM	192	13,434	1993/06–2020/03
	MYS	Malaysia	EM	1,038	111,606	1991/12–2021/12
	NZL	New Zealand	DM	132	13,393	1991/12–2021/12
	PAK	Pakistan	FM	302	23,706	1998/12–2021/12
	PHL	Philippines	EM	211	18,330	1994/07–2021/12
	SGP	Singapore	DM	583	50,663	1991/12–2021/12
	THA	Thailand	EM	676	67,239	1992/09–2021/12
	TWN	Taiwan	EM	1,848	221,514	1994/01–2021/12
	VNM	Vietnam	FM	462	23,696	2013/03–2021/12
Europe	AUT	Austria	DM	83	9,728	1991/12–2021/12
	BEL	Belgium	DM	161	21,367	1991/12–2021/12
	CHE	Switzerland	DM	306	40,494	1991/12–2021/12
	DEU	Germany	DM	915	95,628	1991/12–2021/12
	DNK	Denmark	DM	178	21,708	1991/12–2021/12
	ESP	Spain	DM	217	26,964	1991/12–2021/12
	FIN	Finland	DM	158	21,299	1991/12–2021/12
	FRA	France	DM	931	104,477	1991/12–2021/12
	GBR	United Kingdom	DM	2,268	211,923	1991/12–2021/12
	GRC	Greece	EM	306	23,169	1994/08–2021/12
	ITA	Italy	DM	428	49,129	1991/12–2021/12
	NLD	Netherlands	DM	239	31,617	1991/12–2021/12
	NOR	Norway	DM	245	20,703	1991/12–2021/12
	POL	Poland	EM	538	40,702	1999/06–2021/12
	PRT	Portugal	DM	57	7,057	1996/01–2021/12
	ROU	Romania	FM	51	3,528	2004/10–2021/12
	RUS	Russia	SM	173	8,619	2001/07–2021/12
	SWE	Sweden	DM	538	48,617	1991/12–2021/12
	TUR	Turkey	EM	411	49,245	1996/02–2021/12
Middle East	ARE	United Arab Emirates	EM	54	4,259	2007/05–2021/12
	EGY	Egypt	EM	158	9,990	2002/12–2021/12
	ISR	Israel	DM	349	29,715	2001/01–2021/12
	JOR	Jordan	FM	96	6,795	2000/11–2020/03
	KWT	Kuwait	EM	111	8,208	2007/06–2021/12
	OMN	Oman	FM	61	4,305	2006/10–2021/12
	SAU	Saudi Arabia	EM	166	16,985	2007/08–2021/12
North America	CAN	Canada	DM	1,909	191,319	1988/02–2021/12
	MEX	Mexico	EM	109	10,596	1995/02–2021/12
	USA	United States	DM	13,137	1,935,418	1931/12–2021/12
South America	ARG	Argentina	SM	69	8,005	1995/06–2021/12
	BRA	Brazil	EM	175	13,341	1994/09–2021/12
	CHL	Chile	EM	87	11,120	1995/01–2021/12

Description: This table summarizes the sample of 53 markets. The ISO3 column gives the International Standards Organization (ISO) 3-digit alphabetic country code acquired from https://wits.worldbank.org/wits/wits/witshelp/content/codes/country_codes.htm. The Class column contains the MSCI's 2022 annual market classification, including Developed Markets (DM), Emerging Markets (EM), Frontier Markets (FM), and Standalone Markets (SM). These classifications are available at https://www.msci.com/our-solutions/indexes/market-classification.

Interpretation: Selected markets should have sizable stock markets. For instance, each market must have at least 50 stocks with 5 years of history and a market index with at least 5 constituents over the sampling period. The resulting sample covers 53 markets across Africa, Asia Pacific, Europe, the Middle East, North America, and South America.

Table 2: Summary Statistics of Beta Estimates by Geographical Regions

	bols	bvck	blw	bdim	bsw	bswa	bfp	bmols	bmvck	bmblm	bmlw
Africa (1990–2021, stock-months: 59,268)											
Mean	0.80	0.79	0.80	0.83	0.81	0.81	0.97	0.96	0.93	0.97	0.90
SD	0.31	0.27	0.23	0.36	0.27	0.26	0.22	0.30	0.22	0.20	0.18
0.01	0.07	0.17	0.28	-0.05	0.18	0.23	0.56	0.24	0.38	0.49	0.44
Median	0.78	0.77	0.78	0.82	0.78	0.78	0.94	0.95	0.93	0.96	0.91
0.99	1.62	1.51	1.38	1.81	1.53	1.50	1.68	1.77	1.50	1.51	1.35
Asia Pacific (1990–2021, stock-months: 2,444,861)											
Mean	0.88	0.87	0.85	0.94	0.88	0.88	1.02	1.04	1.01	1.03	0.95
SD	0.39	0.34	0.27	0.47	0.34	0.32	0.28	0.43	0.32	0.28	0.24
0.01	0.08	0.15	0.28	-0.07	0.17	0.19	0.54	0.11	0.25	0.41	0.37
Median	0.87	0.87	0.85	0.92	0.88	0.88	0.99	1.02	1.02	1.01	0.96
0.99	1.91	1.76	1.52	2.19	1.72	1.68	1.92	2.18	1.78	1.79	1.53
Europe (1990–2021, stock-months: 941,262)											
Mean	0.73	0.73	0.75	0.80	0.74	0.74	0.91	0.96	0.93	0.97	0.90
SD	0.37	0.33	0.26	0.42	0.33	0.32	0.24	0.42	0.31	0.28	0.24
0.01	-0.02	0.06	0.22	-0.10	0.09	0.10	0.50	0.08	0.22	0.39	0.36
Median	0.71	0.71	0.73	0.78	0.72	0.72	0.89	0.93	0.93	0.96	0.90
0.99	1.71	1.59	1.43	1.95	1.61	1.59	1.62	2.16	1.75	1.77	1.53
Middle East (1999–2021, stock-months: 92,941)											
Mean	0.91	0.90	0.87	0.98	0.90	0.90	1.08	1.06	1.03	1.04	0.97
SD	0.42	0.39	0.30	0.50	0.37	0.35	0.34	0.44	0.35	0.30	0.27
0.01	0.00	0.05	0.18	-0.08	0.08	0.11	0.50	-0.00	0.15	0.33	0.27
Median	0.88	0.87	0.86	0.95	0.88	0.89	1.03	1.04	1.03	1.03	0.97
0.99	2.05	1.93	1.64	2.38	1.84	1.78	2.09	2.36	2.02	1.91	1.70
North America (1930–2021, stock-months: 2,330,817)											
Mean	0.83	0.82	0.83	0.91	0.83	0.82	1.01	1.03	0.99	1.02	0.95
SD	0.58	0.51	0.39	0.64	0.46	0.45	0.37	0.59	0.44	0.40	0.34
0.01	-0.31	-0.08	0.10	-0.45	0.00	0.01	0.42	-0.15	0.07	0.23	0.23
Median	0.78	0.77	0.80	0.86	0.79	0.79	0.97	0.98	0.97	0.98	0.94
0.99	2.44	2.21	1.89	2.68	2.04	2.00	2.13	2.70	2.12	2.14	1.81
South America (1993–2021, stock-months: 36,736)											
Mean	0.90	0.89	0.89	0.93	0.90	0.90	1.00	1.05	1.03	1.04	0.99
SD	0.29	0.27	0.22	0.34	0.28	0.27	0.20	0.33	0.26	0.22	0.20
0.01	0.25	0.30	0.39	0.21	0.29	0.32	0.62	0.32	0.43	0.55	0.50
Median	0.89	0.89	0.89	0.92	0.89	0.89	0.98	1.03	1.01	1.02	0.99
0.99	1.62	1.54	1.39	1.83	1.57	1.55	1.60	1.98	1.70	1.65	1.49

Description: The above table presents the summary statistics of 11 market beta estimates in 6 geographical regions. Betas estimated from daily or monthly stock returns include OLS betas (bols/bmols), Vasicek (1973) Bayesian shrinkage betas (bvck/bmvck), double shrinkage betas of Levi and Welch (2017) (blw/bmlw), and Blume (1971) adjusted betas (bmblm). Predictors estimated from only daily stock returns are Dimson (1979) synchronicity-adjusted betas (bdim) and hybrid betas of Frazzini and Pedersen (2014) (bfp), and Welch (2022) slope-winsorized betas (bsw) and slope-winsorized betas with age decay (bswa). Daily beta estimators (bols, bvck, blw, bdim, and bsw) utilize rolling windows of 252 days. The age-decayed beta estimator (bswa) has expanding windows of at least 100 days. The bfp estimator uses 5 years of rolling 3-days log returns for estimating correlations and 1 year of rolling 1-day log returns for estimating standard deviations. Monthly beta estimators (bmols, bmvck, bmblm, and bmlw) require 5 years of rolling monthly stock returns. The bracket next to each region label contains the sampling period and the number of observations.

Interpretation: Beta estimators with shrinkage or slope-winsorization usually have smaller dispersion in their market beta estimates. The equal-weight averages of betas estimated from daily returns are below 1, while their counterparts estimated from monthly returns are closer to 1.

Table 3: Predicting 12 Months-Ahead Realized bols by Geographical Regions

	bols	bvck	bdim	bfp	blw	bsw	bswa	bmols	bmvck	bmblm	bmlw
Africa (1991–2021, stock-months: 51,676)											
$\hat{\gamma}_a$	0.34	0.28	0.46	0.27	0.14	0.27	0.22	0.43	0.25	0.24	0.05
$\hat{\gamma}_b$	0.57	0.66	0.41	0.55	0.82	0.66	0.71	0.39	0.59	0.58	0.83
Predictive Regression $\overline{R}^2$	$0.329^{\ddagger\dagger}$	$0.352^{\ddagger\dagger}$	$0.220^{\ddagger\dagger}$	$0.153^{\ddagger\dagger}$	**0.381**	$0.345^{\ddagger\dagger}$	0.375	$0.140^{\ddagger\dagger}$	$0.168^{\ddagger\dagger}$	$0.140^{\ddagger\dagger}$	$0.229^{\ddagger\dagger}$
Direct Comparison RMSE	0.282	0.262	0.342	0.339	**0.243**$^\bigstar$	0.264	0.253	0.372	0.321	0.341	0.290
Asic Pacific (1991–2021, stock-months: 2,187,391)											
$\hat{\gamma}_a$	0.34	0.27	0.47	0.16	0.12	0.25	0.20	0.52	0.38	0.34	0.23
$\hat{\gamma}_b$	0.61	0.70	0.44	0.71	0.90	0.72	0.77	0.35	0.50	0.53	0.69
Predictive Regression $\overline{R}^2$	$0.372^{\ddagger\dagger}$	$0.385^{\ddagger\dagger}$	$0.277^{\ddagger\dagger}$	$0.256^{\ddagger\dagger}$	$0.384^{\ddagger\dagger}$	$0.386^{\ddagger\dagger}$	**0.415**	$0.146^{\ddagger\dagger}$	$0.163^{\ddagger\dagger}$	$0.146^{\ddagger\dagger}$	$0.182^{\ddagger\dagger}$
Direct Comparison RMSE	0.339	0.319	0.421	0.368	0.305	0.317	**0.304**$^\bigstar$	0.473	0.405	0.405	0.363
Europe (1991–2021, stock-months: 835,974)											
$\hat{\gamma}_a$	0.24	0.17	0.33	-0.03	0.00	0.15	0.14	0.41	0.28	0.24	0.09
$\hat{\gamma}_b$	0.68	0.78	0.51	0.84	0.97	0.78	0.81	0.34	0.50	0.51	0.72
Predictive Regression $\overline{R}^2$	$0.451^{\ddagger\dagger}$	$0.479^{\ddagger\dagger}$	$0.325^{\ddagger\dagger}$	$0.287^{\ddagger\dagger}$	$0.490^{\ddagger\dagger}$	$0.483^{\ddagger\dagger}$	**0.504**	$0.147^{\ddagger\dagger}$	$0.173^{\ddagger\dagger}$	$0.147^{\ddagger\dagger}$	$0.216^{\ddagger\dagger}$
Direct Comparison RMSE	0.294	0.273	0.366	0.353	**0.261**$^\bigstar$	0.272	0.264	0.484	0.413	0.430	0.368
Middle East (2000–2021, stock-months: 80,257)											
$\hat{\gamma}_a$	0.38	0.32	0.48	0.42	0.17	0.28	0.22	0.52	0.40	0.34	0.25
$\hat{\gamma}_b$	0.58	0.65	0.44	0.46	0.85	0.70	0.76	0.36	0.50	0.55	0.68
Predictive Regression $\overline{R}^2$	$0.327^{\ddagger\dagger}$	$0.343^{\ddagger\dagger}$	$0.249^{\ddagger\dagger}$	$0.127^{\ddagger\dagger}$	$0.344^{\ddagger\dagger}$	$0.347^{\ddagger\dagger}$	**0.372**	$0.144^{\ddagger\dagger}$	$0.168^{\ddagger\dagger}$	$0.144^{\ddagger\dagger}$	$0.181^{\ddagger\dagger}$
Direct Comparison RMSE	0.387	0.366	0.461	0.462	$0.347^\bigstar$	0.358	**0.345**$^\bigstar$	0.504	0.440	0.433	0.396
North America (1931–2021, stock-months: 2,137,333)											
$\hat{\gamma}_a$	0.26	0.17	0.33	-0.10	-0.03	0.10	0.07	0.35	0.19	0.11	-0.03
$\hat{\gamma}_b$	0.68	0.80	0.55	0.92	1.04	0.88	0.92	0.47	0.66	0.71	0.90
Predictive Regression $\overline{R}^2$	$0.468^{\ddagger\dagger}$	$0.493^{\ddagger\dagger}$	$0.355^{\ddagger\dagger}$	$0.334^{\ddagger\dagger}$	$0.501^{\ddagger\dagger}$	$0.496^{\ddagger\dagger}$	**0.518**	$0.230^{\ddagger\dagger}$	$0.249^{\ddagger\dagger}$	$0.230^{\ddagger\dagger}$	$0.277^{\ddagger\dagger}$
Direct Comparison RMSE	0.460	0.424	0.550	0.506	0.408	0.414	**0.403**$^\bigstar$	0.625	0.545	0.552	0.508
South America (1994–2021, stock-monthss: 32,466)											
$\hat{\gamma}_a$	0.33	0.28	0.43	0.24	0.12	0.29	0.24	0.49	0.37	0.29	0.18
$\hat{\gamma}_b$	0.64	0.70	0.52	0.66	0.89	0.69	0.74	0.40	0.52	0.60	0.73
Predictive Regression $\overline{R}^2$	$0.393^{\ddagger\dagger}$	$0.399^{\ddagger\dagger}$	$0.327^{\ddagger\dagger}$	$0.230^{\ddagger\dagger}$	$0.409^{\ddagger\dagger}$	$0.403^{\ddagger\dagger}$	**0.433**	$0.203^{\ddagger\dagger}$	$0.219^{\ddagger\dagger}$	$0.203^{\ddagger\dagger}$	$0.255^{\ddagger\dagger}$
Direct Comparison RMSE	0.264	0.253	0.303	0.297	**0.239**$^\bigstar$	0.254	0.243	0.378	0.328	0.318	0.285

Description: This table reports estimates from the pooled predictive regressions in the form of $\text{bols}_{i,t} = \gamma_a + \gamma_b \hat{\text{b}}_{i,t-12} + \epsilon_{i,t-12}$ region by region. $\text{bols}_{i,t}$ is the future realized OLS beta in month t, whereas $\hat{\text{b}}_{i,t}$ is one of the seven respective beta estimates in month $t-12$. The $\overline{R}^2$ column shows the average R-squared values while the RMSE column gives the root mean squared errors. Values in **bold** indicate which beta performs best in terms of $\overline{R}^2$ and RMSE. The Fisher's z-test compares the differences between the $\overline{R}^2$ of an estimator and the best performer. Superscripts †, ‡, and ‡† represent rejecting the null hypothesis of the equal correlations at the 10%, 5%, and 1% significance levels under the z-test, respectively. The Borup et al. (2023) multivariate version of the Giacomini and White (2006) test compares the RMSEs of all beta estimators. It then eliminates betas with inferior predictive performance sequentially at the 5% significance level. When the null hypothesis of equal predictive ability across beta estimators is no longer rejected, superscripts ★ indicate which of the remaining beta estimator(s) is/are in the best set of predictor(s) for each predictive horizon. The bracket next to each region label contains the sampling period and the number of observations.

Interpretation: Overall, the Welch (2022) bswa estimator outperforms the other 6 estimators in predicting 12 months-ahead realized bols in 5 out of 6 geographical regions and is consistently ranked among the top 2 by $\overline{R}^2$ and RMSE in all regions.

Table 4: Beta Rankings of Predicting 12 Months-Ahead Realized bols in 53 Markets

	bols	bvck	bsw	bswa	bdim	bfp	blw	bmols	bmvck	bmblm	bmlw
Africa	*Predictive Regression* $\overline{R}^2$ *Ranks*, **Direct Comparison RMSE Ranks**										
MAR (2004–2021, stock-months: 3,066)	3†,6	5‡,3	4†,4	2,2	8‡†,10	11‡†,9	**1,1***	10‡†,11	7‡†,7	9‡†,8	6‡†,5
NGA (2006–2021, stock-months: 8,056)	3‡,5	4‡,3	5‡†,4	2,2	6‡†,9	8‡†,11	**1,1***	11‡†,10	9‡†,7	10‡†,8	7‡†,6
TUN (2004–2021, stock-months: 4,825)	5‡,6	4‡,4	6‡†,5	3,2	10‡†,10	11‡†,8	**1,1***	9‡†,11	7‡†,7	8‡†,9	2,3
ZAF (1991–2021, stock-months: 35,729)	5‡†,5	4‡†,4	3‡†,3	1,2	7‡†,9	9‡†,8	2,**1***	11‡†,11	8‡†,7	10‡†,10	6‡†,6
Asia Pacific											
AUS (1991–2021, stock-months: 97,190)	5‡†,6	4‡†,4	3‡†,3	1,2	6‡†,10	11‡†,9	2,**1***	10‡†,11	8‡†,7	9‡†,8	7‡†,5
BGD (2008–2020, stock-months: 16,054)	2‡†,5	3‡†,2	4‡†,4	**1,1***	6‡†,9	7‡†,11	5‡†,3	10‡†,10	8‡†,8	9‡†,7	11‡†,6
CHN (1998–2021, stock-months: 162,305)	5‡†,5	2‡†,3	3‡†,4	1,2	6‡†,10	7‡†,7	4‡†,**1***	11‡†,11	9‡†,9	10‡†,8	8‡†,6
HKG (1991–2021, stock-months: 143,472)	5‡†,5	4‡†,4	2‡†,3	**1,1***	6‡†,8	7‡†,6	3‡†,**2***	11‡†,11	9‡†,9	10‡†,10	8‡†,7
IDN (1997–2021, stock-months: 27,250)	5‡†,5	3‡†,3	4‡†,4	1,**2***	6‡†,7	8‡†,8	2,**1***	11‡†,11	9‡†,9	10‡†,10	7‡†,6
IND (1994–2021, stock-months: 211,025)	5‡†,5	3‡†,4	2‡†,3	**1,1***	6‡†,10	7‡†,7	4‡†,2	11‡†,11	9‡†,9	10‡†,8	8‡†,6
JPN (1991–2021, stock-months: 768,433)	5‡†,5	4‡†,4	2‡†,3	1,2	7‡†,10	6‡†,6	3‡†,**1***	11‡†,11	9‡†,8	10‡†,9	8‡†,7
KOR (1992–2021, stock-months: 218,081)	5‡†,6	4‡†,3	2‡†,4	**1,1***	6‡†,10	7‡†,7	3‡†,**2***	11‡†,11	9‡†,8	10‡†,9	8‡†,5
LKA (1993–2020, stock-months: 13,434)	2,8	**1**,3	5‡†,6	3,**1***	7‡†,10	6‡†,11	4,2	9‡†,9	10‡†,7	8‡†,4	11‡†,5
MYS (1991–2021, stock-months: 111,606)	3‡†,8	2‡†,4	4‡†,3	**1,1***	7‡†,11	6‡†,10	5‡†,2	10‡†,9	8‡†,6	9‡†,5	11‡†,7
NZL (1991–2021, stock-months: 13,393)	5‡,7	4,5	3,4	1,3	8‡†,10	7‡†,8	2,**1***	11‡†,11	9‡†,6	10‡†,9	6‡†,2
PAK (1998–2021, stock-months: 23,706)	4‡†,5	3‡†,3	2‡†,2	**1,1***	6‡†,10	7‡†,6	5‡†,4	10‡†,11	8‡†,9	9‡†,8	11‡†,7
PHL (1994–2021, stock-months: 18,330)	5‡†,5	4‡†,4	3‡†,3	1,**2***	6‡†,6	7‡†,8	2‡,**1***	11‡†,11	9‡†,10	10‡†,9	8‡†,7
SGP (1991–2021, stock-months: 50,663)	5‡†,6	3‡†,4	2‡†,3	**1,1***	6‡†,9	7‡†,10	4‡†,2	10‡†,11	11‡†,8	9‡†,7	8‡†,5
THA (1992–2021, stock-months: 67,239)	5‡†,5	4‡†,4	3‡†,3	1,2	6‡†,9	8‡†,7	2‡†,**1***	11‡†,11	9‡†,8	10‡†,10	7‡†,6
TWN (1994–2021, stock-months: 221,514)	3‡†,5	5‡†,3	4‡†,4	1,2	7‡†,8	6‡†,6	2‡†,**1***	11‡†,11	9‡†,10	10‡†,9	8‡†,7
VNM (2013–2021, stock-months: 23,696)	4‡†,5	3‡†,3	5‡†,4	2,2	6‡†,8	11‡†,11	**1,1***	9‡†,10	10‡†,7	8‡†,9	7‡†,6
Europe											
AUT (1991–2021, stock-months: 9,728)	5‡†,5	4‡†,4	3‡†,3	1,**2***	6‡†,6	7‡†,8	2,**1***	11‡†,11	9‡†,9	10‡†,10	8‡†,7
BEL (1991–2021, stock-months: 21,367)	5‡†,5	4‡†,4	3‡†,3	1,**2***	6‡†,7	7‡†,9	2,**1***	11‡†,11	9‡†,8	10‡†,10	8‡†,6
CHE (1991–2021, stock-months: 40,494)	5‡†,5	4‡†,4	3‡†,3	**1,1***	6‡†,7	7‡†,6	2‡†,**2***	11‡†,11	9‡†,9	10‡†,10	8‡†,8
DEU (1991–2021, stock-months: 95,628)	5‡†,5	4‡†,4	3‡†,3	1,2	6‡†,9	8‡†,8	2,**1***	11‡†,11	9‡†,7	10‡†,10	7‡†,6
DNK (1991–2021, stock-months: 21,708)	5‡†,5	3‡†,3	4‡†,4	1,2	6‡†,7	7‡†,8	2,**1***	11‡†,11	9‡†,9	10‡†,10	8‡†,6
ESP (1991–2021, stock-months: 26,964)	5‡†,5	3‡†,4	2‡†,3	**1,1***	6‡†,6	7‡†,7	4‡†,2	11‡†,11	9‡†,9	10‡†,10	8‡†,8
FIN (1991–2021, stock-months: 21,299)	5‡†,5	4‡,3	3‡,4	**1,1***	6‡†,6	7‡†,8	2‡,**2***	11‡†,11	9‡†,9	10‡†,10	8‡†,7
FRA (1991–2021, stock-months: 104,477)	5‡†,5	4‡†,4	3‡†,3	**1,1***	6‡†,6	7‡†,7	2‡†,2	11‡†,11	9‡†,9	10‡†,10	8‡†,8
GBR (1991–2021, stock-months: 211,923)	5‡†,5	4‡†,4	3‡†,3	1,2	7‡†,7	6‡†,6	2‡,**1***	11‡†,11	9‡†,9	10‡†,10	8‡†,8
GRC (1994–2021, stock-months: 23,169)	5‡†,5	4‡†,3	3‡†,4	1,2	6‡†,7	8‡†,8	2,**1***	11‡†,11	9‡†,10	10‡†,9	7‡†,6
ITA (1991–2021, stock-months: 49,129)	5‡†,5	4‡†,3	3‡†,4	1,2	6‡†,7	7‡†,6	2‡†,**1***	11‡†,11	9‡†,9	10‡†,10	8‡†,8
NLD (1991–2021, stock-months: 31,617)	5‡†,5	4‡†,4	2‡†,3	**1,1***	7‡†,10	8‡†,7	3‡†,**2***	11‡†,11	9‡†,8	10‡†,9	6‡†,6
NOR (1991–2021, stock-months: 20,703)	5‡†,5	3‡†,4	2‡†,3	**1,1***	6‡†,9	8‡†,7	4‡†,**2***	11‡†,11	9‡†,8	10‡†,10	7‡†,6
POL (1999–2021, stock-months: 40,702)	5‡†,5	3‡†,4	4‡†,3	2,2	6‡†,7	11‡†,10	**1,1***	10‡†,11	8‡†,8	9‡†,9	7‡†,6
PRT (1996–2021, stock-months: 7,057)	5‡†,5	4‡†,4	3‡,3	1,**2***	6‡†,8	7‡†,6	2,**1***	11‡†,11	9‡†,10	10‡†,9	8‡†,7
ROU (2004–2021, stock-months: 3,528)	6‡,7	4†,4	5‡,6	3,3	8‡†,10	11‡†,11	2,**2***	10‡†,9	7‡†,5	9‡†,8	**1,1***
RUS (2001–2021, stock-months: 8,619)	5‡†,5	4‡†,3	3‡†,4	1,2	6‡†,7	8‡†,8	2,**1***	11‡†,11	9‡†,10	10‡†,9	7‡†,6
SWE (1991–2021, stock-months: 48,617)	5‡†,5	4‡†,4	3‡†,3	1,2	6‡†,8	8‡†,7	2†,**1***	11‡†,11	9‡†,9	10‡†,10	7‡†,6
TUR (1996–2021, stock-months: 49,245)	5‡†,6	4‡†,3	3‡†,5	1,**2***	8‡†,10	6‡†,7	2‡†,**1***	11‡†,11	9‡†,8	10‡†,9	7‡†,4

Table 4: Beta Rankings of Predicting 12 Months-Ahead Realized bols in 53 Markets (Continued)

	bols	bvck	bsw	bswa	bdim	bfp	blw	bmols	bmvck	bmblm	bmlw
Middle East	*Predictive Regression* $\overline{R}^2$ *Ranks*, **Direct Comparison RMSE Ranks**										
ARE (2007–2021, stock-months: 4,259)	*5*,5	*3*,4	*4*,3	***1***,2★	$6^{\ddagger\dagger}$,6	$7^{\ddagger\dagger}$,9	*2*,**1**★	$11^{\ddagger\dagger}$,11	$9^{\ddagger\dagger}$,10	$10^{\ddagger\dagger}$,8	$8^{\ddagger\dagger}$,7
EGY (2002–2021, stock-months: 9,990)	$3^{\dagger}$,4	$4^{\ddagger}$,3	*2*,2	***1***,**1**★	$6^{\ddagger\dagger}$,7	$11^{\ddagger\dagger}$,10	$5^{\ddagger\dagger}$,5	$8^{\ddagger\dagger}$,11	$9^{\ddagger\dagger}$,9	$7^{\ddagger\dagger}$,6	$10^{\ddagger\dagger}$,8
ISR (2001–2021, stock-months: 29,715)	$5^{\ddagger\dagger}$,5	$3^{\ddagger\dagger}$,4	$4^{\ddagger\dagger}$,3	***1***,2	$6^{\ddagger\dagger}$,10	$9^{\ddagger\dagger}$,7	*2*,**1**★	$11^{\ddagger\dagger}$,11	$8^{\ddagger\dagger}$,9	$10^{\ddagger\dagger}$,8	$7^{\ddagger\dagger}$,6
JOR (2000–2020, stock-months: 6,795)	$4^{\ddagger\dagger}$,5	*2*,2	$5^{\ddagger\dagger}$,4	*3*,3	$6^{\ddagger\dagger}$,8	$9^{\ddagger\dagger}$,10	***1***,**1**★	$11^{\ddagger\dagger}$,11	$8^{\ddagger\dagger}$,7	$10^{\ddagger\dagger}$,9	$7^{\ddagger\dagger}$,6
KWT (2007–2021, stock-months: 8,208)	*5*,5	*4*,4	*2*,3	***1***,2	$6^{\ddagger\dagger}$,10	$11^{\ddagger\dagger}$,8	*3*,**1**★	$8^{\ddagger\dagger}$,11	$10^{\ddagger\dagger}$,9	$7^{\ddagger\dagger}$,7	$9^{\ddagger\dagger}$,6
OMN (2006–2021, stock-months: 4,305)	$9^{\ddagger\dagger}$,9	$7^{\ddagger\dagger}$,8	*2*,3	***1***,**1**★	$10^{\ddagger\dagger}$,11	$11^{\ddagger\dagger}$,10	$8^{\ddagger\dagger}$,6	$6^{\ddagger\dagger}$,7	$3^{\ddagger\dagger}$,5	$5^{\ddagger\dagger}$,4	$4^{\ddagger\dagger}$,2★
SAU (2007–2021, stock-months: 16,985)	$4^{\ddagger\dagger}$,8	$3^{\ddagger\dagger}$,5	*2*,4	***1***,2	$6^{\ddagger\dagger}$,11	$11^{\ddagger\dagger}$,10	$5^{\ddagger\dagger}$,**1**★	$9^{\ddagger\dagger}$,9	$7^{\ddagger\dagger}$,7	$8^{\ddagger\dagger}$,6	$10^{\ddagger\dagger}$,3
North America											
CAN (1988–2021, stock-months: 191,319)	$5^{\ddagger\dagger}$,6	$4^{\ddagger\dagger}$,4	$2^{\ddagger\dagger}$,3	***1***,**1**★	$6^{\ddagger\dagger}$,10	$9^{\ddagger\dagger}$,8	$3^{\ddagger\dagger}$,2	$11^{\ddagger\dagger}$,11	$8^{\ddagger\dagger}$,7	$10^{\ddagger\dagger}$,9	$7^{\ddagger\dagger}$,5
MEX (1995–2021, stock-months: 10,596)	$5^{\ddagger\dagger}$,5	$3^{\ddagger\dagger}$,3	$4^{\ddagger\dagger}$,4	***1***,2★	$6^{\ddagger\dagger}$,7	$7^{\ddagger\dagger}$,6	$2^{\ddagger}$,**1**★	$11^{\ddagger\dagger}$,11	$9^{\ddagger\dagger}$,10	$10^{\ddagger\dagger}$,9	$8^{\ddagger\dagger}$,8
USA (1931–2021, stock-months: 1,935,418)	$5^{\ddagger\dagger}$,5	$4^{\ddagger\dagger}$,4	$3^{\ddagger\dagger}$,3	***1***,**1**★	$6^{\ddagger\dagger}$,9	$7^{\ddagger\dagger}$,6	$2^{\ddagger\dagger}$,2	$11^{\ddagger\dagger}$,11	$9^{\ddagger\dagger}$,8	$10^{\ddagger\dagger}$,10	$8^{\ddagger\dagger}$,7
South America											
ARG (1995–2021, stock-months: 8,005)	$5^{\ddagger\dagger}$,6	$6^{\ddagger\dagger}$,4	$3^{\ddagger}$,3	***1***,2	$7^{\ddagger\dagger}$,8	$4^{\ddagger\dagger}$,5	*2*,**1**★	$11^{\ddagger\dagger}$,11	$9^{\ddagger\dagger}$,9	$10^{\ddagger\dagger}$,10	$8^{\ddagger\dagger}$,7
BRA (1994–2021, stock-months: 13,341)	$5^{\ddagger\dagger}$,5	$3^{\ddagger\dagger}$,3	$2^{\ddagger\dagger}$,4	***1***,**1**★	$6^{\ddagger\dagger}$,8	$11^{\ddagger\dagger}$,10	$4^{\ddagger\dagger}$,2★	$10^{\ddagger\dagger}$,11	$8^{\ddagger\dagger}$,9	$9^{\ddagger\dagger}$,7	$7^{\ddagger\dagger}$,6
CHL (1995–2021, stock-months: 11,120)	$5^{\ddagger\dagger}$,5	$3^{\dagger}$,3	$4^{\ddagger}$,4	***1***,2	$6^{\ddagger\dagger}$,7	$8^{\ddagger\dagger}$,6	*2*,**1**★	$11^{\ddagger\dagger}$,11	$9^{\ddagger\dagger}$,10	$10^{\ddagger\dagger}$,9	$7^{\ddagger\dagger}$,8

Description: The table show the rankings of different beta estimators in predicting 12 months-ahead realized bols in 53 markets. Each pair of rankings contains predictive regression $\overline{R}^2$ ranking (*italic* number) on the left and the direct comparison RMSE ranking on the right. Values in **bold** indicate which beta performs best in terms of $\overline{R}^2$ and RMSE. The Fisher's z-test compares the differences between the $\overline{R}^2$ of an estimator and the best performer. Superscripts †, ‡, and ‡† represent rejecting the null hypothesis of the equal correlations at the 10%, 5%, and 1% significance levels under the z-test, respectively. The Borup et al. (2023) multivariate version of the Giacomini and White (2006) test compares the RMSEs of all beta estimators. It then eliminates betas with inferior predictive performance sequentially at the 5% significance level. When the null hypothesis of equal predictive ability across beta estimators is no longer rejected, superscripts ★ indicate which of the remaining beta estimator(s) is/are in the best set of predictor(s). The bracket next to each market label contains the sampling period and the number of observations.

Interpretation: The best performer in predictive regressions is bswa, followed by blw and bsw. Based on $\overline{R}^2$ ranks, bswa outperforms the other 10 predictors in 45 markets and is always in the top 3 across 53 markets. Regarding the RMSE metric, blw performs better than bswa as the two estimators go head-to-head in competing for the top 2 spots.

Table 5: Multiyear Predictive Performance of Betas by Geographical Regions, Annual

	Predictive Regression $\overline{R}^2$				Direct Comparison RMSE			
	1y	3y	5y	5yM	1y	3y	5y	5yM
Africa (1995–2021)								
bols	0.32‡†	0.23‡	0.18†	0.12	0.29	0.31	0.31	0.35
bvck	0.34	0.24	0.19	0.13	0.27	0.29	0.29	0.34
bdim	0.21‡†	0.15‡†	0.12‡†	0.1	0.35	0.36	0.36	0.37
bfp	0.14‡†	0.14‡†	0.08‡†	0.09†	0.34	0.33	0.34	0.3
blw	**0.37**	**0.27**	**0.23**	**0.14**	**0.25**$^{\bigstar}$	**0.27**$^{\bigstar}$	**0.27**$^{\bigstar}$	0.32
bsw	0.34†	0.23†	0.18†	0.13	0.27	0.29	0.3	0.34
bswa	0.36	0.24	0.18†	0.13	0.26	0.29	0.3	0.33
bmols	0.14‡†	0.09‡†	0.06‡†	0.08‡	0.37	0.38	0.38	0.33
bmvck	0.17‡†	0.11‡†	0.08‡†	0.09†	0.32	0.33	0.33	0.3
bmblm	0.14‡†	0.09‡†	0.06‡†	0.08‡	0.34	0.35	0.34	0.29
bmlw	0.23‡†	0.17‡†	0.13‡†	0.11	0.29	0.3	0.3	**0.28**$^{\bigstar}$
stock-years	4,465	3,440	2,657	2,657				
Asic Pacific (1995–2021)								
bols	0.36‡†	0.21‡†	0.15‡†	0.14‡†	0.35	0.39	0.4	0.47
bvck	0.37‡†	0.21‡†	0.16‡†	0.14‡†	0.32	0.37	0.39	0.45
bdim	0.27‡†	0.15‡†	0.11‡†	0.14‡†	0.42	0.46	0.47	0.49
bfp	0.25‡†	0.14‡†	0.12‡†	0.14‡†	0.37	0.4	0.39	**0.41**$^{\bigstar}$
blw	0.37‡†	0.22‡†	0.16‡†	0.13‡†	0.31	**0.35**$^{\bigstar}$	**0.36**$^{\bigstar}$	0.45
bsw	0.37‡†	0.22‡†	0.16‡†	0.15‡†	0.32	0.37	0.38	0.45
bswa	**0.42**	**0.23**	**0.18**	**0.17**	**0.3**$^{\bigstar}$	0.36	0.37	0.44
bmols	0.15‡†	0.09‡†	0.07‡†	0.15‡†	0.47	0.49	0.49	0.46
bmvck	0.17‡†	0.1‡†	0.08‡†	0.16	0.41	0.42	0.43	0.42
bmblm	0.15‡†	0.09‡†	0.07‡†	0.15‡†	0.41	0.42	0.42	0.41$^{\bigstar}$
bmlw	0.19‡†	0.12‡†	0.09‡†	0.15‡†	0.36	0.38	0.38	0.41
stock-years	188,865	151,251	120,158	120,158				
Europe (1995–2021)								
bols	0.45‡†	0.3‡†	0.21‡†	0.13	0.3	0.34	0.37	0.48
bvck	0.48‡†	0.31‡†	0.22‡†	0.13	0.28	0.33	0.36	0.47
bdim	0.32‡†	0.22‡†	0.16‡†	0.14	0.37	0.4	0.41	0.47
bfp	0.29‡†	0.2‡†	0.13‡†	0.09‡†	0.36	0.37	0.38	0.39
blw	0.49‡†	**0.33**	**0.25**	0.13	**0.26**$^{\bigstar}$	**0.3**$^{\bigstar}$	**0.33**$^{\bigstar}$	0.43
bsw	0.48‡†	0.31‡†	0.22‡†	0.13	0.27	0.33	0.36	0.46
bswa	**0.51**	0.32†	0.23‡	**0.14**	0.26	0.32	0.35	0.46
bmols	0.15‡†	0.09‡†	0.06‡†	0.1‡†	0.49	0.5	0.51	0.46
bmvck	0.18‡†	0.1‡†	0.06‡†	0.11‡†	0.41	0.43	0.44	0.41
bmblm	0.15‡†	0.09‡†	0.06‡†	0.1‡†	0.43	0.44	0.44	0.39
bmlw	0.22‡†	0.14‡†	0.09‡†	0.12‡	0.37	0.38	0.39	**0.38**$^{\bigstar}$
stock-years	71,737	56,924	45,120	45,120				
Middle East (2005–2021)								
bols	0.32‡†	0.13‡	0.09	0.19	0.4	0.48	0.49	0.46
bvck	0.34‡	0.14	0.09	0.2	0.38	0.45	0.47	0.45
bdim	0.24‡†	0.09‡†	0.06†	0.17	0.47	0.54	0.55	0.49
bfp	0.13‡†	0.08‡†	0.03‡†	0.12‡†	0.46	0.49	0.5	0.45
blw	0.33‡	0.14	0.1	0.19	0.36$^{\bigstar}$	**0.42**$^{\bigstar}$	0.43$^{\bigstar}$	0.44
bsw	0.34	0.14	0.09	0.19	0.37	0.44	0.46	0.45
bswa	**0.37**	**0.17**	0.1	**0.2**	**0.35**$^{\bigstar}$	0.42$^{\bigstar}$	0.45	0.44
bmols	0.14‡†	0.08‡†	0.08	0.16‡	0.51	0.54	0.53	0.47
bmvck	0.16‡†	0.1‡†	0.09	0.17	0.45	0.47	0.47	0.43
bmblm	0.14‡†	0.08‡†	0.08	0.16‡	0.44	0.46	0.46	0.43$^{\bigstar}$
bmlw	0.17‡†	0.11‡†	**0.1**	0.17	0.41	0.43$^{\bigstar}$	**0.43**$^{\bigstar}$	**0.42**$^{\bigstar}$
stock-years	7,023	5,170	3,663	3,663				
North America (1935–2021)								
bols	0.46‡†	0.32‡†	0.25‡†	0.23‡†	0.47	0.53	0.57	0.6
bvck	0.49‡†	0.34‡†	0.26‡†	0.24‡†	0.43	0.49	0.53	0.57
bdim	0.35‡†	0.24‡†	0.19‡†	0.21‡†	0.56	0.61	0.63	0.61
bfp	0.33‡†	0.24‡†	0.18‡†	0.22‡†	0.51	0.53	0.55	**0.5**$^{\bigstar}$
blw	0.49‡†	0.35†	0.27	0.23‡†	0.41	**0.46**$^{\bigstar}$	**0.49**$^{\bigstar}$	0.53
bsw	0.49‡†	0.35‡†	0.27‡†	0.24‡	0.42	0.48	0.51	0.55
bswa	**0.51**	**0.36**	**0.28**	**0.25**	**0.41**$^{\bigstar}$	0.47	0.5	0.54
bmols	0.23‡†	0.16‡†	0.12‡†	0.2‡†	0.63	0.65	0.67	0.59
bmvck	0.25‡†	0.17‡†	0.13‡†	0.21‡†	0.55	0.57	0.59	0.53
bmblm	0.23‡†	0.16‡†	0.12‡†	0.2‡†	0.55	0.57	0.58	0.51
bmlw	0.28‡†	0.2‡†	0.15‡†	0.21‡†	0.51	0.53	0.54	0.5$^{\bigstar}$
stock-years	180,268	152,738	130,329	130,329				

Table 5: Multiyear Predictive Performance of Betas by Geographical Regions, Annual (Continued)

	Predictive Regression $\overline{R}^2$				**Direct Comparison RMSE**			
	1y	3y	5y	5yM	1y	3y	5y	5yM
South America (1998–2021)								
bols	0.38	0.23	0.17	$0.09^{\dagger}$	0.26	0.3	0.31	0.4
bvck	0.39	0.23	0.17	$0.09^{\dagger}$	0.25	0.29	0.3	0.39
bdim	$0.32^{\ddagger\dagger}$	$0.17^{\ddagger\dagger}$	$0.13^{\dagger}$	0.11	0.3	0.33	0.34	0.4
bfp	$0.22^{\ddagger\dagger}$	$0.16^{\ddagger\dagger}$	$0.12^{\ddagger}$	0.11	0.28	0.29	0.3	0.33
blw	0.4	**0.25**	0.19	$0.08^{\ddagger}$	**0.23★**	**0.27★**	**0.28★**	0.38
bsw	0.39	0.23	0.18	$0.09^{\dagger}$	0.25	0.29	0.31	0.39
bswa	**0.42**	0.24	**0.19**	0.1	0.24	0.28	0.29	0.38
bmols	$0.2^{\ddagger\dagger}$	$0.11^{\ddagger\dagger}$	$0.09^{\ddagger\dagger}$	0.14	0.36	0.38	0.37	0.36
bmvck	$0.22^{\ddagger\dagger}$	$0.13^{\ddagger\dagger}$	$0.1^{\ddagger\dagger}$	**0.15**	0.31	0.33	0.32	0.33
bmblm	$0.2^{\ddagger\dagger}$	$0.11^{\ddagger\dagger}$	$0.09^{\ddagger\dagger}$	0.14	0.31	0.32	0.32	**0.32★**
bmlw	$0.26^{\ddagger\dagger}$	$0.16^{\ddagger\dagger}$	$0.13^{\dagger}$	0.14	0.27	0.29	0.29	0.32
stock-years	2,818	2,199	1,681	1,681				

Description: This table reports the region-by-region average R-squared values ($\overline{R}^2$) from the multiyear pooled predictive regressions of $\text{bols}_{i,y} = \gamma_a + \gamma_b \hat{\text{b}}_{i,y-l} + \epsilon_{i,y-l}$ on the left panel and the root mean squared errors (RMSE) in the form of $\sqrt{\frac{1}{NY}\sum_n \sum_y (\text{bols}_{n,y} - \hat{\text{b}}_{n,y-l})^2}$ on the right panel. Candidates predicting $\text{bols}_{i,y}$, the future realized OLS betas in December of year y, include bols, bvck, blw, bswa, and bmols estimated in December of year $y - l$ where $l \in \{1, 3, 5\}$. The last column (5yM) reports the result for predicting bmols over predictive horizons of 5 years. **Bold** figures highlight the best-performing beta's $\overline{R}^2$ and RMSE for each predictive horizon. The Fisher's z-test compares the differences between the $\overline{R}^2$ of an estimator and the best performer. Superscripts †, ‡, and ‡† represent rejecting the null hypothesis of the equal correlations at the 10%, 5%, and 1% significance levels under the z-test, respectively. The Borup et al. (2023) multivariate version of the Giacomini and White (2006) test compares the RMSEs of all beta estimators. It then eliminates betas with inferior predictive performance sequentially at the 5% significance level. When the null hypothesis of equal predictive ability across beta estimators is no longer rejected, superscripts ★ indicate which of the remaining beta estimator(s) is/are in the best set of predictor(s) for each predictive horizon. The bracket next to each region label contains the sampling period. The stock-years row shows the number of observations for each predictive horizon.

Interpretation: From 1y to 2y, bswa usually produces higher $\overline{R}^2$ than its rivalries. For longer horizons, bswa has the overall higher $\overline{R}^2$ in predicting bols across all 4 predictive horizons and predicting 5 years-ahead bmols than other betas in 5 out of 6 geographical regions. As for the RMSE metric, blw has the best overall performance for predictive horizons beyond 1 year.

Table 6: Hedging Performance of Betas by Geographical Regions

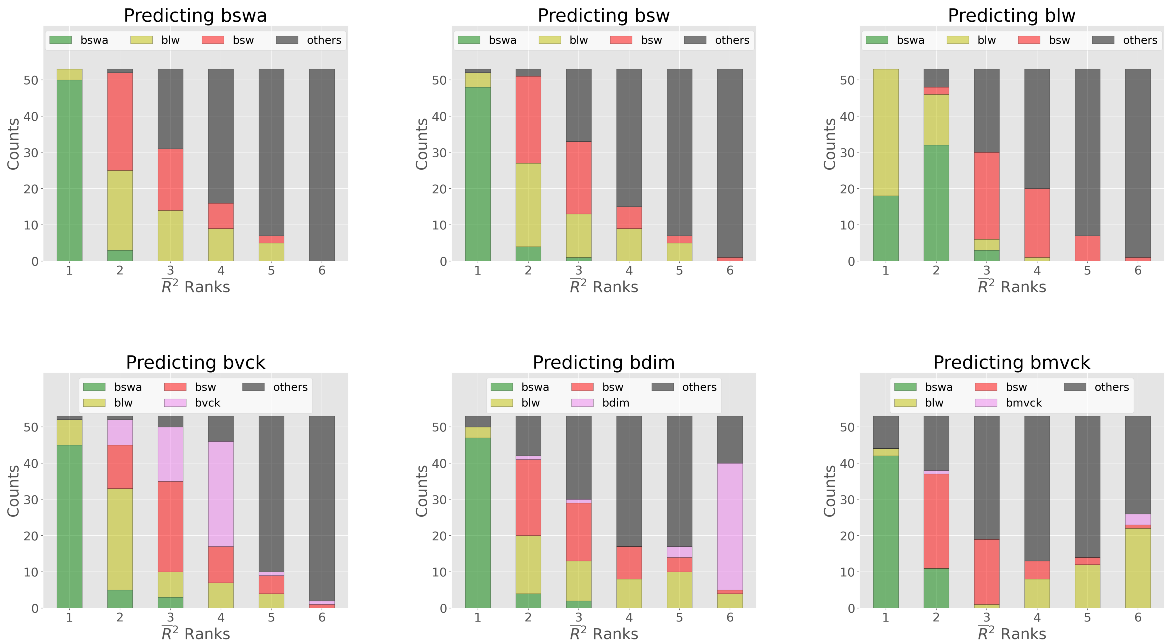
	bols	bvck	bsw	bswa	bdim	bfp	blw	bmols	bmvck	bmblm	bmlw	mret
Africa (1991–2021, stock-days: 1,339,201)												
SD	3.480%***	3.474%	3.474%	3.472%	3.494%***	3.492%***	**3.471%**	3.497%***	3.487%***	3.492%***	3.481%***	3.501%***
SDpct	1.389%	**1.377%**	1.384%	1.381%	1.427%	1.502%	1.380%	1.488%	1.474%	1.502%	1.456%	1.559%
Asia Pacific (1990–2021, stock-days: 57,582,237)												
SD	3.322%***	3.317%	3.316%	**3.314%**	3.341%***	3.331%***	3.316%	3.355%***	3.338%***	3.339%***	3.329%***	3.339%***
SDpct	1.652%	1.645%	1.645%	1.640%	1.685%	1.711%	**1.640%**	1.736%	1.719%	1.724%	1.695%	1.739%
Europe (1991–2021, stock-days: 21,439,536)												
SD	3.492%***	3.487%	3.487%	**3.486%**	3.510%***	3.506%***	3.486%	3.535%***	3.518%***	3.523%***	3.508%***	3.532%***
SDpct	1.378%	1.371%	1.374%	**1.369%**	1.416%	1.456%	1.378%	1.500%	1.488%	1.503%	1.469%	1.544%
Middle East (1999–2021, stock-days: 2,230,800)												
SD	2.500%***	2.496%	2.496%	**2.493%**	2.516%***	2.512%***	2.495%	2.525%***	2.512%***	2.511%***	2.506%***	2.515%***
SDpct	1.336%	1.332%	1.331%	1.331%	1.362%	1.415%	**1.329%**	1.406%	1.396%	1.401%	1.379%	1.419%
North America (1931–2021, stock-days: 53,781,004)												
SD	3.767%***	3.763%***	3.762%***	**3.761%**	3.782%***	3.775%***	3.762%***	3.802%***	3.785%***	3.787%***	3.778%***	3.794%***
SDpct	1.511%	1.500%	1.500%	**1.496%**	1.553%	1.592%	1.502%	1.618%	1.605%	1.614%	1.588%	1.656%
South America (1993–2021, stock-days: 839,218)												
SD	2.793%*	2.790%	2.790%	**2.787%**	2.806%***	2.805%***	2.789%	2.827%***	2.812%***	2.812%***	2.801%***	2.823%***
SDpct	1.548%	1.546%	1.547%	**1.545%**	1.573%	1.612%	1.549%	1.644%	1.630%	1.634%	1.611%	1.648%

Description: For each stock, its daily excess return on day d is hedged by subtracting the product of its beta in the previous month $m-1$ and the market portfolio's daily excess return on day d. The hedged daily stock returns are computed with $r_{i,d} - \hat{b}_{i,m-1} r_{m,d}$. In mret rows, betas are set to 1. This table reports the standard deviations (SD) of hedged daily stock returns in 6 geographical regions. In addition, SDpct of hedged returns is the 70th percentile minus the 30th percentile. Statistics in **bold** represent the lowest standard deviation (SD) and the smallest SDpct of beta-hedged returns in each region. Superscripts *, **, and *** denote 10%, 5%, and 1% significance levels, respectively, for the chi-square test between each standard deviation and the lowest standard deviation within a geographical region. The bracket next to each region label contains the sampling period and the number of observations.

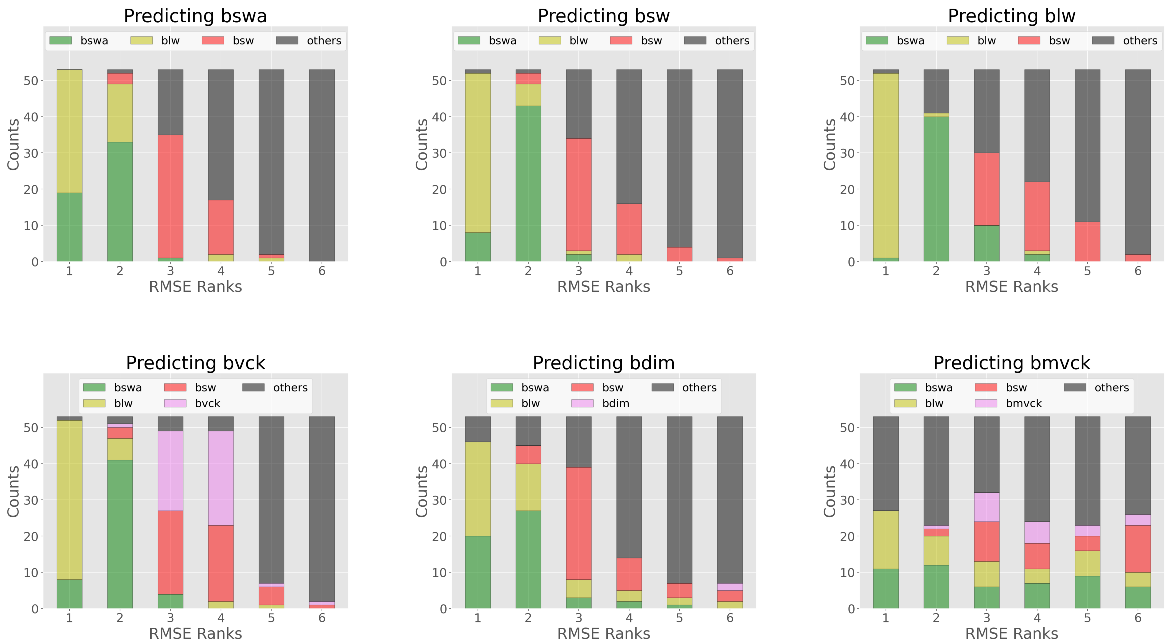
Interpretation: Comparing SD across beta hedged returns, bswa offers the best hedging performances in 5 out of 6 regions, except for Africa. Betas estimated from monthly returns are inferior to their counterparts estimated from daily returns in hedging against market risk. Apart from North America, the hedging performances of bswa and blw are statistically indistinguishable. As measured by SDpct, bswa offers the best hedging outcomes in Europe, North America, and South America.

Figure 1: Betas Rankings of Predicting Future Realized Betas in 53 Markets

Panel A: $\overline{R}^2$ Rankings



Panel B: RMSE Rankings



Description: Each subplot in this figure gives the predictive regression R-squared/direct comparison RMSE rankings of alternative beta estimators in predicting a particular beta estimator's 1 year future realization. The stock-month sample in this figure is identical to those in Table 3.

Interpretation: Based on the R-squared metric, the bswa estimator has the best overall performance in predicting future realizations of itself and other beta estimators, except for blw. As for RMSE, blw generally outperforms other beta estimators.