Improving Volatility-Managed Portfolios in Real Time

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Abstract

Recent studies find poor out-of-sample performance for volatility-managed portfolios. We propose a simple improved strategy based on Moreira and Muir (2017)'s formation of volatility management. The improved volatility management features effective risk scaling, conditional expected return, and intercept from conditional risk return tradeoff. Using this strategy for a comprehensive set of 197 risk factors and anomaly portfolios, we document significant real-time performance improvements including 148 Sharpe ratio increases and 165 positive abnormal returns. The performance survives robustness checks of leverage constraints, transaction costs, and different design choices.

Keywords: Volatility Management, Real-time Performance, Portfolio Choice

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Can volatility management improve investment performance for real-time investors? Comprehensively examining volatility-managed portfolios, Moreira and Muir (2017) find that mean-variance investors achieve a significant abnormal return and a higher Sharpe ratio by simply investing more (less) when factor volatility is low (high). Their findings echo with the exhibition of Barroso and Santa-Clara (2015) and Daniel and Moskowitz (2016) that managing momentum risk remarkably enhances performance. These studies are also related to the rich literature of volatility timing such as Fleming et al. (2001, 2003).

However, recent studies throw doubt on the out-of-sample benefit of volatility management. Liu et al. (2019) and Cederburg et al. (2020) argue that Moreira and Muir (2017)'s specification of variance scaling coefficient suffers from a look-ahead bias. Calibrating this coefficient in real time relegates volatility-managed portfolios to underperformance. Using a broad set of 103 trading anomalies, Cederburg et al. (2020) show that the significant in-sample benefits of volatility management hardly translate into out-of-sample gains.

We contribute to the literature by proposing potential reasons for this real-time underperformance and by developing a simple improved strategy based on Moreira and Muir (2017)'s original formulation. Fully implementable in real time, the improved volatility management reduces to the plain volatility management under certain circumstances. Therefore, an important advantage of our improved strategy is its theoretical compatibility with Moreira and Muir (2017)'s analytical framework, compared to other extensions such as downside-volatility management introduced by Qiao et al. (2020) and Wang and Yan (2021).

Specifically, mean-variance utility is commonly assumed to motivate optimal investment strategies and volatility management (see Daniel and Moskowitz, 2016, Moreira and Muir, 2017, and Cederburg et al., 2020, among others). As managing downside volatility implicitly refers to a mean-downside variance preference, a theoretical reorientation to mean-downside variance utility is necessary to promote downside-volatility management. Comparatively, our improved strategy is fully consistent with Moreira and Muir (2017)'s framework and it does not entail a preference reorientation.

Our improved volatility management features three considerations. The first one is about effective risk scaling. Previous studies use a variance-equating scheme as default to scale factor exposure. Specifically, a coefficient is identified such that the original and managed factors have the same unconditional variance or the same historical variance over prior returns. See Moreira and Muir (2017), Cederburg et al. (2020), Barroso and Detzel (2021), Wang and Yan (2021), and DeMiguel et al. (2021), for the variance-equating scheme.

However, we find that variance-equating coefficients tend to comove with realized variances and deteriorate risk management. For example, we calculate the real-time calibrated variance-equating coefficients and realized variances for the CRSP US total market index from 1926 to 2020. The correlation coefficient is 16% at the 1% significance level. During the Great Depression, volatility management is expected to greatly reduce market exposure, while the concurrent increases in realized variance and variance-equating coefficient impede market exposure confinement and market risk management.[1]

The second consideration is about conditional expected return. The plain volatility management always has positive factor exposure because the factor's unconditional expected return is assumed to be positive (Moreira and Muir, 2017). However, the factor's conditional expected return can importantly differ from its unconditional version (Ehsani and Linnainmaa, 2022, Table 2) and possibly become negative. For instance, the volatility-managed market portfolio sticks to the market even during the Great Depression and Recession, while investors can simply leave the market to evade losses.[2]

The third consideration is about conditional risk return tradeoff. Moreira and Muir

[1]The variance-equating scheme may also hinder the full realization of risk management potential. Consider a factor with near constant return realizations over a short sample. Because the factor volatility is low, the volatility-timing potential which is negatively related to factor variance is high. However, the variance-equating coefficients are trivial, and the changes in factor exposure can be too small to take full advantage of this timing potential.

[2]Prior studies note the importance of positive conditional expected return. For example, Merton (1980) accentuates the restriction of positive prior expected return, and Ferson and Siegel (2001) show that including conditional expected return helps to improve unconditional performance of mean-variance investments. Daniel and Moskowitz (2016) design a dynamic momentum strategy based on estimating conditional expected momentum return, while Barroso and Maio (2021) construct long-short strategies according to the sign of a factor's conditional expected return.

(2017, page 1635) show that when a factor's expected return is independent of its risk, the abnormal return of the managed portfolio is driven by the dynamics of the factor's conditional expected return. In this case, the factor's conditional expected return is equivalent to the intercept from conditional risk return tradeoff. Conditional intercept is generally different from conditional expected return, and the former is informative on its own.

Suppose that the factor has a positive conditional intercept. Investors are interested in the intercept investment, which has a positive return and a minimal variance, thus a large conditional Sharpe ratio. This interest holds even if the factor has a negative conditional expected return. With an implicit assumption of positive conditional intercept, Cederburg et al. (2020, page 100) also point out that investors take aggressive factor exposure when realized variance is low. Once the factor has a negative conditional intercept, investors find it more profitable to turn off volatility management and stick to the original factor.

We formally relate the three considerations to the performance of volatility-managed portfolios in Moreira and Muir (2017)'s continuous-time framework. To guarantee effective risk scaling, we take a constant coefficient corresponding to an annualized volatility level of 12%, which is adopted by Fleming et al. (2001, 2003) and Barroso and Santa-Clara (2015), among others. We employ two dummy variables to opt for factor investing and volatility management. The return dummy takes the value of one if the conditional return is expected to be positive and zero otherwise, while the intercept dummy takes the value of one if the estimated conditional intercept is positive and zero otherwise.

The improved volatility management turns on factor investing when the return dummy is one, and it turns off volatility management when the intercept dummy is zero. We use a one-year rolling window to estimate conditional mean return and a five-year window to estimate conditional intercept. We are interested in the sign of conditional intercept and mean, other than in the estimate. Christoffersen and Diebold (2006) highlight that volatility forecastablility produces sign forecastablility, and a significant sign forecastablility is not at odds with the difficulty of predicting conditional mean. This mitigates the worry about the specification of estimation windows, given the high predictability of volatility at short horizons. Our robustness checks reaffirm that different window choices do not affect the efficacy of the improved volatility management.

Consistent with Moreira and Muir (2017) and Cederburg et al. (2020), we first inspect a collection of common risk factors.[3] To complement Cederburg et al. (2020, Table 5)'s real-time analysis of combination strategies, we directly compare the original and volatility-managed factors. We confirm their finding that the plain volatility management does not benefit real-time investors. Comparatively, our improved volatility management produces a systematic performance improvement: the average Sharpe ratio increase is 0.2 and the average abnormal return is 20%. The ameliorations in Omega ratio, Sortino ratio, and skewness are also material. Compared to the plain strategy, the improved strategy promotes statistically and economically greater enhancements.

The improved strategy also has the edge on alternative risk management strategies. In particular, Wang and Yan (2021) propose to scale factor exposure by downside risk, while Bongaerts et al. (2020) opt for a conditional volatility management only when realized variance is extremely high. We find the improved strategy superior to the two alternative strategies: the average Sharpe ratio surplus is 0.26 over the downside risk management and 0.12 over the conditional volatility management. In addition, we note prevalently significant abnormal returns when regressing the improved strategy on the two alternative strategies.

We extend the analysis to 187 trading anomalies maintained by Hou et al. (2015) and Hou et al. (2021). As Cederburg et al. (2020) highlight, a large factor representation helps to summarize covariance risk and cross-sectional variation. With the extended sample of 197 factors and anomalies, we reaffirm that the improved strategy is advantageous: it promotes 148 Sharpe ratio increases, 165 positive abnormal returns, 153 Sortino ratio enhancements,

[3]These factors are the market (MKT), size (SMB), and value (HML) factors from Fama and French (1993), the momentum (MOM) factor from Carhart (1997), the profitability (RMW) and investment (CMA) factors from Fama and French (2015), the investment (IA) and return on equity (ROE) factors from Hou et al. (2015), the expected growth (EG) factor from Hou et al. (2021), and the betting-against-beta (BAB) factor from Frazzini and Pedersen (2014).

and 132 skewness augmentations, all at the significance level of 1%. Directly comparing the plain and improved strategies, we document 141 Sharpe ratio increases, 161 abnormal return additions, 145 Sortino ratio enhancements, and 136 skewness augmentations, all significant at the 1% level. The material advantage of the improved strategy survives robustness checks of leverage constraints, transaction costs, and different design choices.

The paper proceeds as follows. Section 1 provides the motivation of this paper in improving volatility management. Section 2 describes the data, and Section 3 presents the main results and discussions. Section 4 concludes.

1 Motivation and framework development

1.1 Volatility-managed portfolios in real time

For a factor portfolio or a trading strategy f, let f_t denote its excess return in month t, f_t^j denote its jth daily excess return in month t, $j \in \{1, 2, ..., J_t\}$ where J_t indexes the total number of trading days in this month. In real time, investors manage f such that

$$f_t^{\sigma} = \frac{c_{t-1}}{\hat{\sigma}_{t-1}^2} f_t, \tag{1}$$

where c_{t-1} is a variance scaling coefficient, $\hat{\sigma}_{t-1}^2$ is the realized variance of daily excess returns in month $t-1$. Following Moreira and Muir (2017), we compute realized variance

$$\hat{\sigma}_t^2 = \frac{22}{J_t - 1} \sum_{j=1}^{J_t} (f_t^j - \frac{\sum_{j=1}^{J_t} f_t^j}{J_t})^2. \tag{2}$$

The coefficient of $\frac{22}{J_t - 1}$ adjusts for potential variations in the number of trading days across months, conforming to the treatments of Liu et al. (2019) and Cederburg et al. (2020).

Consistent with Cederburg et al. (2020, Section 4.3.1)'s out-of-sample volatility management strategy, investors calibrate the variance scaling coefficient c_{t-1} at the beginning of month t such that the original and managed factors have equal variances over prior returns.[4]

1.2 Concern of variance-equating coefficients

We notice that variance-equating coefficients usually comove with realized variances. As Equation 1 suggests, the variance-equating coefficient c_{t-1} is a function of previous monthly returns, while the realized variance $\hat{\sigma}^2_{t-1}$ is a function of daily returns in the prior month. Return information of the prior month is the common part, bridging the estimation of contemporary variance-equating coefficient and realized variance. Such a comovement has critical implications on risk scaling and risk management in real time.

For an illustration, we calculate the variance-equating coefficients and realized variances for the CRSP US total market index from 1926 to 2020. The first five years are the initial estimation window, then we consider expanding and rolling estimation windows to calibrate variance-equating coefficients at the beginning of each month. Following Cederburg et al. (2020), we use expanding estimation window as default.

Figure 1 shows the time series of realized variances and variance-equating coefficients. The comovement concern is plainly manifested in Panel A: both realized variances and variance-equating coefficients surge in the Great Depression. As the depression fades, variance-equating coefficients diminish from the 1940s to the 1970s, conforming to a general mitigation of market risk. Nevertheless, the ratio of variance-equating coefficient to realized variance can become so high that investors can hardly implement volatility management in practice. For instance, the ratio of variance-equating coefficient to realized variance is 8.02 in March

[4] Moreira and Muir (2017) specify the variance scaling coefficient to equate unconditional variances of the original and managed factors. From a practical perspective, Liu et al. (2019) and Cederburg et al. (2020) argue that this specification involves a look-ahead bias, as real-time investors have no ex ante information of unconditional variances. Cederburg et al. (2020)'s out-of-sample design adjusts for the look-ahead bias. It is worth noting that the basic motivation of Moreira and Muir (2017) comes from mean variance demand, where the optimal factor weight is positively related to the conditional price of risk and negatively related to investor's risk aversion (see also Daniel and Moskowitz, 2016, Appendix C, Equation 23). Upon constant conditional mean, the variance scaling coefficient is the ratio of conditional mean to risk aversion, which varies across investors due to individual preferences. As Moreira and Muir (2017) focus on a performance comparison with the buy-and-hold case, using other normalization schemes instead of matching unconditional variances makes a trivial difference to the economic risk-return relations and key measures like Sharpe ratio that describe such relations.

1940, and the required leverage is beyond the capacity of an average mean-variance investor.

After the 1970s, variance-equating coefficients experience a mild increase, particularly during the Great Recession. The whole sample period correlation is 16%, statistically significant at the 1% level. A greater comovement is observed with the rolling window in Panel B, and the correlation is 18% at the 1% significance level. Notably, both realized variances and variance-equating coefficients plummet during the market exuberance before 2008 and then swing up as the Great Recession hits.

Volatility management is expected to substantially increase (decrease) factor exposure when realized volatility is low (high). The comovement, however, impedes leveraging (deleveraging) factor exposure and challenges risk management in real time.

1.3 Effective risk scaling and conditional information

We next formally relate the comovement concern to volatility management performance following Moreira and Muir (2017)'s continuous-time theoretical framework. Let R_t denote the total value process of the original factor and r_t^f denote the instantaneous risk-free rate. For the original and managed factors, we have

$$dR_t = (r_t^f + \mu_t)dt + \sigma_t dB_t \quad \text{and} \quad dR_t^\sigma = r_t^f dt + \frac{c_t}{\sigma_t^2}(dR_t - r_t^f dt), \tag{3}$$

where μ_t is the expected excess return of the original factor, σ_t^2 is its conditional variance, B_t is a Brownian process, and c_t is a function of σ_t^2.

Regressing the managed factor returns on the original factor returns, we have the abnormal return

$$\begin{aligned} \alpha &= \mathrm{E}[dR_t^\sigma - r_t^f dt]/dt - \beta \mathrm{E}[dR_t - r_t^f dt]/dt \\ &= \mathrm{cov}(\frac{\mu_t}{\sigma_t^2}, c_t) - \frac{\mathrm{E}[c_t]}{\mathrm{E}[\sigma_t^2]} \cdot \mathrm{cov}(\frac{\mu_t}{\sigma_t^2}, \sigma_t^2). \end{aligned} \tag{4}$$

It shows that the abnormal return is decided by the covariance between the price of risk μ_t/σ_t^2 and risk target c_t as well as between the price of risk and conditional risk σ_t^2. When c_t is constant, $\text{cov}(\mu_t/\sigma_t^2, c_t) = 0$ and Equation 4 reduces to Moreira and Muir (2017)'s Equation 8. We next explain the implication of $\text{cov}(\mu_t/\sigma_t^2, c_t)$ when it is not zero.

The factor's price of risk reflects the importance of risk return tradeoff to volatility management. Consistent with Merton (1980, Model 1), French et al. (1987, Equation 1), and Ghysels et al. (2005, Equation 1), we introduce the following risk return tradeoff

$$\mu_t = \lambda_1 + \lambda_2 \sigma_t^2. \tag{5}$$

Combining Equations 4 and 5, we have

$$\alpha = \underbrace{\frac{\text{E}[\lambda_1]}{\text{E}[\sigma_t^2]}\text{cov}(\text{E}[\sigma_t^2]c_t - \text{E}[c_t]\sigma_t^2, \frac{1}{\sigma_t^2})}_{\text{intercept effect}} + \underbrace{\frac{1}{\text{E}[\sigma_t^2]}\text{cov}(\text{E}[\sigma_t^2]c_t - \text{E}[c_t]\sigma_t^2, \lambda_2)}_{\text{slope effect}}. \tag{6}$$

Following Moreira and Muir (2017), we consider a constant λ_2. The slope effect which describes the comovement between λ_2 and $\text{E}[\sigma_t^2]c_t - \text{E}[c_t]\sigma_t^2$ is zero, and the benefits of volatility management are predominantly driven by the intercept effect. When c_t is independent from σ_t^2, the covariance between $\text{E}[\sigma_t^2]c_t - \text{E}[c_t]\sigma_t^2$ and $1/\sigma_t^2$ is negative. Thus, λ_1 is the key to volatility management performance. This is consistent with Moreira and Muir (2017, page 1635)'s analysis that $\alpha = 0$ when volatility moves together with expected return (i.e., $\lambda_1 = 0$).

We next analyze the effect of c_t dynamics. If $\text{cov}(\text{E}[\sigma_t^2]c_t - \text{E}[c_t]\sigma_t^2, 1/\sigma_t^2)$ is positive, we can rewrite Equation 6 as

$$\alpha = \lambda_1 \cdot \frac{\text{cov}(\text{E}[\sigma_t^2]c_t - \text{E}[c_t]\sigma_t^2, 1/\sigma_t^2)}{\text{E}[\sigma_t^2]}. \tag{7}$$

Straightforwardly, the intercept effect is complementary to previous studies' focus of the slope effect. In particular, it is preferred not to manage factor risk when λ_1 is negative. This implication is of special interest to improve volatility management, which we will discuss in the following subsection. Besides, when λ_1 is positive, the higher the conditional intercept, the greater the abnormal return.

Similar to the slope effect, the term $\text{cov}(\text{E}[\sigma_t^2]c_t - \text{E}[c_t]\sigma_t^2, 1/\sigma_t^2)$ displays the importance of c_t to volatility management. The cross effect between c_t and σ_t^2 is zero when the deviation of conditional variance σ_t^2 from unconditional variance $\text{E}[\sigma_t^2]$ is equal to the deviation of conditional variance scaling coefficient c_t from unconditional variance scaling coefficient $\text{E}[c_t]$. In this case, volatility management assigns constant exposure of $\text{E}[c_t]/\text{E}[\sigma_t^2]$ to the original factor, rendering zero intercept effect and zero slope effect.

We use ρ to characterize the relative deviation: $c_t/\sigma_t^2 = \rho_t \text{E}[c_t]/\text{E}[\sigma_t^2]$. Then,

$$\alpha = \frac{\text{E}[c_t]}{\text{E}[\sigma_t^2]}\big(\text{E}[(\rho_t - 1)\mu_t|\mu_t > 0]P(\mu_t > 0) + \text{E}[(\rho_t - 1)\mu_t|\mu_t \leq 0]P(\mu_t \leq 0)\big), \tag{8}$$

which reflects the interaction between ρ_t and μ_t in affecting volatility management efficacy. Greater factor exposure $\rho_t > 1$ upon high conditional expected return μ_t is favorable. The variance-equating scheme, as Figure 1 shows, provokes adverse adjustments: c_t goes up during the Great Depression and Recession, hindering factor exposure confinement.

Moreover, positive conditional intercept λ_1 does not necessarily come with positive conditional expected return μ_t. To see this point, suppose a constant c across periods and $c < \text{E}[\sigma_t^2]$. Then, the managed factor has a conditional expected return of $c \cdot \mu_t/\sigma_t^2$, which is negative as long as $\mu_t < 0$. Investors can simply choose not to hold the original factor to have zero return, which is better than both μ_t and $c \cdot \mu_t/\sigma_t^2$. By taking this option, investors effectuate a conditional expected return of $\max(c \cdot \mu_t/\sigma_t^2, 0)$ for the managed factor, promoting a higher unconditional expected return and a higher Sharpe ratio.

When expected return is independent of risk, Equation 4 reduces to

$$\alpha = c\frac{\text{cov}(-1/\sigma_t^2, \sigma_t^2)}{\text{E}[\sigma_t^2]}\big(\text{E}[\mu_t|\mu_t > 0]P(\mu_t > 0) + \text{E}[\mu_t|\mu_t \leq 0]P(\mu_t \leq 0)\big). \tag{9}$$

The impact of μ_t is explicit: positive μ_t facilitates volatility management while negative μ_t damages performance. We next show that investors can take advantage of the informative

μ_t and λ_1 to ameliorate volatility management.

1.4 Improving volatility management

The discussions of effective risk scaling and conditional information point to potential improvements for the plain volatility management. First, we take a constant variance target instead of the variance-equating scheme to scale risk. Specifically, we choose the annualized volatility of 12% as c^*, which makes the term $\mathrm{cov}(\mu_t/\sigma_t^2, c_t)$ zero and pacifies the comovement concern.

The choice of 12% conforms to previous studies such as Fleming et al. (2001, 2003) and Barroso and Santa-Clara (2015), among others. Different choice does not affect key performance measures such as Sharpe ratio. Risk scaling is effective and $\alpha = \frac{c^*}{\mathrm{E}[\sigma_t^2]}\mathrm{cov}(-\frac{\mu_t}{\sigma_t^2}, \sigma_t^2)$, whose economic size is proportional to c^* but statistical size unaffected. Besides, $\mathrm{cov}(\mathrm{E}[\sigma_t^2]c_t - \mathrm{E}[c_t]\sigma_t^2, 1/\sigma_t^2) = c^* \cdot \mathrm{cov}(-1/\sigma_t^2, \sigma_t^2) > 0$ guarantees the validity of Equations 7 and 9. Effective risk scaling contributes to raising real-time performance.

Second, we incorporate the constraint of positive conditional intercept based on the implication of Equation 7. We run the following regression to estimate conditional intercept

$$f_t = \lambda_1 + \lambda_2 \hat{\sigma}_{t-1}^2 + \epsilon_t. \tag{10}$$

We adopt a five-year rolling window, which is frequently used in the literature. For example, Wang et al. (2017) use it to construct reference-dependent risk measures and examine the corresponding conditional risk return tradeoff. See also Liu et al. (2018) for the estimation of conditional intercepts with the five-year window.[5]

A concern is that the five-year window may capture more about time-series momentum.

[5] We thank an anonymous referee for pointing out that λ_1 should be considered constant here. When estimation errors regarding λ_1 are a concern, one can conveniently modify the determination of conditional intercept dummy by taking into account the statistical significance of $\hat{\lambda}_1$. The conditional intercept dummy then takes the value of one if the estimated conditional intercept is significantly positive and zero otherwise. Liu et al. (2018) provide further discussions about estimation errors in conditional regression.

However, Kelly et al. (2021, Section A1) show that momentum predictability is significant at one-year length but fades away at five-year length, and considerably less published evidence is for long-term momentum.Ehsani and Linnainmaa (2022) also register significant factor momentum at one-year length. This helps to alleviate the momentum concern.

Third, we include the constraint of positive conditional expected return. By doing so, investors keep from the negative term $\mathrm{E}[\mu_t|\mu_t \leq 0]P(\mu_t \leq 0)$ in Equation 9 to improve performance. Following previous studies, we estimate conditional expected return as the geometric mean of factor excess returns over the prior 12 months. Chordia and Shivakumar (2002, page 997) show that the past one-year information helps to forecast one-month-ahead returns, and Ehsani and Linnainmaa (2022, Table 2) reaffirm that returns in the prior year significantly predict future returns.

It is worth noting that we are interested in the sign of conditional intercept and conditional expected return, rather than in the estimate. Some may fairly argue that the five-year and one-year windows possibly produce noisy estimates. However, Christoffersen and Diebold (2006) find that volatility forecastablility generates sign forecastablility, and a significant sign forecastablility is not at odds with the difficulty of conditional mean forecastablility. This mitigates the worry about window specifications. We conduct robustness checks in the last section and reaffirm that window choice does not make a big difference.

We improve the plain volatility management with the three modifications

$$g_t = \delta_{t-1}(\frac{c^*}{\hat{\sigma}_{t-1}^2})^{\gamma_{t-1}} f_t, \tag{11}$$

where γ_{t-1} (δ_{t-1}) is the conditional intercept (return) dummy which takes the value of one if the estimated conditional intercept (mean) is positive and zero otherwise. Comparing Equations 1 and 11, we see that the improved strategy g reduces to the plain strategy f^σ if conditional intercept and conditional expected return are both positive ($\gamma_{t-1} = \delta_{t-1} = 1$), as long as investors target a prespecified risk level. When investors expect negative conditional intercept and positive conditional mean ($\gamma_{t-1} = 0$ and $\delta_{t-1} = 1$), g turns off volatility management and holds the original factor. Furthermore, g can be more active than f^σ in controlling factor exposure upon negative conditional mean ($\delta_{t-1} = 0$). Factor investing is no longer an obligation, and g holds risk-free asset rather than the depressed original factor.

2 Data description

Consistent with Moreira and Muir (2017) and Cederburg et al. (2020), we first examine the market (MKT), size (SMB), and value (HML) factors from Fama and French (1993), the momentum (MOM) factor from Carhart (1997), the profitability (RMW) and investment (CMA) factors from Fama and French (2015), the investment (IA) and return on equity (ROE) factors from Hou et al. (2015), the expected growth (EG) factor from Hou et al. (2021), and the betting-against-beta (BAB) factor from Frazzini and Pedersen (2014). This collection associates with some of the most widely used multiple-factor models like the Fama-French factor models and q-factor models.

We collect monthly and daily returns for these factors. Specifically, for the MKT, SMB, HML, MOM, RMW, and CMA factors, data are from Kenneth French's data library. For the IA, ROE, and EG factors, data are from Lu Zhang's global-q data library. For the BAB factor, data are from AQR datasets. The factor sample periods are from the earliest available months and all end in December 2020. Precisely, for the MKT, SMB, and HML factors, the sample period begins from July 1926; for the MOM factor, it starts in January 1927; for the RMW and CMA factors, it begins from August 1963; for the IA, ROE, and EG factors, it starts in February 1967; for the BAB factor, the beginning month is December 1930.

Cederburg et al. (2020) note that a broad factor representation helps to summarize covariance risk and cross-sectional variation. They inspect 94 trading anomalies in addition to 9 risk factors to evaluate the real-time performance of volatility management. We further the exploration by investigating 187 trading anomalies maintained by Hou et al. (2015) and Hou et al. (2021), covering 6 categories: momentum (41), value-versus-growth (32), investment (29), profitability (45), intangibles (30), and frictions (10). Monthly and daily return data are from the global-q data library, and the samples are from the earliest available months to December 2020. We provide supplementary information in the Internet Appendix.

In total, we have an extensive sample of 197 factor portfolios and trading anomalies. We believe that such a large representation contributes to comprehensively assessing the real-time efficacy of volatility management strategies.

3 Main results and discussions

3.1 Direct performance comparison

We first report the performance of the 10 risk factors as well as their two managed versions in Table 1. Prior studies use Sharpe ratio as the main indicator in direct performance comparison. We further examine Sortino ratio (Wang and Yan, 2021) and skewness (Barroso and Santa-Clara, 2015; Daniel and Moskowitz, 2016), which are complementary to Sharpe ratio for their specific attention to downside risk. The three ratios are invariant to the choice of constant variance scaling coefficient. In the Internet Appendix, we present more comprehensive performance comparisons including mean return, standard deviation, Omega ratio, upside potential ratio, and kurtosis.

Consistent with Cederburg et al. (2020), we find that the plain strategy weakly improves out-of-sample performance: only the managed MOM and ROE factors witness a statistically significant Sharpe ratio increase. The managed ROE factor has the largest increase of 0.38, with a T statistic of 2.89 from Ledoit and Wolf (2008)'s heteroskedasticity and autocorrelation consistent Sharpe ratio test. It also has the largest amelioration in Sortino ratio (1.30) as well as in skewness (2.45). A notable failure is for the SMB factor, as the three ratios are substantially deteriorated. In particular, the Sharpe ratio reduction is about 40% (0.09/0.22), and the managed SMB has a poor Sortino ratio of 0.20 ($0.37 - 0.17$).

By contrast, the improved strategy materially improves performance, as the average Sharpe ratio change escalates from 0.09 by the plain strategy to 0.20. Specifically, the improved strategy promotes eight Sharpe ratio increases, and five of them are statistically significant. This out-of-sample performance is even better than the in-sample performance of the plain strategy, which only achieves four significant Sharpe ratio increases (see Cederburg et al., 2020, Table 1). The changes in Sortino ratio and skewness are also favorable.

Comparing the two strategies reaffirms the edge of the improved strategy. We note four significant Sharpe ratio increases and only two trivial decreases (0.01 for the MKT factor and 0.02 for the ROE factor). Particularly, the improved strategy brings a remarkable surplus of 0.20 in Sharpe ratio, 0.95 in Sortino ratio, and 3.77 in skewness when the MOM factor is under management. The literature broadly recognizes the success of the plain strategy for momentum, while the improved strategy offers greater benefits to manage momentum risk. Similarly, BAB investors find it profitable to switch to the improved strategy.

To exhibit the performance difference, we plot the cumulative returns on the MKT, HML, and BAB factors as well as their managed versions in Figure 2. For the MKT factor, the improved strategy circumvents the early 1940s drawdown concerning the plain strategy. The cumulative return of the plain strategy is $0.66 in April 1942, much lower than that of the improved strategy at $1.26. This observation is consistent with Table 1 that the improved strategy has a higher skewness by 0.67. The terminal gains of the three investments are close, while the improved strategy presents the most robust performance.

For the HML and BAB factors, it is clear that the improved strategy is the most profitable for investors. Its cumulative return is higher than that of the original factor and the plain strategy. For example, the terminal gain is $19.69 for the HML factor and $22.93 for the plainly managed-HML factor. By contrast, the improved strategy contributes a considerable return of $285.55, eclipsing its peers. The performance difference is even more striking for the BAB factor. The improved strategy permits a terminal payoff of $5,4607.24, much beyond the plain strategy benefit of $2,856.89.

3.2 Spanning regression analysis

We complement the direct performance comparison with spanning regression analysis. Our examination involves three time-series spanning regressions: regressing the plain strategy on the original factor ($f^\sigma \sim f$), the improved strategy on the original factor ($g \sim f$), and the improved strategy on the plain strategy ($g \sim f^\sigma$). Besides, we consider the following regression to measure the abnormal return surplus of the improved strategy

$$h_t = \alpha + \eta_\alpha I_t + (\beta + \eta_\beta I_t) f_t + \epsilon_t, \tag{12}$$

where the strategy dummy variable I_t takes the value of one if $h_t = g_t$ and zero if $h_t = f_t^\sigma$. The estimate of η_α corresponds to the additional benefit by switching to the improved strategy.

Table 2 demonstrates that the improved strategy is superior to its peers in promoting abnormal returns. Specifically, the plain strategy gives rise to an average annualized abnormal return of 3.68%, and six abnormal returns are significant at the level of 5%. Consistent with Moreira and Muir (2017, Table 1), managing SMB and CMA factors fails to arrive at significant abnormal returns. For instance, the managed SMB factor has a trivial abnormal return of 0.31% with a T statistic of 0.24.

Comparatively, the improved strategy brings an average abnormal return of 21.13%, nearly six times that of the plain strategy. There are nine (seven) significant abnormal returns at the 5% (1%) level. For instance, managing the MOM factor with the improved strategy raises the abnormal return from 12.39% to 31.41%, and this surplus of 19.02% is highly significant (T statistic is 3.16). Besides, the improved strategy produces better abnormal returns for all factors, an unambiguous evidence that it is a superior choice. The average surplus is a considerable 16.45%, and eight abnormal return increases are significant at the 5% level. The largest surplus is 30.71% (T statistic is 4.81) for the EG factor.

Spanning regressions between the two strategies reaffirm the advantage of the improved strategy. The average abnormal return is 9.87%, and seven abnormal returns are significant at the 5% level. The largest one is 18.68% (T statistic is 4.48) for the BAB factor, and even the minimum one is a non-trivial 1.36%. Further controlling for Fama and French (1993)'s three factors does not change the results. For instance, the improved strategy attains an additional abnormal return of 26.09% (T statistic is 3.50) for the RMW factor.

Furthermore, we conduct Gibbons et al. (1989, GRS)'s test to examine the null hypothesis that all of the abnormal returns are jointly zero in the presence of Fama and French (1993)'s three common factors. The GRS test statistics help to summarize the performance of volatility management strategies and facilitate the comparison. As we can see, the GRS test rejects the null hypothesis for both the plain and improved strategies at the 1% significance level. Besides, the edge of the improved strategy over the plain strategy remains significant. For instance, the F-statistic is 3.16 in the direct comparison, corroborating that investors are benefited from the switch to the improved strategy.

An important reason for the edge of the improved strategy is effective risk scaling. In the Internet Appendix, we plot the correlation between realized variances and variance-equating coefficients. The factors are prevalently exposed to the comovement concern, and the RMW factor suffers the most. The improved strategy is free from such a concern, and its risk scaling is effective. We advance performance attribution in the next subsection.

3.3 Performance attribution

To inspect how the modifications contribute individually and interactively, we exhaust all combinations of them. We first consider the individual contribution of each modification: $g_t^C = \frac{c^*}{\hat{\sigma}_{t-1}^2} f_t$ focuses on effective risk scaling; $g_t^M = \delta_{t-1} \cdot \frac{c_{t-1}}{\hat{\sigma}_{t-1}^2} f_t$ concentrates on positive conditional mean; $g_t^I = (\frac{c_{t-1}}{\hat{\sigma}_{t-1}^2})^{\gamma_{t-1}} f_t$ pinpoints positive conditional intercept. We also include the interactive contribution in pairs: $g_t^{CM} = \delta_{t-1} \cdot \frac{c^*}{\hat{\sigma}_{t-1}^2} f_t$ characterizes the effect of constant risk scaling and positive conditional mean; $g_t^{MI} = \delta_{t-1}(\frac{c_{t-1}}{\hat{\sigma}_{t-1}^2})^{\gamma_{t-1}} f_t$ marks the effect of positive conditional mean and positive conditional intercept; $g_t^{CI} = (\frac{c^*}{\hat{\sigma}_{t-1}^2})^{\gamma_{t-1}} f_t$ specifies the effect of constant risk scaling and positive conditional intercept.

Table 3 compares these strategies in terms of Sharpe ratio improvements. The plain strategy increases Sharpe ratio by 0.09 on average, while the modifications contribute to greater improvements: the average Sharpe ratio increase is 0.15 by g^C, 0.16 by g^M, and 0.12 by g^I, displaying stronger benefits by departing from the plain strategy. We note that the three modifications boost performance from different aspects: g^C raises more significant Sharpe ratio changes, yet it fails to manage the SMB factor given a decrease of 0.16 (T statistic is -1.70); g^M acquires the highest average Sharpe ratio increase, but it inaptly manages the MOM factor; g^I extends the best protection from Sharpe ratio declines, though its average Sharpe ratio amelioration is the lowest.

Coupling the modifications leads to interesting observations. The average Sharpe ratio improvements of the three interactive strategies–g^{CM}, g^{MI}, and g^{CI}–are 0.21, 0.17, and 0.14, dominating that of the plain strategy. One may argue that the constraint of positive conditional mean is the foremost performance driver by capturing factor momentum. However, we find g^{CI}, which is without factor momentum, the most successful in managing the MOM factor. It produces the greatest Sharpe ratio increase of 0.55 (T statistic is 4.25). By contrast, the constraint of positive conditional mean substantially underperforms, individually or jointly with other modifications, for the MOM factor. This observation suggests that using factor momentum is detrimental to manage momentum factor. As momentum risk management is a prominent arena of risk management,it is necessary to synthesize the three modifications to reach a comprehensive improvement.

Table 4 evaluates the individual and interactive effects with spanning regressions on the plain strategy. Panel A shows that the average abnormal return is 3.77% for g^C, 1.28% for g^M, and 1.32% for g^I. Combining two modifications further raises average abnormal returns to 9.65% for g^{CM}, 2.09% for g^{MI}, and 6.09% for g^{CI}. Therefore, it is beneficial for investors to deviate from the plain strategy, consistent with the Sharpe ratio examination. Besides, strategies without factor momentum, like g^C and g^{CI}, achieve much better performance in managing the MOM factor. For instance, g^{CI} attains the highest abnormal return of 19.24%

(T statistic is 3.47), suggesting that factor momentum does not necessarily guarantee the best performance amelioration. The improved strategy outperforms its reduced versions, as its average abnormal return is the highest 9.87%.

Cederburg et al. (2020) notice that a positive abnormal return does not necessarily signal performance enhancement because reverse spanning regression can render a positive intercept, too. We run reverse spanning regressions and report the results in Panel B. The intercepts are hardly significant, reaffirming that the improved strategy is advantageous. For example, the average intercept is -0.44% for g^C, suggesting that investors lose by following the plain strategy. Notably, g^M incurs a significant intercept of 2.95% for the MOM factor, corroborating that factor momentum deteriorates momentum risk management.

In the Internet Appendix, we explore why factor momentum fails the MOM factor. A key reason is that momentum have time-varying systematic risk exposure (Grundy and Martin, 2001 and Daniel and Moskowitz, 2016), and factor momentum poorly forecasts momentum returns (Ehsani and Linnainmaa, 2022, Table 2). However, the MOM factor has stable conditional intercepts, extending a favorable condition to adjust volatility management. Consequently, the constraint of positive conditional intercept is more successful than the constraint of positive conditional mean.

In short, the three modifications are complementary to each other, and they enhance performance from different aspects. Integrating their advantages by using the improved strategy is particularly relevant when investors manage a broad range of risk factors.

3.4 Comparison with alternative risk management

The advantage of the improved strategy is not only on the plain strategy. Recent studies propose alternative risk management, and two representative strategies are Wang and Yan (2021)'s downside risk management as well as Bongaerts et al. (2020)'s conditional volatility management. We next show that the improved strategy also has the edge on them.

Wang and Yan (2021) use downside risk to scale factor exposure and their downside risk management can be formulated as

$$f_t^{\sigma D} = \frac{c_{t-1}}{\hat{\sigma}_{D,t-1}^2} f_t, \tag{13}$$

where $\hat{\sigma}_{D,t}^2 = \frac{22}{J_t - 1} \sum_{j=1}^{J_t} \min(f_t^j, 0)^2$ is the realized downside variance. Wang and Yan (2021) choose downside volatility for risk scaling while we employ downside variance to be consistent with the formation of the plain and improved strategies.

Bongaerts et al. (2020) opt for a conditional volatility management only when realized variance is extreme. The conditional strategy goes as

$$f_t^{\sigma C} = \mathcal{V} \cdot \frac{d_{t-1}}{\hat{\sigma}_{t-1}^2} \cdot f_t + (1 - \mathcal{V}) \cdot f_t, \tag{14}$$

where $\mathcal{V}$ is the risk indicator taking the value of one if $\hat{\sigma}_{t-1}^2$ falls in the extreme quintile of realized variances up to month $t-1$ $\{\hat{\sigma}_\tau^2 | \tau = 1, 2, \ldots, t-1\}$ and zero otherwise, while d_{t-1} is the long-term realized variance computed with all daily returns prior to month t.

Compared with Bongaerts et al. (2020, Equation 3), we stick to scaling factor exposure by realized variance and ignore the leverage constraint in accordance with the formation of the plain and improved strategies. This treatment tends to overstate the strategy performance. As a balance, the extreme risk indicator focuses on the high-risk state.

Figure 3 displays that the improved strategy outperforms the two alternative strategies. The average Sharpe ratio surplus is 0.26 over the downside risk management and 0.12 over the conditional volatility management. Particularly, the improved strategy exhibits a significant advantage in managing the MOM factor: the Sharpe ratio surplus is 0.52 (T statistic is 6.06) over the downside strategy and 0.25 (T statistic is 2.81) over the conditional strategy. Our results are complementary to Wang and Yan (2021, Table 7) as their examination is with respect to the combination strategy consisting of the risk-free asset, original factor, and managed factor.

Regressing the improved strategy on the two alternative strategies, we focus on the statistical significance of abnormal performance as T statistics are invariant to changes in the constant risk target. The improved strategy promotes seven (eight) significant abnormal returns at the 5% level over the downside (conditional) strategy. These observations of abnormal performance agree with those of Sharpe ratio comparisons, reaffirming that the improved strategy provides greater benefits to real-time investors.

3.5 Extended sample

We expand our analysis to the 187 trading anomalies maintained by Hou et al. (2015) and Hou et al. (2021). Together with the 10 risk factors, our extended sample consists of 197 factor portfolios and trading anomalies. We believe that this extended sample facilitates the characterization of cross-sectional return variations and covariance risk.

Following Cederburg et al. (2020), we concentrate on the number of improved indicators to assess volatility management efficacy.[6] Table 5 shows that the plain strategy does not strongly enhance Sharpe ratio in general (p-value is 0.12), but it significantly excels at momentum anomalies. Specifically, the momentum category contributes the most to the 110 Sharpe ratio increases by the plain strategy, in line with Cederburg et al. (2020)'s finding that this strategy's success is principally attributable to momentum. By contrast, the plain strategy reduces Sharpe ratio for the other anomalies. For instance, it results in a significant performance deterioration (p-value is below 0.01) in managing intangibles anomalies.

The improved strategy systematically improves performance. For value-versus-growth anomalies, the number of Sharpe ratio improvements is 29 (p-value is below 0.01), much greater than its counterpart of 16 (p-value is 1.00) by the plain strategy. Even for momentum anomalies, the improved strategy is on a par with the plain strategy in boosting performance. Moreover, the improved strategy prevalently enhances the statistical significance of abnormal

[6] Consistent with Cederburg et al. (2020), we use a binomial distribution test to infer statistical significance for the performance improvement of volatility management. This test takes the null hypothesis of equal likelihood of positive or negative performance difference brought by volatility management. Rejecting the null suggests that investors should favor volatility management in practice.

returns across categories. As skewness is important in revealing factor crashes (see Barroso and Santa-Clara, 2015 and Daniel and Moskowitz, 2016), the 132 skewness additions attest that the improved strategy notably mitigates crash risk.

In the direct comparison, the improved strategy significantly outperforms the plain strategy along all indicators. For instance, there are 161 abnormal return additions (p-value is below 0.01), a strong proof that real-time investors are better off with the improved strategy. Particularly, it is always favorable to manage value-versus-growth and intangibles anomalies following the improved strategy, and the advantage is predominant. Even for momentum anomalies at which the plain strategy excels, the improved strategy still ameliorates 32 abnormal returns with a p-value below 0.01.

The improved strategy achieves 141 higher Sharpe ratios. For instance, 31 out of 32 value-versus-growth anomalies are better managed by the improved strategy. Nonetheless, only 11 Sharpe ratio increases are documented for momentum anomalies. A reason for this relative underperformance, as aforementioned, is that factor momentum poorly predicts conditional mean for momentum anomalies. Meanwhile, we interpret this relative disadvantage with caution because the improved strategy significantly raises 32 (33) abnormal returns (skewness) out of 41, with a p-value well below 0.01.

In addition to the statistical significance, Table 6 confirms that the performance advantage is also economically important. On average, the plain strategy obtains a Sharpe ratio increase of 0.03, an abnormal return of 1.27%, a Sortino ratio augmentation of 0.12, and a skewness addition of 0.20. The improved strategy dominates the plain strategy along each indicator: the increase is 0.14 in Sharpe ratio, 0.32 in Sortino ratio, and 0.60 in skewness, while the abnormal return is elevated to 5.22%. Notably, the plain strategy materially deteriorates performance for intangibles and frictions anomalies, while the improved strategy strongly ameliorates all the performance indicators.

3.6 Leverage constraint and transaction cost

Real-time investors can be constrained to employ leverage in implementing volatility management strategies. We next consider if the improved strategy remains superior in the presence of leverage constraints. We specify three scenarios: when factor exposure has to be below 1 (i.e., no leverage is allowed), below 1.5 (leverage below 0.5), and below 2 (leverage below 1). The first two scenarios follow Moreira and Muir (2017) and Cederburg et al. (2020), while we add the third one because the initial margin requirement of 50% for stock purchases implies a maximal exposure of 2 (see Ang et al., 2011).

Leverage constraint binds factor exposure is risk scaling. For example, the effective factor exposure is $\min(\frac{c_{t-1}}{\hat{\sigma}^2_{t-1}}, 2)$ for the plain strategy and $\min\big(\delta_{t-1}(\frac{c^*}{\hat{\sigma}^2_{t-1}})^{\gamma_{t-1}}, 2\big)$ for the improved strategy if leverage is up to 1. Figure 4 shows that the improved strategy produces higher abnormal returns for eight factors when factor exposure is below 2. The largest surplus is 4.88% for the BAB factor, while the only notable decline is −1.73% for the MOM factor. The size of abnormal return surplus tends to be smaller when leverage constraint is more binding. However, the advantage of the improved strategy is not materially affected. Even if leverage is inaccessible at all, the improved strategy still manages to reward investors with seven better abnormal returns.

Table 7 summarizes the effects of leverage constraint on the extended sample. The edge of the improved strategy keeps highly significant in all the three scenarios. Even when leverage is unavailable, the improved strategy leads to 114 increases in Sharpe ratio (p-value is 0.03) and 124 increases in abnormal return (p-value is below 0.01) over the plain strategy. Directly regressing the improved strategy on the plain strategy, we find 160 positive abnormal returns with a p-value well below 0.01. In particular, investors occupied managing the 10 risk factors should practice the improved strategy rather than the plain strategy.

The advantage of the improved strategy grows with better leverage. When factor exposure is below 2, the improved strategy promotes 128 Sharpe ratio increases and 131 higher abnormal returns, more than in the case of unavailable leverage. The plain strategy seems to better serve momentum anomalies as leverage is more limited. However, the improved strategy obtains positive abnormal returns for most momentum anomalies in the direct spanning regression, regardless of leverage constraints. That is to say, when investors take the original momentum anomalies as performance benchmark, the improved strategy is less advantageous; when the benchmark is the plain strategy, investors stick to the improved strategy. For the other factors and anomalies, the edge of the improved strategy is unambiguous.

The improved strategy turns off factor investing when the factor's conditional return is expected to be negative. This feature reduces factor exposure and mitigates leverage constraint. Meanwhile, it also promotes exposure adjustments such as more frequent in-and-out than the plain strategy, so turnover and transaction costs can be a practical concern. We next examine if the advantage of the improved strategy survives transaction costs.[7]

Moreira and Muir (2017) inspect three assumptions of transaction cost: 1bp, 10bps, and 14bps. We further add an assumption of 40bps. Table 8 reports the performance comparison when transaction costs are 10bps, 14bps, and 40bps, respectively. The edge of the improved strategy survives transaction costs. With the 40bps assumption, the improved strategy produces 150 higher Sharpe ratios and 152 higher abnormal returns, both significant at the 1% level. Especially, investors always find the improved strategy preferred in managing the 10 risk factors: it offers 8 better Sharpe ratios, 10 better abnormal returns, and 10 positive direct abnormal returns. Such benefits are persistent across transaction cost assumptions.

3.7 Different design choice

Implementing the plain and improved strategies involves three estimation windows: the one-month window for realized variance, the one-year window for conditional expected re-

[7]Because our focus is the comparison between the original and managed factors, we consider factor level turnover for the two volatility management strategies. As Barroso and Detzel (2021) point out, a cost-effective option for tracking factor portfolios is via ETFs. For instance, Fidelity and Blackrock offer diverse ETF choices covering size, momentum, value, low volatility, and quality. Barroso and Detzel (2021) also comprehensively examine the effect of transaction cost at stock level when common risk factors are under management.

turn, and the five-year window for conditional intercept. We next perform robustness checks on these design choices.

The literature specifies three main choices to estimate realized variance: Moreira and Muir (2017) and Cederburg et al. (2020) adopt one-month window, Barroso and Santa-Clara (2015) and Daniel and Moskowitz (2016) use six-month window, while Moskowitz et al. (2012) and Eisdorfer and Misirli (2020) choose 12-month window. Although we take one-month window as default, Table 9 shows that the advantage of the improved strategy is robust to different choices of variance estimation window. There are 163 abnormal return increases with the six-month window and 161 increases with the 12-month window, both significant at the 1% level. Besides, the numbers of positive direct abnormal returns are 159 and 154 respectively, with the same statistical significance. For the anomaly categories, the results are also highly similar across different window specifications.

Although the improved strategy does not require precise estimation for conditional expected return and conditional intercept,[8] we consider two alternative estimation windows for them respectively. In terms of conditional expected return, Daniel and Moskowitz (2016) adopt a two-year window to determine factor state, and we add a five-year window in case of more persistent effects. Therefore, the conditional expected return is alternatively estimated over the prior two-year and five-year windows. The improved strategy renders 170 abnormal return increases with the two-year window and 161 increases with the five-year window, both at a significance level of 1%. The results remain similar across anomaly categories.

Regarding conditional risk return tradeoff, we consider a 10-year window and a 20-year window in addition to the 5-year window used in the main analysis. The improved strategy keeps its advantage. There are 165 abnormal return increases with the 10-year window and 118 increases with the 20-year window, both at a significance level of 1%. The numbers of positive direct abnormal returns are 165 and 150 respectively, at the same significance level. Still, the 10 risk factors are better managed with the improved strategy, regardless

[8]Sign dependence, as Christoffersen and Diebold (2006) highlight, is not inconsistent with unpredictable conditional mean. This reaffirms a mitigated concern of noisy estimates to our main design choices.

of estimation windows. Thus, we have a support that the edge of the improved strategy is robust to different design choices.

4 Conclusion

Recent studies argue that implementing Moreira and Muir (2017)'s volatility management strategy involves a look-ahead bias. Once the bias is corrected, volatility management does not systematically improve real-time performance, and the impressive in-sample benefits do not readily translate into out-of-sample gains.

We find the real-time underperformance largely due to the comovement between realized variances and variance-equating coefficients. Correspondingly, we propose a simple improved strategy to mitigate the comovement concern. Besides, the improved strategy uses two dummy variables to opt for factor investing and volatility management. The return dummy is responsible for turning off factor investing when conditional expected return is negative. The intercept dummy is responsible for exiting volatility management when the intercept from conditional risk return tradeoff is negative.

For a comprehensive sample of 197 factor portfolios and trading anomalies, our improved strategy significantly enhances Sharpe ratio, abnormal return, Sortino ratio, and skewness. The improved strategy has the edge on the original factor, the plain volatility management strategy, and alternative risk management strategies. Additionally, its benefits are robust to leverage constraints and transaction costs.

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Figures

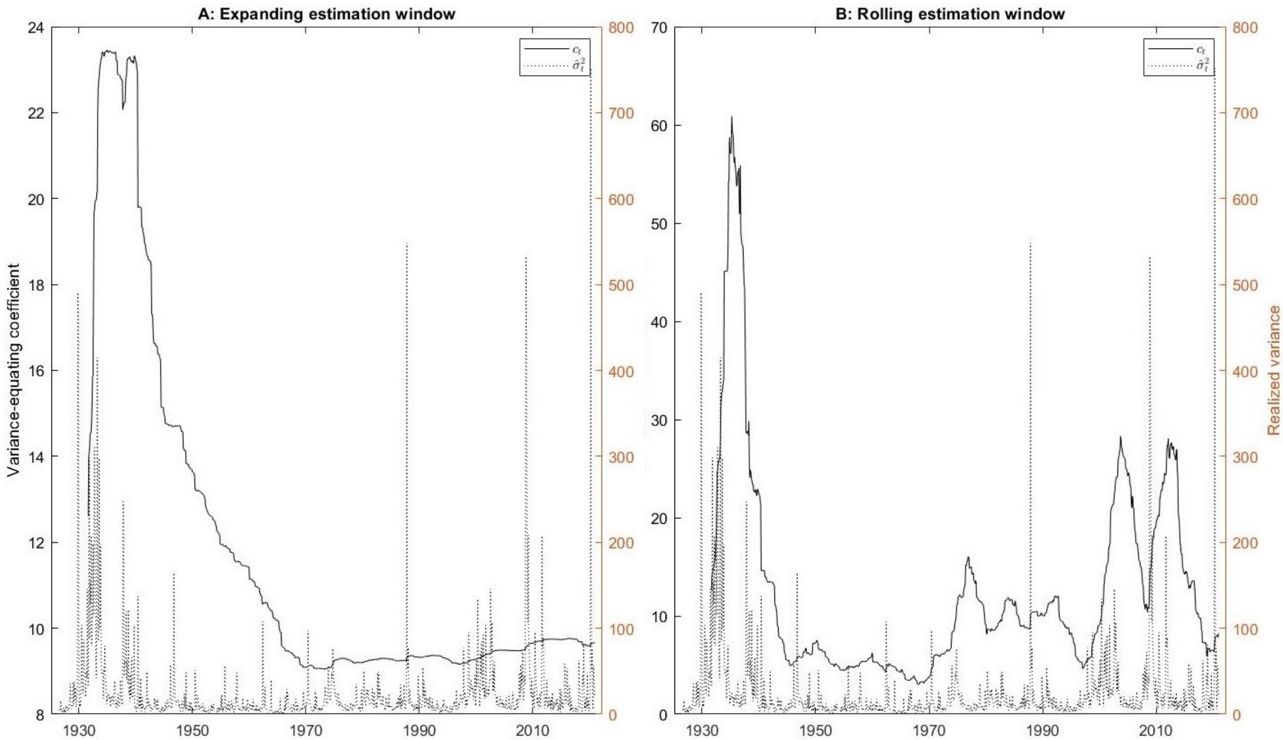

Figure 1: Comovement concern of the variance-equating scheme

Description: This figure shows the time series of realized variance (by dotted line) and variance-equating coefficient (by solid line) for the CRSP US total market index from July 1926 to December 2020. The calculation of realized variance and variance-equating coefficient follows Equations 1 and 2. The first five years are the initial estimation window for variance-equating coefficients, and the window either expands as in Panel A or rolls over time as in Panel B.

Interpretation: The variance-equating coefficients often positively comove with realized variances.

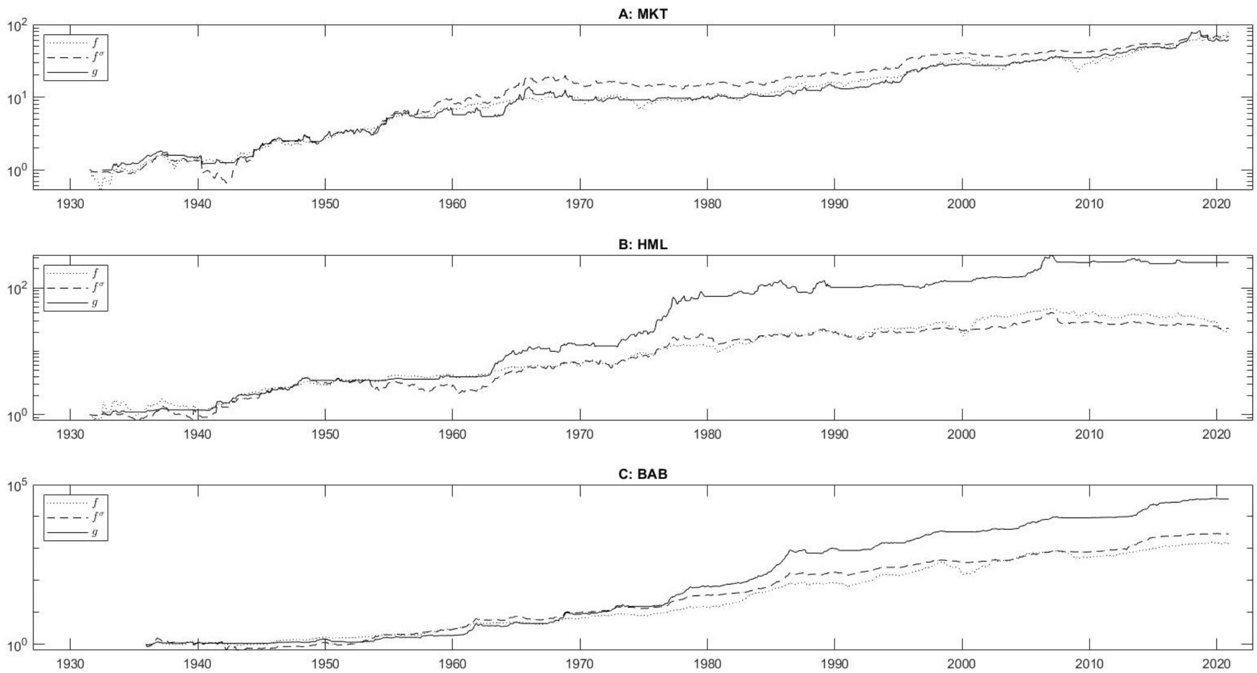

Figure 2: Cumulative performance of volatility management

Description: This figure plots the cumulative returns on three risk factors (f) and their managed versions. The factors are the market (MKT) and value (HML) factors from Fama and French (1993), as well as the betting-against-beta (BAB) factor from Frazzini and Pedersen (2014). The managed versions are the plain volatility management strategy (f^σ, Equation 1) and the improved volatility management strategy (g, Equation 11). The sample period begins from July 1926 for the MKT and HML, and from December 1930 for the BAB. The sample periods end in December 2020.

Interpretation: The improved volatility management strategy outperforms the original factor and the plain strategy.

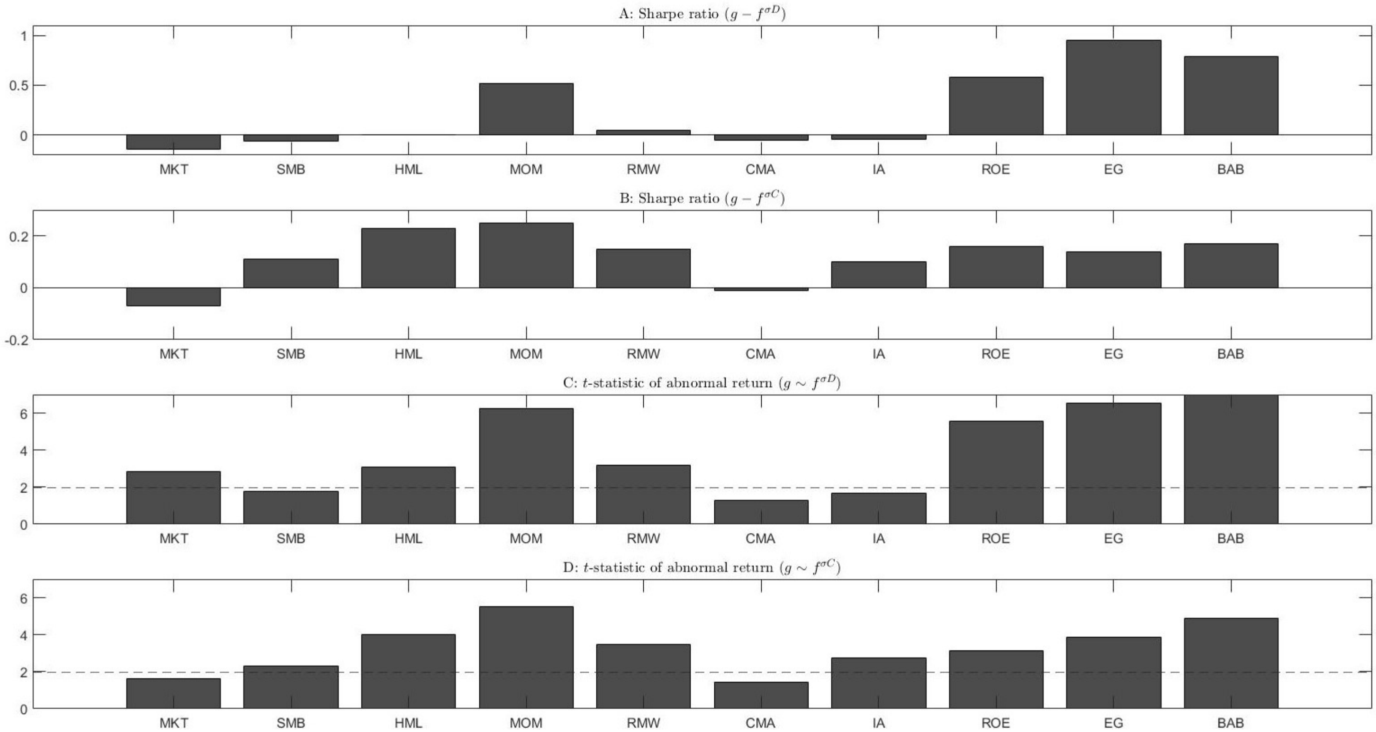

Figure 3: Comparison with alternative risk management

Description: This figure compares the performance between the improved volatility management strategy (g, Equation 11) and two alternative risk management strategies for 10 risk factors. The risk factors are the market (MKT), size (SMB), and value (HML) factors from Fama and French (1993), the momentum (MOM) factor from Carhart (1997), the profitability (RMW) and investment (CMA) factors from Fama and French (2015), the investment (IA) and return on equity (ROE) factors from Hou et al. (2015), the expected growth (EG) factor from Hou et al. (2021), and the betting-against-beta (BAB) factor from Frazzini and Pedersen (2014). The alternative risk management strategies are the downside risk management strategy ($f^{\sigma D}$, Equation 13) and the conditional volatility management strategy ($f^{\sigma C}$, Equation 14). Panels A and B depict the excess Sharpe ratio of the improved volatility management strategy over the alternative risk management strategies. Panels C and D depict the robust Newey and West (1987) T statistics associated with the abnormal return by regressing the improved volatility management strategy on the alternative risk management strategies. The dashed line denotes a cut-off T statistic of 1.96.

Interpretation: The improved volatility management strategy outperforms the downside risk management strategy and the conditional volatility management strategy.

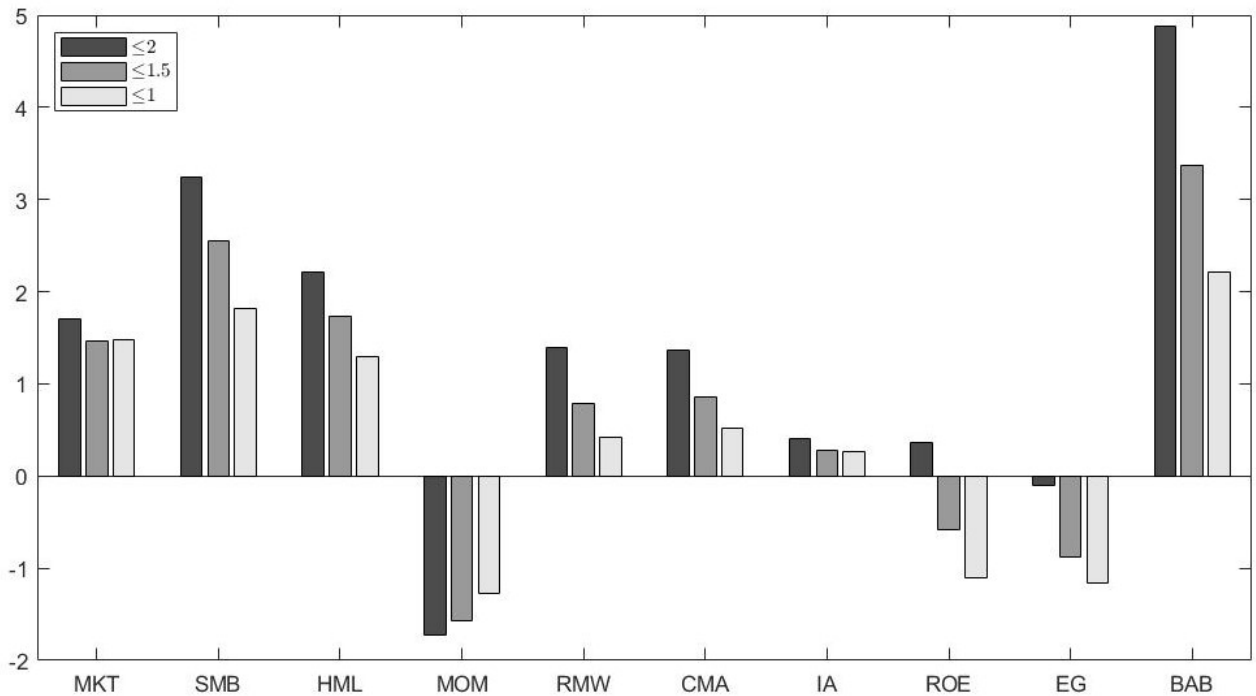

Figure 4: Leverage constraint and volatility management performance

Description: This figure shows the excess abnormal return of the improved volatility management strategy over the plain volatility management strategy for 10 risk factors in the presence of leverage constraints. The risk factors are the market (MKT), size (SMB), and value (HML) factors from Fama and French (1993), the momentum (MOM) factor from Carhart (1997), the profitability (RMW) and investment (CMA) factors from Fama and French (2015), the investment (IA) and return on equity (ROE) factors from Hou et al. (2015), the expected growth (EG) factor from Hou et al. (2021), and the betting-against-beta (BAB) factor from Frazzini and Pedersen (2014). We consider three leverage constraints: when factor exposure is below 2, below 1.5, and below 1.

Interpretation: The performance of the improved volatility management strategy is robust to leverage constraints.

Tables

Table 1: Performance comparison of volatility management strategies

	MKT	SMB	HML	MOM	RMW	CMA	IA	ROE	EG	BAB
Panel A: Performance of original factor (f)										
SR	0.44	0.22	0.32	0.48	0.40	0.45	0.63	0.68	1.42	0.73
Sortino	0.66	0.37	0.56	0.62	0.61	0.74	1.05	1.02	2.77	1.10
Skewness	0.17	1.88	2.09	–2.98	–0.33	0.31	0.15	–0.90	0.10	–0.72
Panel B: Performance difference by the plain strategy (f^{σ} vs. f)										
SR	–0.01	–0.09	0.01	0.29	0.20	–0.10	0.07	0.38	0.14	0.06
	[–0.11]	[–1.00]	[0.11]	[1.77]	[1.50]	[–1.03]	[0.62]	[2.89]	[1.25]	[0.54]
Sortino	–0.02	–0.17	0.00	0.52	0.48	–0.18	0.19	1.30	1.14	0.18
Skewness	–0.88	–0.63	–0.06	1.07	1.10	0.36	0.49	2.45	1.02	0.30
Panel C: Performance difference by the improved strategy (g vs. f)										
SR	–0.02	0.11	0.23	0.49	0.28	–0.06	0.08	0.36	0.19	0.31
	[–0.22]	[1.05]	[2.12]	[3.61]	[1.78]	[–0.45]	[0.61]	[2.23]	[1.62]	[2.60]
Sortino	–0.03	0.20	0.54	1.47	0.87	–0.07	0.35	1.42	1.83	1.39
Skewness	–0.21	–0.74	–0.13	4.84	2.29	0.70	1.35	2.96	1.24	2.36
Panel D: Performance difference between volatility management strategies (g vs. f^{σ})										
SR	–0.01	0.20	0.22	0.20	0.08	0.04	0.01	–0.02	0.04	0.25
	[–0.18]	[1.96]	[2.49]	[1.87]	[0.77]	[0.40]	[0.12]	[–0.24]	[0.65]	[2.25]
Sortino	–0.01	0.37	0.53	0.95	0.39	0.11	0.16	0.12	0.69	1.21
Skewness	0.67	–0.11	–0.07	3.77	1.18	0.34	0.86	0.52	0.22	2.06

Description: This table compares the performance of 10 risk factors (f) with their managed versions following the plain volatility management strategy (f^{σ}, Equation 1) and the improved volatility management strategy (g, Equation 11). The risk factors are the market (MKT), size (SMB), and value (HML) factors from Fama and French (1993), the momentum (MOM) factor from Carhart (1997), the profitability (RMW) and investment (CMA) factors from Fama and French (2015), the investment (IA) and return on equity (ROE) factors from Hou et al. (2015), the expected growth (EG) factor from Hou et al. (2021), and the betting-against-beta (BAB) factor from Frazzini and Pedersen (2014). Performance measures include Sharpe ratio, Sortino ratio, and skewness. Figures in brackets below Sharpe ratio differences are T statistics from Ledoit and Wolf (2008)'s heteroskedasticity and autocorrelation consistent Sharpe ratio test.

Interpretation: The improved volatility management strategy outperforms the plain strategy in direct performance comparison.

Table 2: Abnormal returns of volatility management strategies

	MKT	SMB	HML	MOM	RMW	CMA	IA	ROE	EG	BAB	GRS
Panel A: Spanning regressions											
$f^\sigma \sim f$	4.19	0.31	2.71	12.39	1.77	0.38	1.59	4.64	3.31	5.54	
	[1.80]	[0.24]	[1.74]	[4.55]	[3.03]	[0.63]	[2.29]	[4.91]	[4.44]	[3.42]	
$g \sim f$	4.30	8.05	14.64	31.41	27.78	7.56	14.34	32.83	34.02	26.41	
	[2.19]	[2.27]	[4.05]	[5.86]	[3.86]	[1.45]	[2.87]	[4.91]	[5.36]	[5.74]	
η_α	0.11	7.74	11.93	19.02	26.01	7.18	12.75	28.20	30.71	20.87	
	[0.03]	[2.05]	[3.03]	[3.16]	[3.60]	[1.37]	[2.53]	[4.17]	[4.81]	[4.28]	
$g \sim f^\sigma$	1.36	8.65	12.52	16.63	12.07	6.88	5.74	8.69	7.45	18.68	
	[1.13]	[2.73]	[4.34]	[3.21]	[2.64]	[1.63]	[1.63]	[2.11]	[2.16]	[4.48]	
Panel B: Spanning regressions controlling for Fama and French (1993)'s three factors											
$f^\sigma \sim f$	4.37	0.17	3.07	10.17	1.99	–0.05	1.06	4.54	2.94	5.89	23.96
	[1.86]	[0.13]	[1.86]	[3.75]	[3.21]	[–0.08]	[1.57]	[4.91]	[3.53]	[3.92]	(0.00)
$g \sim f$	4.49	7.45	16.24	25.53	28.09	5.88	11.83	30.18	27.80	24.84	24.87
	[2.25]	[2.10]	[4.39]	[5.39]	[3.78]	[1.06]	[2.46]	[4.48]	[3.97]	[5.46]	(0.00)
η_α	0.13	7.28	13.17	15.36	26.09	5.93	10.77	25.64	24.86	18.94	23.97
	[0.04]	[1.93]	[3.25]	[2.81]	[3.50]	[1.06]	[2.22]	[3.76]	[3.53]	[3.95]	(0.00)
$g \sim f^\sigma$	1.41	7.23	13.22	15.67	10.24	6.64	5.79	7.76	5.31	15.89	3.16
	[1.19]	[2.35]	[4.39]	[2.92]	[2.10]	[1.50]	[1.63]	[1.61]	[1.51]	[3.87]	(0.00)

Description: This table compares the abnormal return from spanning regression of volatility management strategies on 10 risk factors (f). The risk factors are the market (MKT), size (SMB), and value (HML) factors from Fama and French (1993), the momentum (MOM) factor from Carhart (1997), the profitability (RMW) and investment (CMA) factors from Fama and French (2015), the investment (IA) and return on equity (ROE) factors from Hou et al. (2015), the expected growth (EG) factor from Hou et al. (2021), and the betting-against-beta (BAB) factor from Frazzini and Pedersen (2014). The plain volatility management strategy (f^σ) is formulated by Equation 1, and the improved volatility management strategy (g) is formulated by Equation 11. We run three time-series regressions: $f^\sigma \sim f$ represents the abnormal return from the spanning regression of $f_t^\sigma = \alpha + \beta f_t + \epsilon_t$; $g \sim f$ represents the abnormal return from the regression of $g_t = \alpha + \beta f_t + \epsilon_t$; and $g \sim f^\sigma$ represents the abnormal return from the regression of $g_t = \alpha + \beta f_t^\sigma + \epsilon_t$. To estimate the excess abnormal return of g over f^σ, we run $h_t = \alpha + \eta_\alpha I_t + (\beta + \eta_\beta I_t) f_t + \epsilon_t$, where the strategy dummy variable I_t takes the value of one if $h_t = g_t$ and zero if $h_t = f_t^\sigma$. Panel A reports the univariate spanning regression, and Panel B further controls for Fama and French (1993)'s three factors. Figures in brackets below annualized abnormal returns are robust Newey and West (1987) T statistics. The last column in Panel B reports the F-statistic and the corresponding p-value in parenthesis from Gibbons et al. (1989, GRS)'s test.
Interpretation: The improved volatility management strategy outperforms the plain strategy in terms of abnormal performance.

Table 3: Marginal contribution to Sharpe ratio improvements

	MKT	SMB	HML	MOM	RMW	CMA	IA	ROE	EG	BAB
f^σ	–0.01	–0.09	0.01	0.29	0.20	–0.10	0.07	0.38	0.14	0.06
	[–0.11]	[–1.00]	[0.11]	[1.77]	[1.50]	[–1.03]	[0.62]	[2.89]	[1.25]	[0.54]
g	–0.02	0.11	0.23	0.49	0.28	–0.06	0.08	0.36	0.19	0.31
	[–0.22]	[1.05]	[2.12]	[3.61]	[1.78]	[–0.45]	[0.61]	[2.23]	[1.62]	[2.60]
g^C	0.02	–0.16	0.01	0.53	0.22	–0.08	0.08	0.40	0.19	0.29
	[0.21]	[–1.70]	[0.08]	[3.83]	[1.74]	[–0.76]	[0.75]	[3.00]	[1.71]	[2.96]
g^M	–0.06	0.10	0.25	0.22	0.31	–0.08	0.17	0.35	0.18	0.16
	[–0.56]	[1.02]	[2.43]	[1.33]	[2.03]	[–0.64]	[1.39]	[2.43]	[1.58]	[1.25]
g^I	–0.03	–0.01	0.06	0.41	0.19	–0.06	0.00	0.35	0.14	0.18
	[–0.33]	[–0.16]	[0.90]	[2.88]	[1.82]	[–0.66]	[0.04]	[2.62]	[1.25]	[1.77]
g^{CM}	–0.04	0.08	0.26	0.45	0.32	–0.06	0.18	0.37	0.22	0.36
	[–0.36]	[0.79]	[2.29]	[3.19]	[2.12]	[–0.48]	[1.46]	[2.49]	[1.95]	[3.18]
g^{MI}	–0.07	0.11	0.27	0.34	0.23	–0.01	0.14	0.33	0.19	0.23
	[–0.61]	[1.40]	[2.74]	[2.19]	[1.63]	[–0.06]	[1.14]	[2.17]	[1.66]	[1.94]
g^{CI}	0.02	–0.05	0.02	0.55	0.21	–0.10	–0.01	0.38	0.16	0.24
	[0.22]	[–0.51]	[0.19]	[4.25]	[1.53]	[–0.84]	[–0.11]	[2.60]	[1.44]	[2.27]

Description: This table compares the Sharpe ratio improvement of volatility management strategies relative to 10 risk factors (f). The risk factors are the market (MKT), size (SMB), and value (HML) factors from Fama and French (1993), the momentum (MOM) factor from Carhart (1997), the profitability (RMW) and investment (CMA) factors from Fama and French (2015), the investment (IA) and return on equity (ROE) factors from Hou et al. (2015), the expected growth (EG) factor from Hou et al. (2021), and the betting-against-beta (BAB) factor from Frazzini and Pedersen (2014). The plain volatility management strategy (f^σ) is formulated by Equation 1, and the improved volatility management strategy (g) is formulated by Equation 11. We consider six reduced strategies of g: $g_t^C = \frac{c^*}{\hat{\sigma}_{t-1}^2} f_t$, $g_t^M = \delta_{t-1} \cdot \frac{c_{t-1}}{\hat{\sigma}_{t-1}^2} f_t$, $g_t^I = (\frac{c_{t-1}}{\hat{\sigma}_{t-1}^2})^{\gamma_{t-1}} f_t$, $g_t^{CM} = \delta_{t-1} \cdot \frac{c^*}{\hat{\sigma}_{t-1}^2} f_t$, $g_t^{MI} = \delta_{t-1} (\frac{c_{t-1}}{\hat{\sigma}_{t-1}^2})^{\gamma_{t-1}} f_t$, and $g_t^{CI} = (\frac{c^*}{\hat{\sigma}_{t-1}^2})^{\gamma_{t-1}} f_t$. Figures in brackets below Sharpe ratio differences are T statistics from Ledoit and Wolf (2008)'s heteroskedasticity and autocorrelation consistent Sharpe ratio test.

Interpretation: The three modifications of the improved volatility management strategy, both individually and interactively, raise Sharpe ratio.

Table 4: Marginal contribution to abnormal returns

	MKT	SMB	HML	MOM	RMW	CMA	IA	ROE	EG	BAB
Panel A: Spanning regressions										
g	1.36	8.65	12.52	16.63	12.07	6.88	5.74	8.69	7.45	18.68
	[1.13]	[2.73]	[4.34]	[3.21]	[2.64]	[1.63]	[1.63]	[2.11]	[2.16]	[4.48]
g^C	1.20	–3.37	1.32	15.23	2.19	1.71	0.71	1.88	2.97	13.83
	[1.18]	[–1.21]	[0.55]	[4.33]	[1.08]	[1.93]	[0.88]	[1.95]	[3.39]	[4.26]
g^M	–0.10	2.51	4.50	0.02	0.86	0.58	1.04	0.38	0.58	2.43
	[–0.09]	[3.33]	[4.95]	[0.01]	[2.74]	[1.30]	[2.29]	[1.01]	[1.59]	[2.30]
g^I	0.46	1.56	1.98	4.58	0.81	0.71	–0.10	0.27	0.13	2.77
	[0.44]	[1.85]	[1.81]	[1.80]	[1.24]	[1.57]	[–0.36]	[0.79]	[0.80]	[2.55]
g^{CM}	0.82	9.29	13.85	13.21	13.22	6.24	8.55	4.44	7.78	19.12
	[0.71]	[2.64]	[4.81]	[3.55]	[2.94]	[1.54]	[2.40]	[1.43]	[2.25]	[4.76]
g^{MI}	0.12	3.11	5.13	3.58	1.13	1.34	1.15	0.63	0.75	3.92
	[0.10]	[3.33]	[5.09]	[1.45]	[2.13]	[2.35]	[2.37]	[1.28]	[1.91]	[3.19]
g^{CI}	1.97	3.17	4.08	19.24	4.46	3.58	–0.70	6.53	3.52	15.04
	[1.46]	[0.94]	[1.15]	[3.47]	[1.19]	[1.07]	[–0.30]	[1.87]	[2.25]	[3.46]
Panel B: Reverse spanning regressions										
g	2.12	–1.04	0.04	0.82	0.44	0.65	0.79	1.56	0.36	1.04
	[1.18]	[–0.84]	[0.02]	[0.44]	[0.87]	[1.02]	[1.34]	[1.97]	[0.91]	[0.83]
g^C	–0.22	1.17	0.54	–3.53	–0.06	–0.15	–0.03	–0.14	–0.20	–1.83
	[–0.22]	[1.77]	[0.72]	[–1.85]	[–0.37]	[–1.58]	[–0.28]	[–1.17]	[–2.11]	[–1.77]
g^M	2.33	–1.58	–1.62	2.95	0.10	0.54	–0.03	0.80	0.16	0.15
	[1.70]	[–1.70]	[–1.30]	[2.50]	[0.26]	[0.89]	[–0.07]	[1.92]	[0.52]	[0.21]
g^I	1.26	–0.54	0.39	–0.46	0.73	0.10	0.66	0.62	0.18	0.04
	[1.28]	[–0.65]	[0.39]	[–0.37]	[1.82]	[0.28]	[1.98]	[1.41]	[1.37]	[0.05]
g^{CM}	2.20	–1.06	–1.16	–0.32	0.09	0.45	–0.03	0.70	0.02	–1.11
	[1.34]	[–1.03]	[–0.86]	[–0.16]	[0.21]	[0.75]	[–0.06]	[1.61]	[0.06]	[–0.96]
g^{MI}	2.74	–1.16	–0.55	2.20	0.88	0.53	0.44	1.39	0.29	0.47
	[1.68]	[–1.03]	[–0.35]	[1.49]	[1.74]	[0.82]	[0.78]	[2.37]	[0.86]	[0.45]
g^{CI}	0.63	0.10	1.35	–1.33	0.32	0.40	0.84	0.90	0.17	1.05
	[0.49]	[0.09]	[1.08]	[–0.76]	[0.87]	[0.97]	[2.10]	[1.29]	[0.60]	[0.90]

Description: This table compares the abnormal return of the improved volatility management strategy (g, Equation 11) on the plain volatility management strategy (f^σ, Equation 1). We consider six reduced strategies of g: $g_t^C = \frac{c^*}{\hat{\sigma}_{t-1}^2} f_t$, $g_t^M = \delta_{t-1} \cdot \frac{c_{t-1}}{\hat{\sigma}_{t-1}^2} f_t$, $g_t^I = (\frac{c_{t-1}}{\hat{\sigma}_{t-1}^2})^{\gamma_{t-1}} f_t$, $g_t^{CM} = \delta_{t-1} \cdot \frac{c^*}{\hat{\sigma}_{t-1}^2} f_t$, $g_t^{MI} = \delta_{t-1} (\frac{c_{t-1}}{\hat{\sigma}_{t-1}^2})^{\gamma_{t-1}} f_t$, and $g_t^{CI} = (\frac{c^*}{\hat{\sigma}_{t-1}^2})^{\gamma_{t-1}} f_t$. Panel A regresses g and its reduced strategies on f^σ, and Panel B reverses the regression. Figures in brackets below annualized abnormal returns are robust Newey and West (1987) T statistics.
Interpretation: The three modifications of the improved volatility management strategy, both individually and interactively, raise abnormal performance.

Table 5: Extended performance comparison of volatility management strategies

		SR	α	Sortino	Skewness	SR	α	Sortino	Skewness	SR	δ_α	Sortino	Skewness
		f^σ vs. f				g vs. f				g vs. f^σ			
Total	197	110	142	114	107	148	165	153	132	141	161	145	136
		[0.12]	[0.00]	[0.03]	[0.25]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
Factors	10	7	10	7	7	8	10	8	7	8	10	9	8
		[0.34]	[0.00]	[0.34]	[0.34]	[0.11]	[0.00]	[0.11]	[0.34]	[0.11]	[0.00]	[0.02]	[0.11]
Anomaly	187	103	132	107	100	140	155	145	125	133	151	136	128
portfolios		[0.19]	[0.00]	[0.06]	[0.38]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
Momentum	41	33	40	33	35	31	38	34	39	11	32	16	33
		[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.21]	[0.00]
Value-versus-	32	16	25	17	12	29	31	29	20	31	31	31	24
growth		[1.00]	[0.00]	[0.86]	[0.22]	[0.00]	[0.00]	[0.00]	[0.22]	[0.00]	[0.00]	[0.00]	[0.01]
Investment	29	21	11	22	10	27	19	27	16	28	18	28	16
		[0.02]	[0.26]	[0.01]	[0.14]	[0.00]	[0.14]	[0.00]	[0.71]	[0.00]	[0.26]	[0.00]	[0.71]
Profitability	45	23	37	24	26	30	37	32	32	29	33	27	29
		[1.00]	[0.00]	[0.77]	[0.37]	[0.04]	[0.00]	[0.01]	[0.01]	[0.07]	[0.00]	[0.23]	[0.07]
Intangibles	30	5	14	6	12	15	23	15	11	25	29	25	19
		[0.00]	[0.86]	[0.00]	[0.36]	[1.00]	[0.01]	[1.00]	[0.20]	[0.00]	[0.00]	[0.00]	[0.20]
Frictions	10	5	5	5	5	8	7	8	7	9	8	9	7
		[1.00]	[1.00]	[1.00]	[1.00]	[0.11]	[0.34]	[0.11]	[0.34]	[0.02]	[0.11]	[0.02]	[0.34]

Description: This table compares the performance of 197 factor portfolios and trading anomalies (f) with their volatility-managed versions following the plain volatility management strategy (f^σ, Equation 1) and the improved volatility management strategy (g, Equation 11). The 187 trading anomalies, maintained by Hou et al. (2015) and Hou et al. (2021), consist of 6 categories: momentum, value-versus-growth, investment, profitability, intangibles, and frictions. Performance measures include Sharpe ratio, abnormal return, Sortino ratio, and skewness. Figures denote the counts of positive differences of f^σ over f, g over f, or g over f^σ. In brackets below counts are corresponding two-tailed p-values from a binomial distribution test with the null hypothesis of equal likelihood of a positive or negative performance difference.
Interpretation: For the comprehensive set of 197 factor portfolios and trading anomalies, the improved volatility management strategy leads to statistically significant enhancements.

Table 6: Average performance improvement of the extended comparison

	SR	α	Sortino	Skewness	SR	α	Sortino	Skewness	SR	δ_α	Sortino	Skewness
	f^σ vs. f				g vs. f				g vs. f^σ			
Total	0.03	1.27	0.12	0.20	0.14	5.22	0.32	0.60	0.11	3.95	0.21	0.41
Factors	0.09	3.68	0.34	0.52	0.20	20.13	0.80	1.47	0.10	16.45	0.45	0.94
Anomaly	0.03	1.15	0.10	0.18	0.14	4.42	0.30	0.56	0.11	3.28	0.19	0.38
Momentum	0.18	3.89	0.48	1.38	0.12	8.19	0.41	1.84	–0.06	4.31	–0.07	0.47
Value-versus-growth	0.01	0.70	0.02	–0.17	0.23	5.26	0.42	0.33	0.22	4.56	0.39	0.51
Investment	0.04	–0.67	0.06	–0.18	0.27	0.97	0.36	0.08	0.24	1.64	0.30	0.26
Profitability	0.02	1.57	0.09	0.01	0.06	4.33	0.21	0.15	0.03	2.76	0.12	0.14
Intangibles	–0.11	–0.11	–0.21	–0.40	0.02	2.23	0.01	0.00	0.13	2.35	0.22	0.40
Frictions	–0.03	–1.52	–0.03	–0.10	0.25	3.28	0.47	0.84	0.28	4.80	0.50	0.94

Description: This table compares the performance of 197 factor portfolios and trading anomalies (f) with their volatility-managed versions following the plain volatility management strategy (f^σ, Equation 1) and the improved volatility management strategy (g, Equation 11). The 187 trading anomalies, maintained by Hou et al. (2015) and Hou et al. (2021), consist of 6 categories: momentum, value-versus-growth, investment, profitability, intangibles, and frictions. Performance measures include Sharpe ratio, abnormal return, Sortino ratio, and skewness. Figures denote the average performance improvement of f^σ over f, g over f, or g over f^σ.

Interpretation: For the comprehensive set of 197 factor portfolios and trading anomalies, the improved volatility management strategy leads to economically significant enhancements.

Table 7: Leverage constraint and performance comparison

		SR	δ_α	α	SR	δ_α	α	SR	δ_α	α
		Exposure below 1			Exposure below 1.5			Exposure below 2		
Total	197	114	124	160	121	128	175	128	131	174
		[0.03]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
Factors	10	7	7	10	8	7	10	8	8	10
		[0.34]	[0.34]	[0.00]	[0.11]	[0.34]	[0.00]	[0.11]	[0.11]	[0.00]
Anomaly	187	107	117	150	113	121	165	120	123	164
portfolios		[0.06]	[0.00]	[0.00]	[0.01]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
Momentum	41	2	3	26	3	5	36	6	8	38
		[0.00]	[0.00]	[0.12]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
Value-versus-	32	30	31	32	31	31	32	30	30	31
growth		[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
Investment	29	28	27	19	28	27	18	28	28	16
		[0.00]	[0.00]	[0.14]	[0.00]	[0.00]	[0.26]	[0.00]	[0.00]	[0.71]
Profitability	45	18	23	36	21	24	40	23	24	41
		[0.23]	[1.00]	[0.00]	[0.77]	[0.77]	[0.00]	[1.00]	[0.77]	[0.00]
Intangibles	30	21	26	30	23	26	30	25	25	30
		[0.04]	[0.00]	[0.00]	[0.01]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
Frictions	10	8	7	7	7	8	9	8	8	8
		[0.11]	[0.34]	[0.34]	[0.34]	[0.11]	[0.02]	[0.11]	[0.11]	[0.11]

Description: This table compares the performance of 197 factor portfolios and trading anomalies (f) with their volatility-managed versions following the plain volatility management strategy (f^σ, Equation 1) and the improved volatility management strategy (g, Equation 11). The 187 trading anomalies, maintained by Hou et al. (2015) and Hou et al. (2021), consist of 6 categories: momentum, value-versus-growth, investment, profitability, intangibles, and frictions. We consider three leverage constraints: the factor exposure has to be below 1, below 1.5, and below 2. Performance comparisons include Sharpe ratio difference, abnormal return difference, and direct abnormal return by regressing the improved strategy on the plain strategy. Figures denote the counts of positive differences of the improved strategy over the plain strategy. In brackets below counts are corresponding two-tailed p-values from a binomial distribution test with the null hypothesis of equal likelihood of a positive or negative performance difference.

Interpretation: For the comprehensive set of 197 factor portfolios and trading anomalies, the performance enhancements by using the improved volatility management strategy are robust to leverage constraints.

Table 8: Trading cost and performance comparison

		SR	δ_α	α	SR	δ_α	α	SR	δ_α	α
		10 bps			14bps			40bps		
Total	197	145	166	159	146	164	159	150	152	149
		[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
Factors	10	8	10	10	8	10	10	8	10	10
		[0.11]	[0.00]	[0.00]	[0.11]	[0.00]	[0.00]	[0.11]	[0.00]	[0.00]
Anomaly portfolios	187	137	156	149	138	154	149	142	142	139
		[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
Momentum	41	13	31	32	14	30	32	16	21	29
		[0.03]	[0.00]	[0.00]	[0.06]	[0.00]	[0.00]	[0.21]	[1.00]	[0.01]
Value-versus-growth	32	31	31	31	31	31	31	31	31	31
		[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
Investment	29	28	26	18	28	26	18	28	26	12
		[0.00]	[0.00]	[0.26]	[0.00]	[0.00]	[0.26]	[0.00]	[0.00]	[0.46]
Profitability	45	29	33	31	29	32	31	29	28	31
		[0.07]	[0.00]	[0.02]	[0.07]	[0.01]	[0.02]	[0.07]	[0.14]	[0.02]
Intangibles	30	27	27	29	27	27	29	28	27	29
		[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
Frictions	10	9	8	8	9	8	8	10	9	7
		[0.02]	[0.11]	[0.11]	[0.02]	[0.11]	[0.11]	[0.00]	[0.02]	[0.34]

Description: This table compares the performance of 197 factor portfolios and trading anomalies (f) with their volatility-managed versions following the plain volatility management strategy (f^σ, Equation 1) and the improved volatility management strategy (g, Equation 11). The 187 trading anomalies, maintained by Hou et al. (2015) and Hou et al. (2021), consist of 6 categories: momentum, value-versus-growth, investment, profitability, intangibles, and frictions. We consider three assumptions of transaction cost: 10 bps, 14bps, and 40 bps. Performance comparisons include Sharpe ratio difference, abnormal return difference, and direct abnormal return by regressing the improved strategy on the plain strategy. Figures denote the counts of positive differences of the improved strategy over the plain strategy. In brackets below counts are corresponding two-tailed p-values from a binomial distribution test with the null hypothesis of equal likelihood of a positive or negative performance difference. **Interpretation:** For the comprehensive set of 197 factor portfolios and trading anomalies, the performance enhancements by using the improved volatility management strategy are robust to transaction costs.

Table 9: Different design choices and performance comparison

		Realized variance				Conditional expected return				Conditional intercept			
		6 months		12 months		2 years		5 years		10 years		20 years	
		δ_α	α	δ_α	α	δ_α	α	δ_α	α	δ_α	α	δ_α	α
Total	197	163	159	161	154	170	166	161	150	165	165	118	150
		[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.01]	[0.00]
Factors	10	10	10	10	10	9	10	9	9	10	10	9	10
		[0.00]	[0.00]	[0.00]	[0.00]	[0.02]	[0.00]	[0.02]	[0.02]	[0.00]	[0.00]	[0.02]	[0.00]
Anomaly	187	153	149	151	144	161	156	152	141	155	155	109	140
portfolios		[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.03]	[0.00]
Momentum	41	30	29	30	23	37	35	37	30	29	35	13	28
		[0.00]	[0.01]	[0.00]	[0.53]	[0.00]	[0.00]	[0.00]	[0.00]	[0.01]	[0.00]	[0.03]	[0.03]
Value-versus-	32	31	32	31	32	30	31	28	28	31	31	18	28
growth		[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.60]	[0.00]
Investment	29	23	20	24	21	26	20	22	15	26	20	27	19
		[0.00]	[0.06]	[0.00]	[0.02]	[0.00]	[0.06]	[0.01]	[1.00]	[0.00]	[0.06]	[0.00]	[0.14]
Profitability	45	34	33	32	34	37	35	35	36	33	33	23	33
		[0.00]	[0.00]	[0.01]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[1.00]	[0.00]
Intangibles	30	28	28	27	27	22	26	21	24	27	28	21	24
		[0.00]	[0.00]	[0.00]	[0.00]	[0.02]	[0.00]	[0.04]	[0.00]	[0.00]	[0.00]	[0.04]	[0.00]
Frictions	10	7	7	7	7	9	9	9	8	9	8	7	8
		[0.34]	[0.34]	[0.34]	[0.34]	[0.02]	[0.02]	[0.02]	[0.11]	[0.02]	[0.11]	[0.34]	[0.11]

Description: This table compares the performance of 197 factor portfolios and trading anomalies (f) with their volatility-managed versions following the plain volatility management strategy (f^σ, Equation 1) and the improved volatility management strategy (g, Equation 11). The 187 trading anomalies, maintained by Hou et al. (2015) and Hou et al. (2021), consist of 6 categories: momentum, value-versus-growth, investment, profitability, intangibles, and frictions. For realized variance, we specify two alternative estimation windows: 6-month window and 12-month window. For conditional expected return, we use two alternative estimation windows: 2-year window and 5-year window. For conditional intercept, we use two alternative estimation windows: 10-year window and 20-year window. Performance comparisons include abnormal return difference and direct abnormal return by regressing the improved strategy on the plain strategy. Figures denote the counts of positive differences of the improved strategy over the plain strategy. In brackets below counts are corresponding two-tailed p-values from a binomial distribution test with the null hypothesis of equal likelihood of a positive or negative performance difference.
Interpretation: For the comprehensive set of 197 factor portfolios and trading anomalies, the performance enhancements by using the improved volatility management strategy are robust to different design choices.