The Cross-Section of Volatility and Expected Returns: Then and Now*

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Abstract

We successfully replicate the main results of Ang, Hodrick, Xing, and Zhang (2006): Aggregate-volatility risk and idiosyncratic volatility (IV) are each priced in the cross-section of stock returns from 1963 to 2000. We also examine the pricing of volatility outside the original time period and under more recent asset-pricing models. With the exception of NASDAQ stocks, aggregate-volatility risk continues to be priced in the years following the Ang et al. (2006) sample period, and none of the more recent asset-pricing models we consider consistently accounts for the pricing of aggregate-volatility risk. The difference in abnormal returns between stocks with high and low IV decreases but remains significant out of sample. More recent asset-pricing models do not resolve the IV anomaly for the Ang et al. (2006) sample, but the four-factor model of Stambaugh and Yuan (2017) and the six-factor model of Barillas and Shanken (2018) resolve the anomaly out of sample and over the extended period of 1967 to 2016. Finally, both models eliminate the arbitrage asymmetry that Stambaugh, Yu, and Yuan (2015) propose as an explanation of the IV anomaly.

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Volatility plays a central role in asset pricing. In an influential paper, Ang, Hodrick, Xing, and Zhang (2006) examine the relationship between aggregate and idiosyncratic volatility and the cross-section of expected returns. They find that stocks with large positive exposure to changes in aggregate volatility have significantly lower abnormal returns than stocks with low or negative exposure. This finding is consistent with predictions by finance theory since aggregate volatility adversely affects investment opportunities and should therefore have a negative risk premium (e.g., Merton, 1973; Campbell, 1993, 1996; Chen, 2002). However, Ang et al. (2006) also find that stocks with high idiosyncratic volatility (IV) relative to the Fama and French (1993) three-factor model (FF3) have significantly lower returns than stocks with low IV, a finding that contrasts with finance theory, which predicts no or a positive relationship (e.g., Merton, 1987). Recent studies refer to this finding as the "idiosyncratic volatility puzzle."[1] In this paper, we attempt to replicate the findings of Ang et al. (2006), extend their analysis to other time periods as well as more recent asset-pricing models, and address competing explanations of their findings.

In the first part of the paper, we examine the pricing of aggregate-volatility risk, measured by exposure of returns to daily changes in the CBOE Volatility Index (VIX), in the cross-section of stock returns. We successfully replicate the results of Ang et al. (2006) on the pricing of volatility risk over their original sample period of 1986 to 2000. That is, controlling for the CAPM and FF3 factors, we find significantly lower abnormal returns for stocks in the highest quintile of VIX exposures than stocks in the lowest quintile. This finding is consistent with aggregate volatility being a negatively priced risk factor that is not captured by the CAPM or FF3. We find similar results for the extended time period of 1986 to 2016. But, in the post–Ang et al. (2006) period of 2001 to 2016, the estimated spread in CAPM and FF3 abnormal returns between stocks with high and low VIX betas attenuates and becomes statistically insignificant. However, recent studies posit that nonsynchronous price movements and illiquidity can cause estimation error when using daily data to estimate exposures of returns to systematic factors such as the market return and aggregate volatility. One way to mitigate this error is to exclude NASDAQ stocks (e.g., Lewellen and Nagel, 2006; Barras and Malkhozov, 2016). When we exclude NASDAQ stocks, we again find a significant spread in CAPM and FF3 abnormal returns between stocks with high and low VIX exposures

[1] See, e.g., Hou and Loh (2016) for a recent review of papers that attempt to explain this puzzle.

over the post–Ang et al. (2006) period. Moreover, this spread is not explained by the more recent asset-pricing models of Fama and French (2015, 2018), Hou, Xue, and Zhang (2015), Stambaugh and Yuan (2017), or Barillas and Shanken (2018). In summary, our results are consistent with aggregate volatility being a negatively priced risk factor that is not captured by recent models.

The second part of the paper examines the idiosyncratic volatility puzzle. We are again able to replicate the results of Ang et al. (2006) over their original sample period of 1963 to 2000. That is, the FF3 abnormal return of a "high-minus-low IV" portfolio, which is long stocks in the highest IV quintile and short stocks in the lowest IV quintile, is significantly negative and within a few basis points of the corresponding estimate reported by Ang et al. (2006). Moreover, FF3 abnormal returns remain significantly negative for almost two years following portfolio formation. Furthermore, the IV puzzle exists in both the pre-Ang et al. (2006) period of 1926 to 1963 and the post-Ang et al. (2006) period of 2001 to 2016. However, the abnormal returns of the high-minus-low IV portfolio are lower by approximately half in each of these two periods relative to those in the original sample period. Importantly, the IV puzzle holds when we consider only medium- and large-cap stocks, thus confirming existing research that has ruled out microstructure-related explanations of the puzzle (e.g., Chen, Jiang, Xu, and Yao, 2012; Stambaugh, Yu, and Yuan, 2015), but also countering the size-related explanation proposed by Hou, Xue, and Zhang (2018). Like Hou et al. (2018), we find that among larger stocks, unadjusted average returns of a high-minus-low IV portfolio are not significantly different from zero. However, corresponding FF3 abnormal returns, which Hou et al. (2018) do not consider, remain statistically significant in all time periods, even for stocks in the largest size quintile.

We also investigate two explanations of the IV puzzle. First, Ang, Hodrick, Xing, and Zhang (2009) and Chen and Petkova (2012) propose that the original IV puzzle is due to a risk factor that is missing from the FF3. To test this hypothesis, we evaluate whether the more recent asset-pricing models of Fama and French (2015), Hou et al. (2015), Stambaugh and Yuan (2017), and Barillas and Shanken (2018) can price portfolios formed on IV relative to the FF3. Consistent with the missing risk factor hypothesis, we find that the four-factor model of Stambaugh and Yuan (2017) (SY4) and the six-factor model of Barillas and Shanken (2018) (BS6) explain the IV puzzle over the extended time period of 1967 to 2016. Over the full sample, based on the estimated loadings and risk premia, we find the profitability factor (ROE) from the Hou et al. (2015) and Barillas and Shanken (2018) models explains most (-0.63% per month) of the -0.87% average return on the high-minus-low IV portfolio. Further, exposures to size, value, investment, and profitability factors model suggest that high-IV stocks earn low returns because they behave like small-cap growth stocks with low profitability and high levels of real investment. Moreover, the BS6 does not give rise to a new IV puzzle; portfolios sorted on IV with respect to the BS6 do not earn BS6 abnormal returns. However, the Ang et al. (2006) sample demonstrates an unusually strong IV effect relative to both the pre-1963 and post-2000 periods. For the sub-period that overlaps with the Ang et al. (2006) sample of 1963 to 2000, the recent models attenuate the IV puzzle by approximately half, but none of them resolves the IV puzzle.

Second, we examine the alternative explanation of the IV puzzle proposed by Stambaugh et al. (2015), which is based on two common asset-pricing assumptions. First, IV represents arbitrage risk that prevents traders from eliminating mispricing because they must trade off the benefit of earning a specific asset's abnormal return against the cost of bearing that asset's IV (e.g., Pontiff, 2006). Second, short-sale constraints create an "arbitrage asymmetry" in which traders devote less capital to exploiting overpricing than underpricing. As a result, the effect of IV should be greater on overpricing than underpricing, yielding a negative abnormal return on a high-minus-low IV portfolio. Consistent with the arbitrage-risk principle, Stambaugh et al. (2015) show that abnormal returns of underpriced stocks increase with IV, while abnormal returns of overpriced stocks decrease, i.e., become more negative, as IV increases. Consistent with arbitrage asymmetry, Stambaugh et al. (2015) further document that the high-minus-low IV spread in abnormal returns is larger in absolute value among overpriced stocks than underpriced stocks. As a result, aggregating across over- and underpriced stocks yields the IV puzzle. Using FF3 as the asset-pricing model, we are able to replicate the arbitrage asymmetry of Stambaugh et al. (2015) for the period of 1967 to 2016. However, when using the SY4 or the BS6, the arbitrage asymmetry disappears. That is, using the more recent models, IV does not seem to exacerbate overpricing more than underpricing, and, as a result, there is no unconditional cross-sectional correlation between IV with respect to these models and abnormal returns.

In summary, we are able to replicate the original results of Ang et al. (2006) with respect to aggregate as well as idiosyncratic volatility. We find that results outside the original sample period are somewhat weaker, but overall consistent with the original findings of Ang et al. (2006). While the adoption of more recent asset-pricing models does not fundamentally alter conclusions with respect to aggregate volatility, we find that, at least for the extended time period of 1967 to 2016, the four-factor model of Stambaugh and Yuan (2017) and the six-factor model of Barillas and Shanken (2018) resolve the IV puzzle of Ang et al. (2006) and eliminate the arbitrage asymmetry of Stambaugh et al. (2015).

1. Data

The main results of Ang et al. (2006) are based on data sources that are still widely available and updated. We use the same sources for the replication and extension of these results. Specifically, monthly and daily individual stock data come from CRSP, accounting data come from COMPUSTAT, and analysts' earnings forecast data come from IBES.[2] Data on the CBOE implied volatility index, VIX, come from the CBOE via WRDS.[3] Finally, we obtain the time series of liquidity innovations of the U.S. stock market from the website of Ľuboš Pástor.[4]

The original results of Ang et al. (2006) related to aggregate volatility use data between January 1986 and December 2000, while their results related to IV use data from June 1963 to December 2000. In addition, we collect data for the period following the end of the Ang et al. (2006) sample period, that is, between January 2001 and December 2016. For some of our IV results, we also collect data for the time period preceding the Ang et al. (2006) sample, July 1926 to June 1963.

Ang et al. (2006) use the Fama and French (1993) three-factor model to construct IV and estimate abnormal returns. However, since the publication of Ang et al. (2006), multiple studies propose new empirical asset-pricing models that outperform the FF3. In this study, we also employ the following five more "recent" models:

[2] We adjust CRSP stock returns for delisting bias following Shumway (1997), which is now common practice, although Ang et al. (2006) do not specify how they handle delisting returns. Untabulated results show that this adjustment is inconsequential for our results.

[3] To be clear, Ang et al. (2006) use the SP100-based implied volatility index, which has a ticker of VXO, for all tests reported in this paper. In 1990, the CBOE introduced a new VIX index, which has a ticker VIX, that is based on SP500 stock options. Untabulated results show inferences are virtually identical using either index.

[4] http://faculty.chicagobooth.edu/lubos.pastor/research/.

- FF5: The five-factor model of Fama and French (2015) and Fama and French (2016). FF5 includes the market (MKT) and value (HML) factors of FF3, as well as factors based on size ($SMB5$), robust-minus-weak profitability (RMW), and conservative (low)-minus-aggressive (high) real investment (CMA).
- FF6: The six-factor model of Fama and French (2018) constructed as FF5 augmented by MOM, the momentum factor of Carhart (1997).
- HXZ4: The four-factor q-model of Hou et al. (2015). HXZ4 adds three factors to the market factor: size (ME), investment (IA) and profitability (ROE).
- SY4: The four-factor mispricing model of Stambaugh and Yuan (2017). SY4 includes the market (MKT) and size ($SMBSY$) factors and two "mispricing" factors: $MGMT$, which is based on six anomaly variables that represent quantities that firms' managements can affect directly, and $PERF$, which is based on five anomaly variables that are related more to performance and less directly controlled by management.
- BS6: The six-factor model of Barillas and Shanken (2018). Barillas and Shanken (2018) use a Bayesian technique to compare the factors of FF6, HXZ4, and $HML(m)$, which is the monthly-updated value factor of Asness and Frazzini (2013). They conclude that the dominant model includes six factors: MKT, $SMB5$, IA, ROE, MOM, and $HML(m)$.

Table 1 provides the means, standard deviations, and Newey and West (1987) t-statistics for the means of the returns on each factor over 1967 to 2016 (Panel A), 1967 to 2000 (Panel B), and 2001 to 2016 (Panel C). The sample periods start only in 1967 because HXZ4 and BS6 data start in January 1967. Panel A shows that, with the exception of SMB and SMB5, all of the spread factors have statistically significant average returns over the extended period of 1967 to 2016. Panel B shows the subsample that overlaps with the AHXZ sample (1967 to 2000), and Panel C shows the post–Ang et al. (2006) subsample (2001 to 2016).

2. Aggregate volatility

Ang et al. (2006) start their analysis of the relation between volatility and expected returns by studying the pricing of aggregate-volatility risk in the cross-section of stock returns. The intertemporal capital asset-pricing model predicts that aggregate volatility should be a negatively priced risk factor because it adversely affects investment opportunities (e.g., Merton, 1973; Campbell 1993, 1996; Chen, 2002; Campbell, Giglio, Polk, and Turley, 2018). Furthermore, a long literature estimates negative volatility and variance risk premia in options and other variance-based derivatives, notably variance swaps (e.g., Coval and Shumway, 2001; Bollerslev, Tauchen, and Zhou, 2009; Carr and Wu, 2009).

Ang et al. (2006) define the volatility risk premium in the context of a general expected return-beta model

$$\begin{aligned} r^e_{it+1} =& E_t(r^e_{it+1}) + \beta_{mkt,it}(r^e_{mkt,t+t} - E_t(r^e_{mkt,t})) + \beta_{v,it}(v_{t+1} - E_t(v_{t+1})) \\ &+ \sum_{k=1}^{K} \beta_{k,it}(f_{kt+1} - E_t(f_{kt+1})) + \epsilon_{it+1} \end{aligned} \tag{1}$$

$$E_t(r^e_{it+1}) = \beta_{mkt,it}\lambda_{mkt,t} + \beta_{v,it}\lambda_{v,t} + \sum_{k=1}^{K} \beta_{k,it}\lambda_{k,t}. \tag{2}$$

r^e_{it} denotes the excess return on asset i, $r^e_{mkt,t}$ denotes the market excess return, v_t denotes the volatility of the market return, and f_{kt} denotes an arbitrary risk factor. The corresponding λ denote the factor risk premia. Standard asset pricing results (e.g. Cochrane, 2005) imply that $\lambda_{v,t}$ may also be written as

$$\lambda_{v,t} = -R_{ft} cov_t(m_{t+1}, v_{t+1}) = E_t(v_{t+1}) - E^Q_t(v_{t+1}), \tag{3}$$

where $E_t(v_{t+1})$ and $E^Q_t(v_{t+1})$ are the expected value of aggregate volatility under the physical and risk-neutral probability measures, respectively; m_{t+1} is the stochastic discount factor from time t to $t+1$; and $R_{ft} = E^{-1}_t(m_{t+1})$ is the gross risk-free rate from time t to $t+1$.

Ang et al. (2006) estimate betas with respect to innovations in volatility and then infer the risk premium from spreads in returns generated by spreads in betas as predicted by Eq. (2). Specifically, Ang et al. (2006) measure volatility innovations as daily changes in VIX throughout their analysis. Their first step is to estimate, in each month m, the exposure of each stock i to aggregate volatility with the regression

$$r^e_{i,m,t} = \beta_0 + \beta_{MKT,i,m} MKT_t + \beta_{\Delta VIX,i,m} \Delta VIX_t + \epsilon_{it}, \tag{4}$$

where $r^e_{i,m,t}$ is the return of stock i on day t in excess of the risk-free rate, MKT_t is the market excess return, and ΔVIX_t is the change in the VIX index between day t and $t-1$. The factor loadings, $\beta_{MKT,i,m}$ and $\beta_{\Delta VIX,i,m}$, measure the exposures of stock i's return to the market and ΔVIX in month m, respectively. To capture the time variation in the factor loadings, Ang et al. (2006) run the regression above for every stock-month in their sample with more than 17 daily observations. At the end of each month, Ang et al. (2006) sort stocks into quintiles, based on the $\beta_{\Delta VIX,i,m}$ estimated over the past month. Firms in quintile 1 have the lowest $\beta_{\Delta VIX,i,m}$, while firms in quintile 5 have the highest loadings. Within each quintile portfolio, Ang et al. (2006) value weight the stocks to form one series of post-ranking portfolio returns.

Bollerslev et al. (2009) and the recent variance risk premium literature use a different approach from Ang et al. (2006) to estimate the variance risk premium. They directly infer the risk-neutral expected variance ex-ante using derivatives prices and define the variance risk premium as $E^Q_t(rv_{t+1}) - E_t(rv_{t+1})$, which equals $-\lambda_{rv,t}$ by a similar logic as in Eq. (3), where rv_{t+1} denotes the realized variance of the market return. While the two approaches are similar, they differ in three notable ways. First, the variance risk premium literature focuses on market variance instead of volatility, although the corresponding risk premia are closely related in theory.[5] Second, Ang et al. (2006) proxy for changes in market volatility, $\Delta E_t(v_{t+1})$, using changes in the VIX index, ΔVIX, while the variance risk premium literature uses VIX^2 to measure $E^Q_t(rv_{t+1})$. More generally, VIX is derived from option prices and hence ΔVIX proxies for $\Delta E^Q_t(v_{t+1})$ instead of $\Delta E_t(v_{t+1})$. Thus, a limitation of the Ang et al. (2006) approach is that it is unclear whether daily innovations in VIX represent changes in expected market volatility, $E_t(v_{t+1})$, or innovations in the risk premium, $\lambda_{v,t}$. However, under the assumption that the volatility risk premium is constant, changes in VIX are driven by changes in volatility and the Ang et al. (2006) approach is consistent with that of the variance risk premium literature. Another empirical difference between Ang et al.

[5] To see this, we can follow the same logic as in Eq. (3) to write $\lambda_{rv,t} = -cov_t(R_{ft} m_{t+1}, rv_{t+1})$. Then, a simple first-order Taylor expansion of $rv_{t+1} = v^2_{t+1}$ about $E_t(v_{t+1})$ yields $\lambda_{rv,t} \approx 2 \cdot E_t(v_{t+1}) \cdot \lambda_{v,t}$.

(2006) and the variance risk premium literature comes from the fact that the former estimates a volatility risk premium in equities while the latter focuses on derivatives markets. These markets may be segmented and have different volatility risk premia (e.g., Barras and Malkhozov, 2016).

Panel A of Table 2 replicates the results of Ang et al. (2006). All the results in this panel are qualitatively similar to those in Ang et al. (2006). The column "Mean" reports the time-series mean of monthly returns of each quintile portfolio. The difference in mean returns is significant at the 1% level and only three basis points different from the corresponding estimates in Ang et al. (2006) (-1.01 vs. -1.04 in Ang et al., 2006, with corresponding t-statistics of -3.79 vs. -3.90). The columns "CAPM Alpha" and "FF3 Alpha" report the time-series alphas, i.e., average abnormal returns, of each quintile portfolio relative to the CAPM and to the FF3 model, respectively. As in Ang et al. (2006), the difference in the alphas of portfolios 5 and 1 (row "$5-1$") is statistically significant at the 1% level, which is consistent with aggregate volatility being a priced risk factor that is not captured by the CAPM or FF3. Moreover, our alpha estimates are only three basis points different from those reported in Ang et al. (2006) (CAPM: -1.12 vs. -1.15 in Ang et al., 2006 with t-statistics of -3.46 vs. -3.54; FF3: -0.80 vs. -0.83 with t-statistics of -2.93 vs. -2.76). The column "Pre-formation $\beta_{\Delta VIX}$" shows the time-series mean of the monthly value-weighted averages of the $\beta_{\Delta VIX}$ coefficients estimated via Eq. (4), while the column "Next month post-formation $\beta_{\Delta VIX}$" shows the time-series mean of the monthly value-weighted average $\beta_{\Delta VIX}$ over the months following the portfolio formation. Both the pre-formation and the post-formation $\beta_{\Delta VIX}$ increase monotonically from portfolio 1 to 5. Overall, the evidence in Panel A indicates that stocks that strongly and positively covary with changes in VIX have a negative alpha with respect to the CAPM and FF3, which suggests that the price of volatility risk is negative.

Following Ang et al. (2006), we also report post-ranking exposures to a portfolio, $FVIX$, that mimics the daily changes in VIX. Specifically, denoting the excess returns over day t on the five quintile portfolios in Table 2 by

$$X_t = (x_{1t}, x_{2t}, x_{3t}, x_{4t}, x_{5t})', \tag{5}$$

we construct $FVIX_t$ as $\hat{b}'X_t$, where $\hat{b}$ is estimated each month using daily data via

$$\Delta VIX_t = a + b'X_t + \epsilon_t. \tag{6}$$

The final column in Panel A of Table 2, "Full sample post-formation β_{FVIX}," reports post-ranking loadings on $FVIX$ for each portfolio estimated from the four-factor model that consists of the FF3 along with $FVIX$. Ang et al. (2006) argue that β_{FVIX} is a more direct proxy for unconditional exposure to aggregate-volatility risk than are post-ranking betas with respect to changes in VIX measured at the monthly frequency.

Panel B of Table 2 repeats the analysis in Panel A using the post-Ang et al. (2006) period of 2001 to 2016, while Panel C displays the results using the entire sample period during which VIX is available, 1986 to 2016. The differences between the CAPM and FF3 alphas of portfolio 5 and 1 are statistically significant at the 1% level in Panel C, while they are not statistically significant in Panel B at the usual significance levels. Even though the difference in FF3 alphas between portfolios 5 and 1 in Panel B (−38 bps) is about half of that in Panel A (−80 bps), an untabulated Wald test reveals that these two figures are not statistically different across panels (p=0.279). On the other hand, the differences between the mean returns and CAPM alphas of portfolios 5 and 1 in Panel B are significantly less than the corresponding differences in Panel A (p=0.026 and 0.041, respectively).

At first glance, the results in Panel B suggest that the volatility risk premium vanishes in the most recent sample period. However, it is not completely clear that this is the case for at least two reasons. First, the 2001 to 2016 period is only 16 years, which is relatively short for asset-pricing tests, and the whole-sample results in Panel C demonstrate evidence of a risk premium that is significant at the 1% level. Second, prior studies argue that factor loadings estimated with daily data, such as β_{MKT} and $\beta_{\Delta VIX}$, contain noise due to nonsynchronous trading and illiquidity (e.g., Lewellen and Nagel, 2006; Barras and Malkhozov, 2016). This noise would attenuate the post-ranking spread in alphas between portfolios 1 and 5, even if the volatility risk premium is not zero. These studies further argue this noise can be reduced by excluding NASDAQ stocks.[6] Hence, Table

[6]Barras and Malkhozov (2016) explicitly exclude NASDAQ stocks when estimating exposures to SP500 volatility shocks.

3 repeats the analysis from Table 2 after removing NASDAQ stocks from our sample. The results in Panel A of Table 3 are qualitatively similar to those in Panel A of Table 2 and are consistent with aggregate volatility being a negatively priced risk factor over the Ang et al. (2006) sample period of 1986 to 2000.[7] In contrast, the results in Panel B of Table 3 indicate that removing NASDAQ stocks changes the conclusion that aggregate-volatility risk is not priced over the 2001 to 2016 period. Specifically, the $5-1$ CAPM and FF3 alphas in Panel B of Table 3 are statistically significant at the 1% level. Moreover, these alphas are relatively large and are not significantly different from those in Panel A using untabulated Wald tests (p=0.846 and 0.367, respectively). Similar to Panel C of Table 2, Panel C of Table 3 shows that, over the extended period of 1986 to 2016, the differences in average returns along with CAPM and FF3 alphas between portfolios 1 and 5 is significant at the 1% level after excluding NASDAQ stocks.

Overall, the evidence in Tables 2 and 3 is consistent with aggregate-volatility risk being a priced risk factor that is not captured by the FF3 model. A natural question is therefore whether the more recent asset-pricing models capture this risk. To address this question, we regress the $5-1$ portfolio returns from Tables 2 and 3 on the factors of the five recent asset-pricing models listed in Section 1. The results are reported in Table 4, along with FF3 alphas for comparison. Panel A of Table 4 shows that during the Ang et al. (2006) sample period, when including all stocks, the recent-model alphas are all economically large, ranging from -74 to -38 bps. However, only the FF3 and FF5 alphas are significant at the 5% level. Panel B shows that, when using all stocks, the alphas of all models decline in the 2001 to 2016 period to -29 to -12 bps and become insignificant. Over the whole 1986 to 2016 period, when using all stocks, the recent-model alphas range from -57 to -41 bps, which is not small, but they are mostly only marginally significant with t-statistics ranging from -2.03 to -1.64. In contrast, Panels A and B of Table 4 show that excluding NASDAQ stocks lowers the recent-model $5-1$ alphas during the 1986 to 2000 period and renders them all insignificant, but increases them during the 2001 to 2016 period. Moreover, in the latter period, these alphas are all large, ranging from -84 to -68 bps, and significant at the 5% level. Panel

[7] The difference in average returns and alphas between portfolios 1 and 5 are larger in absolute value in Panel A of Table 2 than those in Panel A of Table 3. However, it is important to note that spreads in alphas frequently decrease with size (e.g., Fama and French, 1993) for reasons not necessarily related to risk, and NASDAQ stocks have a smaller market capitalization on average than the NYSE and AMEX stocks. Thus, the absolute values of alphas are not necessarily directly comparable between Tables 2 and 3.

C shows that, over the whole sample, the non-NASDAQ $5-1$ alphas are all large, ranging from -69 to -55 bps, and significant at the 5% level. Assuming that the estimation of $\beta_{\Delta VIX}$ in Eq. (4) is affected by non-synchronous trading, and excluding NASDAQ stocks mitigates these effects, the results in Table 4 suggest that, like the CAPM and FF3, the recent asset-pricing models we consider do not capture aggregate-volatility risk. In section 3, we extend these results by showing $\beta_{\Delta VIX}$ predicts the cross-section of individual stock returns controlling for other characteristics.

Tables A1, A2, and A3 in the Appendix replicate, respectively, Tables III, IV, and V of Ang et al. (2006). The first two of these tables show the inferences from our Table 2 are unchanged controlling for growth-stock effects, liquidity, and past returns. Table A3 estimates risk premia via cross-sectional regressions for portfolios formed on β_{MKT} and $\beta_{\Delta VIX}$. In Table A3, we successfully replicate the negative volatility risk premium of Ang et al. (2006), but not its statistical significance. We do not interpret the lack of significance as evidence that the volatility risk premium is zero, given the strong evidence in Tables 2 and 3 coupled with the fact that cross-sectional regressions with a small number of portfolios are highly sensitive to portfolio construction (e.g., Lewellen, Nagel, and Shanken, 2010).

3. Idiosyncratic volatility

In this section, we replicate the IV findings of Ang et al. (2006) and extend their analysis to earlier and later time periods. We then introduce several extensions that are related to two prominent explanations of the IV puzzle.

3.1. Replication

We follow Ang et al. (2006) and construct a monthly measure of a stock's idiosyncratic volatility relative to the Fama and French (1993) three-factor model. We do so for the same sample period as Ang et al. (2006), i.e., July 1963 to December 2000, as well as for an earlier period, from July 1926 to June 1963, and for a later period, from January 2001 to December 2016.

Specifically, for every month, m, and every stock, i, we perform separate time-series regressions of daily excess returns, $r^e_{i,m,t}$, on daily returns of the three FF3 factors

$$r^e_{i,m,t} = \alpha_{i,m} + \beta_{MKT,i,m} MKT_t + \beta_{SMB,i,m} SMB_t + \beta_{HML,i,m} HML_t + \varepsilon^{FF3}_{i,m,t}. \tag{7}$$

We measure stock i's idiosyncratic return volatility relative to the FF3 in month m, $IV^{FF3}_{i,m}$, as the standard deviation of the residuals from Eq. (7)

$$IV^{FF3}_{i,m} = \sqrt{\frac{1}{T_{i,m} - 1} \sum_{t=1}^{T_{i,m}} (\varepsilon^{FF3}_{i,m,t})^2}, \tag{8}$$

where $T_{i,m}$ denotes the number of daily observations during the month. At the beginning of every month, we sort all stocks by their idiosyncratic volatility, $IV^{FF3}_{i,m}$, during the previous month into five portfolios from low (1) to high (5) idiosyncratic volatility. We also construct a "high-minus-low IV" $(5 - 1)$ portfolio that invests in the highest IV-quintile portfolio by short-selling the lowest IV-quintile portfolio.

For each portfolio and sample period, Table 5 reports the time-series mean and standard deviation of the total portfolio return, each portfolio's market share, and the average size and book-to-market ratio of the stocks included in each portfolio. Finally, the table reports CAPM and FF3 alphas.

Panel A reports results for the sample period of Ang et al. (2006). The summary statistics as well as the average returns and alphas are very similar to the ones reported in Table VI in Ang et al. (2006). The replication confirms that the FF3 does not price the two highest IV^{FF3} portfolios or the $5 - 1$ portfolio. In particular, the $5 - 1$ portfolio produces an economically and statistically significant FF3 alpha of -137 bps, which is very close to the corresponding estimate of Ang et al. (2006), -131 bps (with a corresponding t-statistics of -6.93 vs. -7.00).[8]

Panel B of Table 5 reports the corresponding results for the post-Ang et al. (2006) period of 2001 to 2016. The average return of the $5 - 1$ portfolio is much smaller than during the original sample period and is no longer statistically significantly different from zero. However, controlling

[8] Ang et al. (2006) also report very similar mean returns (-1.06 vs. our -1.08 with corresponding t-statistics of -3.18 vs. -3.17) and CAPM alphas (-1.38 vs. our -1.40 with corresponding t-statistics of -4.56 vs. -4.58) on the $5 - 1$ portfolio.

for the FF3 factors, we continue to find a significantly negative monthly alpha of −81 bps and the mean returns and alphas are not significantly different from those reported in Panel A using untabulated Wald tests (p=0.163 for the mean return, and p=0.143 and 0.118 for the CAPM and FF3 alphas, respectively).

Finally, in Panel C, we repeat the analysis for the 1926 to 1963 period, using about 37 years of data before the original sample period. While we fail to find a significantly negative average return for the $5-1$ portfolio, we again find that it has a significantly negative FF3 alpha of −74 bps.

In sum, we successfully replicate the original result of Ang et al. (2006). We also find significantly negative, even though in absolute magnitude smaller, FF3 alphas for the $5-1$ portfolio before and after the original sample period.

Ang et al. (2006) provide several robustness tests for their IV results, which control for a large number of cross-sectional characteristics. In Table 6, we report replications of those robustness checks related to firm size. Controlling for size appears particularly important, given a recent study by Hou et al. (2018), which suggests that a large number of asset pricing anomalies disappear, at least in the form of significant average returns, once researchers control for size, specifically by excluding microcap stocks.[9] In particular, Hou et al. (2018) find that forming decile portfolios based on on IV^{FF3} does not yield a significant spread in average returns after excluding micro-cap stocks.

Panel A of Table 6 shows results for the original sample period of Ang et al. (2006). As before, we are able to closely replicate the corresponding results reported in Table VII of Ang et al. (2006). Size does not explain the FF3 alphas of the $5-1$ portfolio. Interestingly, and not previously reported in Ang et al. (2006), we find that the average returns of the $5-1$ portfolios consisting of NYSE stocks only and of the largest stocks only are small and statistically insignificant, but the corresponding FF3 alphas remain negative and statistically significantly different from zero, at the 5% level and 10% level, respectively.

In Panel B, we report the corresponding results for the more recent out-of-sample period, while Panel C reports results for the earlier out-of-sample period. In all cases, we find significantly

[9] Bali and Cakici (2008) and Han and Lesmond (2011) also argue that the IV puzzle arises from size-related effects, such as microstructure effects and sensitivity of IV-sorted portfolios to breakpoint choices. However, Chen et al. (2012) and Stambaugh et al. (2015) document that these effects are mitigated by excluding penny stocks.

negative FF3 alphas for the $5-1$ portfolio.[10] We again find, in absolute magnitude, smaller and statistically insignificant average returns for some of these $5-1$ portfolios, reconciling our conclusion with the evidence presented by Hou et al. (2018), which focuses on unadjusted average returns of the idiosyncratic volatility anomaly.

We report replications of results controlling for additional cross-sectional characteristics, including book-to-market, leverage, and liquidity, in Appendix Table A4. We are again able to replicate all the findings of Ang et al. (2006) for their original sample period and find that, with the exception of controlling for book-to-market, FF3 alphas of the $5-1$ portfolio remain statistically significant in the more recent out-of-sample period.[11] The characteristic-control tests of Ang et al. only control for one characteristics at a time. We extend these results in Table 7, which presents results of Fama-Macbeth regressions of individual stock returns on prior-month IV^{FF3}, together with $\beta_{\Delta VIX}$ and several common control characteristics.[12] Following Ball, Gerakos, Linnainmaa, and Nikolaev (2016), we report results using both all stocks and excluding "microcap" stocks whose market capitalization is below the NYSE 20$^{\text{th}}$ percentile. Panel A presents results for the 1986 to 2016 period during which both $\beta_{\Delta VIX}$ and IV^{FF3} exist. Panel A shows that both $\beta_{\Delta VIX}$ and IV^{FF3} predict the cross-section of returns controlling for each other, as well as the other cross-sectional return predictors, and this finding is robust to excluding microcap stocks. Panel B, which presents results over the July 1963 to December 2016 sample period, shows that IV^{FF3} predicts the cross-section of returns at the 1% significance level, controlling for all the characteristics, and for both all stocks and non-microcap stocks. These results are consistent with the hypothesis that IV^{FF3} captures a missing risk factor. We examine this hypothesis in the next section.

Appendix Tables A5 and A6 also report successful replications of Tables IX and X of Ang et al. (2006), respectively. These tables show that the IV puzzle is robust to controlling for aggregate-volatility risk, using holding periods of 12 months, estimating IV over horizons of up to 12 months, and including a lag of one month between the estimation of IV and formation of portfolios based

[10] Unlike the pricing of aggregate volatility, removing NASDAQ stocks does not have a significant effect on the FF3 alpha of the $5-1$ IV portfolio in 2001 to 2016.

[11] We do not conduct robustness checks for the earlier out-of-sample period due to lack of data.

[12] These characteristics are defined following Fama and French (2015, 2016) and include the natural logarithm of market capitalization and book-to-market ratio, the prior-12-month-excluding-prior-month return, the prior-month return, operating profitability, and investment.

on IV. Figure 1 extends this analysis by investigating the horizon at which IV predicts abnormal returns. Specifically, each month t we form value-weighted quintile portfolios and a $5-1$ portfolio by sorting stocks based on IV^{FF3} from the previous month. We then follow the $5-1$ portfolio for a period of l months, where $l = 0$, 1, 2, ..., or 120. Figure 1 plots the FF3 alphas and 95% confidence intervals of the resulting $5-1$ portfolios as a function of l. The sample period of the $5-1$ returns is July 1963 to December 2016 and is kept constant for the portfolios, regardless of l, by estimating IV^{FF3} as early as 1953. The figure shows that sorting on IV^{FF3} yields significant alphas for up to 24 months, and yields strictly negative alphas for up to 54 months. This finding suggests that IV^{FF3} does not represent a mispricing that is rapidly arbitraged away, and is perhaps surprising given that IV^{FF3} is estimated over a relatively short horizon of one month.

3.2. The idiosyncratic volatility puzzle and a missing risk factor

Ang et al. (2009) and Chen and Petkova (2012) propose that the IV puzzle of Ang et al. (2006) could be explained by a missing risk factor. Moreover, since the publication of Ang et al. (2006), several asset-pricing models have been introduced that dominate the FF3, such as the recent models listed in Section 1. In this section, we evaluate the ability of these models to explain the IV^{FF3} puzzle of Ang et al. (2006). We also investigate whether these more recent models can price portfolios sorted on IV relative to themselves.

3.2.1. Pricing IV^{FF3}-sorted portfolios with more recent asset-pricing models

To test for a possibly missing risk factor, we consider the five recent asset-pricing models of Fama and French (2015, 2018), Hou et al. (2015), Stambaugh and Yuan (2017), and Barillas and Shanken (2018). Table 8 reports regressions of the $5-1$ IV^{FF3} portfolio return on the factors of the CAPM, FF3, and more recent models over 1967 to 2016 (Panel A), 1967 to 2000 (Panel B), and 2001 to 2016 (Panel C). Panel A shows that, for the extended period of 1967 to 2016, the newer models attenuate the abnormal returns. The alpha estimate decreases substantially from -130 bps (for FF3) to -65 bps (for FF5), to about -35 bps (for HXZ4 and SY4), and to -21 bps (for BS6). While all estimates of alpha are negative, the SY4 alpha and the BS6 alpha are statistically indistinguishable from zero at conventional levels, suggesting that both models can explain the IV^{FF3} puzzle over

1967 to 2016.

Panel B shows that while the newer models attenuate the IV puzzle over 1967 to 2000, which maximally overlaps with the Ang et al. (2006) sample, none of them solve the puzzle. While the alpha decreases by about 50%, from -147 bps (FF3) to about -72 bps (HXZ4 and BS6), all the alphas are significant at 1%. Hence, the original findings of Ang et al. (2006) are not fully explained by the more recent asset-pricing models.

Panel C reports the corresponding regression results over 2001 to 2016. Consistent with our finding that the anomaly is weaker in that period than in 1967 to 2000 (the FF3 alpha decreases from -147 to -81 bps), each of the recent models can explain the anomaly by generating alphas that are economically very small and statistically indistinguishable from zero. Indeed, all of the more recent models except BS6 yield positive alphas.

To summarize, our tests suggest that the more recent asset-pricing models substantially attenuate the abnormal returns of portfolios sorted on idiosyncratic volatility relative to the FF3. Nevertheless, none of them can fully explain the IV^{FF3} puzzle over the sample of Ang et al. (2006), where this puzzle was unusually strong relative to the pre-1963 and post-2000 periods. Subsequently, as the FF3 alpha is much smaller out-of-sample, over the entire period of 1967 to 2016, the SY4 and BS6 generate negative alphas that are much smaller than the FF3 alpha and are statistically indistinguishable from zero at conventional levels.

Inspection of the factor loadings in Table 8 reveals how exposure to each factor of the SY4 and BS6 contribute to explaining the average return of the $5-1$ IV^{FF3} portfolio, which Panel A shows is -87 bps over 1967 to 2016. The average return of each factor we consider is positive in Panel A of Table 1, so negative loadings contribute in the "right direction" to account for the $5-1$ return, and vice versa. In the SY4, the loadings on $MGMT$ and $PERF$ are negative, and therefore primarily responsible for the ability of the SY4 to explain the IV puzzle. These two factors are designed to capture two latent common factors driving 11 prominent characteristics-based FF3-anomaly returns. That their loadings are both economically and statistically significant is consistent with the common finding that prominent anomalies are driven by relatively few common factors (e.g., Fama and French, 2016; Kozak, Nagel, and Santosh, 2018), and that the IV^{FF3} puzzle represents exposure to at least two sources of common variation in returns unaccounted for by the FF3.[13]

In the BS6, the loadings in Panel A of Table 8 on the market and size factor are positive and significant at the 1% level. Thus, exposure to these factors works in the opposite direction from accounting for the returns on the $5-1$ portfolio. This finding can be seen further by observing the $5-1$ FF3 alpha (-130 bps) that is larger in magnitude than the corresponding unadjusted average return.[14] In contrast to those on the market and size factors, the loadings on the remaining four BS6 factors ($HML(m)$, MOM, ROE, and IA) are all negative, and all but the MOM loading are significant at the 1% level. Thus, given the positive average factor returns, these four factors account for the ability of the BS6 to explain the negative returns on the $5-1$ portfolio. The profitability factor ROE is the most important; using the BS6 ROE loading of -1.17, and the average ROE return of 54 bps from Panel A of Table 1, the ROE factor accounts for -63 (-1.17×54) bps, or about 73%, of the -87 bps average return of the $5-1$ portfolio return over 1967 to 2016. Overall, the BS6 factor exposures in Panel A of Table 8 provide an intuitive explanation for why the returns of high-IV^{FF3} stocks are so low: they behave like small-cap growth stocks with low profitability and high levels of real investment.

3.2.2. Idiosyncratic volatility relative to more recent pricing models

Given the performance of the more recent asset-pricing models with respect to pricing portfolios formed on IV^{FF3}, we calculate IV relative to each of these models, by using residuals from regressions similar to Eq. (7), but where we use the factors from a more recent model instead of those from the FF3. We then form quintile portfolios sorted by IV estimated from each of these models and examine whether the corresponding $5-1$ IV portfolio is priced by the more recent models. Specifically, for each model and separately over 1967 to 2016, 1967 to 2000, and 2001 to 2016, we test whether the $5-1$ portfolio based on the model's own IV is priced by the model itself as well

[13] In untabulated results, we also investigate whether the $5-1$ IV^{FF3} portfolio can price the factors from more recent models. Adding the $5-1$ IV^{FF3} to the FF3 model eliminates the alpha of RMW over the period of July 1963 to December 2016, but does not eliminate the alphas of any factor aside from RMW. This finding is robust to including the momentum factor, MOM, and including a $5-1$ $\beta_{\Delta VIX}$ portfolio.

[14] Indeed, untabulated results show that excluding the size factor or the size and market factors from BS6 produces positive alphas for the $5-1$ FF3-IV portfolio in each of the periods, including the original period of Ang et al. (2006). This does not mean that the nested four-factor model is a better model than BS6. As noted by Barillas and Shanken (2017), when examining a test asset in isolation, it is possible for a nested model to produce a smaller pricing error than its broader nesting model even when the broader model is the true model.

as by the SY4 and BS6.

Table 9 reports the results for FF5 (Panel A), HXZ4 (Panel B), SY4 (Panel C), and BS6 (Panel D). Our main results are: First, over 1967 to 2000, the $5 - 1$ portfolios of all four models have negative average returns and negative alphas that are economically and statistically significant. Hence, the relatively strong IV^{FF3} anomaly documented in the Ang et al. (2006) sample extends to IV estimated from each of the more recent asset-pricing models we consider. Second, none of the models produce average returns and alphas for the $5 - 1$ portfolios that are statistically different from zero over the subsequent period of 2001 to 2016. Indeed, average returns and alphas decrease by more than 60% compared to the original sample period. Third, over the entire period 1967 to 2016, all models yield negative average returns for the $5 - 1$ portfolio that are economically and statistically significant. Own-model alphas and SY4 alphas are also economically and statistically significant at conventional levels (at 5% for FF5 and HXZ4 and 10% for SY4). However, we cannot reject the null hypothesis that BS6 alphas over 1967 to 2016 are zero for the $5 - 1$ portfolio of any of the four models including BS6.

To summarize, we find that while recent models attenuate the alphas of the IV^{FF3} puzzle documented in Ang et al. (2006) over 1967 to 2000, they cannot fully explain it. Moreover, each of the recent models also produces its own anomalous $5 - 1$ IV portfolio return over that period. Hence, the IV puzzle of Ang et al. (2006) cannot be accounted for by the recent models over their sample. However, for all models we consider, the anomaly is economically and statistically much weaker over the subsequent period of 2001 to 2016. Over the entire period of 1967 to 2016, the four-factor model of Stambaugh and Yuan (2017) and the six-factor model of Barillas and Shanken (2018) successfully price portfolios sorted on IV relative to the FF3. The six-factor model also prices portfolios formed on IV with respect to each of the recent asset-pricing models, including itself.

3.3. Idiosyncratic volatility and the arbitrage asymmetry of Stambaugh et al. (2015)

Stambaugh et al. (2015) provide an alternative explanation of the IV puzzle of Ang et al. (2006), according to which the negative average return of high IV^{FF3} portfolios arises due to short-selling constraints coupled with the fact that IV represents risk that prevents arbitrageurs from eliminating mispricing (e.g., Pontiff, 2006). Consistent with this explanation, Stambaugh et al. (2015) show that between 1966 and 2011, FF3 alphas of underpriced stocks increase with IV^{FF3}, while FF3 alphas of overpriced stocks decrease, i.e., become more negative, as IV increases. Moreover, the change in the absolute magnitudes is larger for overpriced stocks than for underpriced stocks. Aggregating across under- and overpriced stocks, this "arbitrage asymmetry" results in the observed negative FF3 alpha of the $5-1$ portfolio sorted on IV^{FF3}.

In Table 10, we reexamine the mechanism proposed by Stambaugh et al. (2015), when idiosyncratic volatility is calculated relative to FF3 (IV^{FF3}), the four-factor mispricing model of Stambaugh and Yuan (2017) (IV^{SY4}), and the six-factor model of Barillas and Shanken (2018) (IV^{BS6}), the latter two being the recent models that seem most successful in pricing IV^{FF3}-sorted portfolios. Using data for the period from 1967 to 2016, we sort stocks by their idiosyncratic volatility as well as the mispricing measure of Stambaugh and Yuan (2017), which is an updated version of the Stambaugh et al. (2015) measure.[15] We measure the degree of arbitrage asymmetry by the sum of the two $5-1$ IV alphas in the "Most Underpriced" and "Most Overpriced" columns, and report this statistic, "Asymmetry," in the bottom-right corner of each panel. An Asymmetry of zero indicates that the IV effect is the same for both underpriced and overpriced stocks. A negative value of Asymmetry indicates that the effect of IV is greater for overpriced stocks.

In Panel A, we report the results when the FF3 is used to calculate IV as well as the alphas of the returns on the 25 (5x5) portfolios sorted independently on IV^{FF3} and the mispricing measure. As in Stambaugh et al. (2015), we find that, when comparing Low to High IV portfolios, the FF3 alphas increase, in absolute terms, more for the "Most Overpriced" stocks (by 152 bps) compared to the "Most Underpriced" stocks (by 25 bps). The Asymmetry of -127 bps is economically large and statistically significantly different from zero at the 1% level.

In Panel B, we report the corresponding results when the SY4 is used to calculate IV as well as to price the 25 portfolios sorted independently on IV^{SY4} and the mispricing measure. We find that the arbitrage Asymmetry is reduced to -33 bps and is no longer significantly different from zero. We now also find that the $5-1$ IV alpha for the "Most Underpriced" stocks is significantly

[15]We obtain this measure from Robert Stambaugh's website (http://finance.wharton.upenn.edu/~stambaug/).

positive, consistent with the hypothesis that IV represents risk for both long and short arbitrage positions.

Finally, in Panel C, we report similar results when the BS6 is used to estimate IV as well as price the 25 portfolios sorted independently on IV^{BS6} and the mispricing measure. The $5-1$ IV alphas are higher than in Panel A in each of the five mispricing columns, alpha for the "Most Underpriced" stocks is significantly positive and the $5-1$ alpha for the "Most Overpriced" stocks is significantly negative, and the arbitrage asymmetry is further reduced to -19 bps and is again not significantly different from zero.

Our results suggest that controlling for additional risk factors in the construction and pricing of portfolios sorted on IV and the mispricing measure eliminates the arbitrage asymmetry initially documented by Stambaugh et al. (2015). This finding is consistent with the hypothesis that one or more risk factors, which are missing from the FF3, lower the returns of stocks with high IV^{FF3}, as initially documented by Ang et al. (2006), and thereby create the arbitrage asymmetry documented by Stambaugh et al. (2015). Indeed, consistent with this missing risk factor perspective, we find that the $5-1$ IV alphas are lower in Panel A than in Panels B and C for each of the five mispricing columns. Once the missing risk factors are accounted for, in both the construction and pricing of the portfolios, the average mispricing increases as IV increases and does so by the same amount (in absolute terms) for the most over- and underpriced stocks. We can therefore no longer reject the null hypothesis that the absolute values of the $5-1$ IV alphas of the most over- and underpriced stocks are equal.

4. Conclusions

The relationship between volatility and expected returns is one of the most fundamental lines of inquiry in asset pricing. Intertemporal asset-pricing theory predicts that aggregate-volatility risk should be negatively priced in the cross-section of stock returns. Ang et al. (2006) document evidence that this is indeed the case and we are able to closely replicate these findings. Moreover, after excluding NASDAQ stocks to mitigate estimation errors in betas that stem from the use of daily data, we find that the volatility risk premium in equities remains significantly negative out of sample and is unexplained by recent factor models.

Ang et al. (2006) also document evidence that stocks' idiosyncratic volatility (IV), which they measure relative to the Fama and French (1993) three-factor model (FF3), negatively predicts the cross-section of returns. Moreover, this predictability is not explained by the FF3. Unlike the aggregate volatility result, this finding runs counter to classical asset-pricing theory, which predicts either no cross-sectional relationship between IV and returns or a positive one (e.g., Fama and MacBeth, 1973; Merton, 1987).

We are able to closely replicate the idiosyncratic volatility results of Ang et al. (2006) and show that the significantly negative FF3 abnormal returns of high-IV stocks also exist out of sample. Consistent with the missing risk-factor-based explanation of the IV puzzle, we show that the recent asset-pricing models proposed by Stambaugh and Yuan (2017) and Barillas and Shanken (2018) explain the abnormal returns on IV-sorted portfolios over the extended period of 1967 to 2016. The six-factor model also explains returns on portfolios formed by sorting stocks based on IV relative to itself. This evidence contrasts with the asymmetric arbitrage risk-based explanation of the IV anomaly proposed by Stambaugh et al. (2015). This explanation relies on short-selling frictions increasing the impact of IV on overpricing more than on underpricing to yield a negative average relationship between IV and abnormal returns in the cross section. When we repeat the analysis of Stambaugh et al. (2015) using IV and abnormal returns relative to the more recent factor models, we find that, in absolute value, abnormal returns of underpriced stocks increase with IV by a similar magnitude as those of overpriced stocks decrease with IV.

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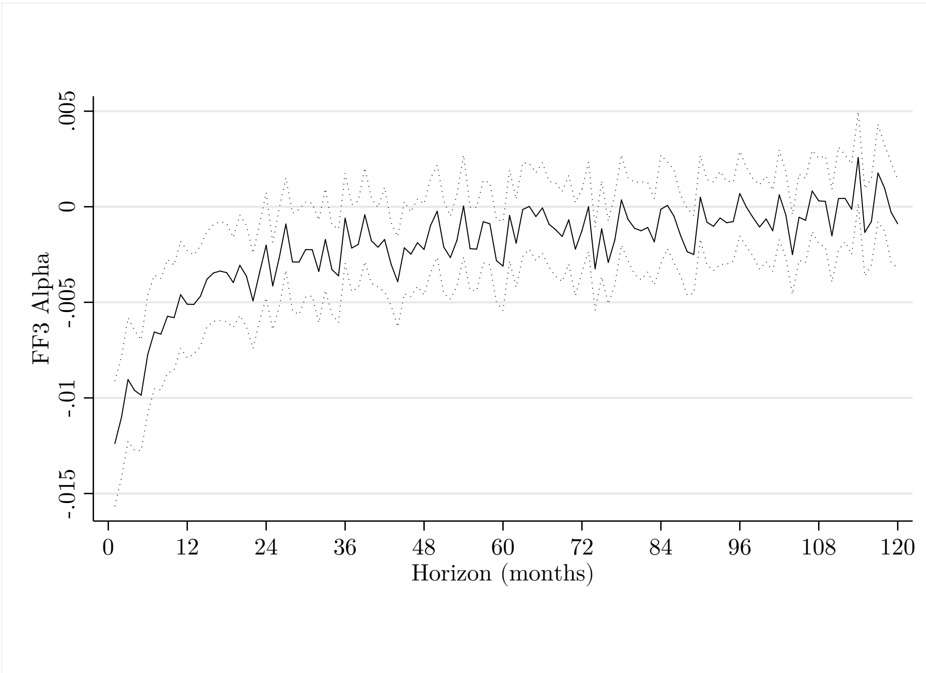

Figure 1: Fama-French three-factor alphas of long-short portfolios sorted on lags of idiosyncratic volatility

Description: We form value-weighted quintile portfolios every month t by sorting stocks based on idiosyncratic volatility relative to the Fama and French (1993) three-factor model (FF3) estimated using daily data over month $t - l$, where $l = 1,\ 2, ...,\ 120$. Portfolio 1 (5) is the portfolio of stocks with the lowest (highest) volatilities. For each lag l (given by the x-axis), the figure reports FF3 alphas and 95% confidence intervals of the $5 - 1$ long-short portfolio based on IV lagged l months. Confidence intervals are based on robust Newey and West (1987) standard error using three lags. The sample period is July 1963 to December 2016.

Interpretation: The $5 - 1$ idiosyncratic volatility portfolio earns significant alphas for up to 24 months, and strictly negative alphas for up to 54 months.

Table 1: Summary statistics of factors from various asset-pricing models

Description: This table presents means, standard deviations, and Newey and West (1987) t-statistics based on three lags of the returns on factors from the asset-pricing models used in this paper. MKT denotes the excess return on the CRSP value-weighted index. SMB and HML denote the Fama and French (1993) size and value factors. $HML(m)$ denotes the monthly-updated HML factor $HML(m)$ of Asness and Frazzini (2013). $SMB5$, RMW, and CMA denote, respectively, the size, profitability, and investment factors from the Fama and French (2015) five-factor model. MOM denotes the Fama-French momentum factor. ME, IA, and ROE denote the Hou et al. (2015) size, investment, and profitability factors, respectively. $SMB(SY)$, $MGMT$, and $PERF$ denote the Stambaugh and Yuan (2017) size and two "mispricing" factors, respectively. In Panel A, the sample period is January 1967 to December 2016. In Panels B and C, respectively, the sample periods are January 1967 to December 2000 and January 2001 to December 2016.

Interpretation: All factors have positive average excess returns. With the exception of SMB, these average returns are statistically significant over the full sample period.

	Panel A: 1967 to 2016			Panel B: 1967 to 2000			Panel C: 2001 to 2016		
	Mean	Std. Dev.	t	Mean	Std. Dev.	t	Mean	Std. Dev.	t
MKT	0.52	4.53	2.72	0.54	4.60	2.37	0.47	4.38	1.36
SMB	0.22	3.13	1.66	0.16	3.37	0.90	0.34	2.55	2.05
$SMB5$	0.25	3.08	1.93	0.18	3.29	1.06	0.40	2.61	2.25
$SMB(SY)$	0.44	2.88	3.60	0.41	3.06	2.60	0.48	2.47	2.88
ME	0.31	3.09	2.41	0.27	3.27	1.65	0.38	2.66	2.08
HML	0.37	2.89	2.72	0.43	2.94	2.51	0.24	2.79	1.14
$HML(m)$	0.34	3.51	2.13	0.30	3.40	1.62	0.42	3.74	1.43
MOM	0.65	4.32	3.62	0.92	3.73	5.01	0.07	5.32	0.18
RMW	0.26	2.28	2.51	0.22	2.25	1.71	0.35	2.36	1.94
ROE	0.54	2.55	5.22	0.71	2.34	6.04	0.19	2.92	0.91
CMA	0.33	2.03	3.52	0.37	2.09	3.05	0.25	1.90	1.74
IA	0.41	1.87	4.93	0.49	1.89	4.71	0.23	1.83	1.73
$MGMT$	0.61	2.88	4.73	0.73	2.93	4.46	0.35	2.76	1.79
$PERF$	0.68	3.85	4.22	0.70	3.17	4.29	0.62	5.02	1.69

Table 2: (Replication of Table I of Ang et al., 2006) Portfolios sorted by exposure to aggregate volatility shocks

Description: Each month, we estimate $\beta_{\Delta VIX}$ as the slopes from regressions of individual excess stock returns on ΔVIX, controlling for the MKT factor as in Eq. (4), using daily data over the month. We then sort stocks at the beginning of each following month into value-weighted quintile portfolios based on the coefficient $\beta_{\Delta VIX}$ from lowest (quintile 1) to highest (quintile 5). The statistics in the columns labeled Mean and Std. Dev. are measured in monthly percentage terms and apply to total, not excess, simple returns. Size reports the average log market capitalization for firms within the portfolio and B/M reports the average book-to-market ratio. The row "5 − 1" refers to the difference in monthly returns between portfolio 5 and portfolio 1. The Alpha columns report alpha with respect to the CAPM or the Fama and French (1993) three-factor model. The pre-formation betas refer to the value-weighted average of the $\beta_{\Delta VIX}$ or β_{FVIX} within each quintile portfolio at the start of the month. We report the pre-formation $\beta_{\Delta VIX}$ and β_{FVIX} averaged across the whole sample period defined by the panel heading. The second to last column reports the time-series average of $\beta_{\Delta VIX}$ loadings estimated over the post-ranking month with daily data. The last column reports ex post β_{FVIX} factor loadings over the whole sample, where $FVIX$ is the factor mimicking aggregate-volatility risk. To correspond with the Fama-French alphas, we compute the ex post betas by running a four-factor regression with the three Fama and French (1993) factors together with the $FVIX$ factor that mimics aggregate-volatility risk. Robust Newey and West (1987) t-statistics based on three lags are reported in square brackets. In Panels A, B, and C, respectively, the sample periods are February 1986 to December 2000, January 2001 to December 2016, and February 1986 to December 2016.

Interpretation: The results in Panel A closely match those of Ang et al. (2006). Over their original sample period, as well as the extended sample, average excess returns and alphas are significantly lower for the portfolio of stocks in the top pre-formation $\beta_{\Delta VIX}$ quintile than in the bottom quintile, although this significance diminishes in the post–Ang et al. (2006) period.

Panel A: Ang et al. (2006) sample, 1986 to 2000											
								Factor loadings			
Rank	Mean	Std. Dev.	%Mkt Share	Size	B/M	CAPM Alpha	FF3 Alpha	Pre-formation $\beta_{\Delta VIX}$	Pre-formation β_{FVIX}	Next month post-formation $\beta_{\Delta VIX}$	Full sample post-formation β_{FVIX}
1	1.65	5.54	9.1	3.76	0.85	0.25	0.30	−1.33	−1.59	−0.030	−10.40
						[1.50]	[1.70]				[−4.00]
2	1.35	4.41	28.8	4.90	0.79	0.12	0.05	−0.42	−0.77	−0.022	−6.48
						[1.40]	[0.63]				[−3.83]
3	1.32	4.44	30.4	4.93	0.83	0.07	0.01	0.03	−0.31	0.013	−2.57
						[0.78]	[0.14]				[−1.57]
4	1.26	4.74	24.2	4.88	0.78	−0.04	−0.06	0.50	0.11	0.031	6.64
						[−0.52]	[−0.64]				[3.73]
5	0.64	6.64	7.4	3.77	0.84	−0.87	−0.51	1.49	0.76	0.041	14.99
						[−3.40]	[−2.70]				[4.12]
5 − 1	−1.01					−1.12	−0.80				
	[−3.79]					[−3.46]	[−2.76]				

Table 2: Continued

Panel B: 2001 to 2016

Rank	Mean	Std. Dev.	%Mkt Share	Size	B/M	CAPM Alpha	FF3 Alpha	Factor loadings: Pre-formation $\beta_{\Delta VIX}$	Factor loadings: Pre-formation β_{FVIX}	Factor loadings: Next month post-formation $\beta_{\Delta VIX}$	Factor loadings: Full sample post-formation β_{FVIX}
1	0.61	5.81	10.9	5.32	0.95	−0.08	−0.07	−1.06	−1.29	0.046	1.18
						[−0.48]	[−0.47]				[0.15]
2	0.64	4.06	29.4	6.42	0.81	0.10	0.12	−0.33	−0.66	0.011	−2.93
						[1.47]	[1.60]				[−0.79]
3	0.59	4.14	30.6	6.58	0.80	0.04	0.03	0.05	−0.36	0.017	3.37
						[0.60]	[0.43]				[0.97]
4	0.66	4.71	21.3	6.30	0.81	0.05	0.02	0.44	−0.10	0.030	−0.85
						[0.57]	[0.27]				[−0.23]
5	0.43	6.98	7.8	5.33	0.94	−0.37	−0.45	1.25	0.43	0.080	13.76
						[−2.16]	[−2.61]				[1.20]
5 − 1	−0.18					−0.29	−0.38				
	[−0.68]					[−1.15]	[−1.50]				

Panel C: 1986 to 2016

Rank	Mean	Std. Dev.	%Mkt Share	Size	B/M	CAPM Alpha	FF3 Alpha	Factor loadings: Pre-formation $\beta_{\Delta VIX}$	Factor loadings: Pre-formation β_{FVIX}	Factor loadings: Next month post-formation $\beta_{\Delta VIX}$	Factor loadings: Full sample post-formation β_{FVIX}
1	1.11	5.70	10.0	4.57	0.90	0.07	0.11	−1.19	−1.43	0.009	−5.70
						[0.59]	[0.99]				[−2.62]
2	0.98	4.24	29.1	5.69	0.80	0.12	0.11	−0.38	−0.71	−0.005	−5.43
						[2.05]	[1.94]				[−2.82]
3	0.94	4.29	30.5	5.78	0.82	0.05	0.03	0.04	−0.33	0.015	−1.78
						[1.04]	[0.59]				[−0.79]
4	0.94	4.73	22.7	5.61	0.80	0.00	−0.02	0.47	0.00	0.031	4.53
						[0.02]	[−0.40]				[2.33]
5	0.53	6.81	7.6	4.58	0.89	−0.62	−0.57	1.37	0.59	0.061	15.64
						[−4.02]	[−3.89]				[3.01]
5 − 1	−0.58					−0.69	−0.69				
	[−2.99]					[−3.36]	[−3.27]				

Table 3: Portfolios sorted by exposure to aggregate volatility shocks, excluding NASDAQ stocks

Description: Each month, we estimate $\beta_{\Delta VIX}$ as the slopes from regressions of individual excess stock returns on ΔVIX, controlling for the MKT factor as in Eq. (4), using daily data over the month. We then sort NYSE and AMEX stocks at the beginning of each following month into value-weighted quintile portfolios based on the coefficient $\beta_{\Delta VIX}$ from lowest (quintile 1) to highest (quintile 5). The statistics in the columns labeled Mean are time-series averages of monthly percentage total, not excess, simple returns. The row "5 − 1" refers to the difference in monthly returns between portfolio 5 and portfolio 1. The Alpha columns report alpha with respect to the CAPM or the Fama and French (1993) three-factor model. Robust Newey and West (1987) t-statistics based on three lags are reported in square brackets. In Panels A, B, and C, respectively, the sample periods are February 1986 to December 2000, January 2001 to December 2016, and February 1986 to December 2016.

Interpretation: After excluding NASDAQ stocks, the portfolio of stocks in the top $\beta_{\Delta VIX}$ quintile has significantly lower average excess returns and alphas than in the bottom quintile over the whole sample period and each subsample.

	Panel A: 1986 to 2000			Panel B: 2001 to 2016			Panel C: 1986 to 2016		
Rank	Mean	CAPM Alpha	FF3 Alpha	Mean	CAPM Alpha	FF3 Alpha	Mean	CAPM Alpha	FF3 Alpha
1	1.42	0.16	−0.04	0.75	0.14	0.12	1.07	0.14	0.07
		[0.93]	[−0.32]		[0.89]	[0.73]		[1.20]	[0.63]
2	1.35	0.17	−0.01	0.67	0.17	0.17	1.00	0.17	0.12
		[1.28]	[−0.11]		[1.86]	[1.93]		[2.19]	[1.74]
3	1.35	0.17	−0.03	0.51	−0.01	−0.03	0.92	0.08	0.00
		[1.12]	[−0.29]		[−0.08]	[−0.45]		[0.99]	[−0.02]
4	1.22	−0.01	−0.15	0.64	0.06	0.03	0.92	0.02	−0.05
		[−0.08]	[−1.64]		[0.55]	[0.29]		[0.25]	[−0.75]
5	0.86	−0.44	−0.48	0.19	−0.53	−0.63	0.51	−0.50	−0.61
		[−2.96]	[−3.34]		[−2.76]	[−3.54]		[−3.67]	[−4.79]
5 − 1	−0.56	−0.60	−0.44	−0.56	−0.66	−0.75	−0.56	−0.64	−0.69
	[−2.91]	[−2.64]	[−1.96]	[−2.06]	[−2.50]	[−2.77]	[−3.26]	[−3.49]	[−3.42]

Table 4: Alphas with respect to recent asset-pricing models of the $5-1$ $\beta_{\Delta VIX}$ portfolio

Description: This table presents alphas with respect to the asset-pricing models defined by the column headings. In the first five columns, the left-hand-side return is that of the high-minus-low-$\beta_{\Delta VIX}$-quintile portfolio defined in Table 2 based on all U.S. stocks. In the last five columns, the left-hand-side return is that of the high-minus-low-$\beta_{\Delta VIX}$-quintile portfolio defined in Table 3 based on all U.S. stocks except those on the NASDAQ. Panels A, B, and C, respectively, use the samples February 1986 to December 2000, January 2001 to December 2016, and February 1986 to December 2016. FF3 denotes the Fama and French (1993) three-factor model. FF5 denotes the Fama and French (2015) five-factor model. FF6 denotes the FF5 model augmented with MOM. HXZ4 denotes the Hou et al. (2015) four-factor model. SY4 denotes the Stambaugh and Yuan (2017) four-factor model. BS6 denotes the Barillas and Shanken (2018) six-factor model. Newey and West (1987) t-statistics based on three lags are in brackets.

Interpretation: After excluding NASDAQ stocks, no model explains the abnormal returns of the $5-1$ $\beta_{\Delta VIX}$ portfolio. Without excluding NASDAQ stocks, the $5-1$ portfolio still generally earns at least marginally significant alpha with respect to each model over the extended sample period.

All stocks						Excluding NASDAQ stocks					
FF3	FF5	FF6	HXZ4	SY4	BS6	FF3	FF5	FF6	HXZ4	SY4	BS6
Panel A: Ang et al. (2006) sample, 1986 to 2000											
−0.80	−0.74	−0.73	−0.38	−0.72	−0.49	−0.44	−0.37	−0.23	−0.14	−0.31	−0.08
[−2.78]	[−2.09]	[−1.92]	[−1.03]	[−1.69]	[−1.22]	[−1.98]	[−1.48]	[−0.94]	[−0.54]	[−1.03]	[−0.28]
Panel B: 2001 to 2016											
−0.38	−0.13	−0.12	−0.29	−0.22	−0.29	−0.75	−0.70	−0.68	−0.81	−0.68	−0.84
[−1.51]	[−0.50]	[−0.48]	[−0.98]	[−0.75]	[−0.94]	[−2.80]	[−2.42]	[−2.51]	[−2.56]	[−2.08]	[−2.67]
Panel C: 1986 to 2016											
−0.68	−0.41	−0.44	−0.42	−0.57	−0.42	−0.69	−0.55	−0.57	−0.60	−0.62	−0.62
[−3.28]	[−1.87]	[−1.89]	[−1.71]	[−2.03]	[−1.64]	[−3.43]	[−2.67]	[−2.63]	[−2.52]	[−2.40]	[−2.51]

Table 5: (Replication of Table VIB of Ang et al., 2006) Portfolios sorted by idiosyncratic volatility

Description: We form value-weighted quintile portfolios every month by sorting stocks based on idiosyncratic volatility relative to the Fama and French (1993) three-factor model (FF3) estimated using daily data over the previous month. Portfolio 1 (5) is the portfolio of stocks with the lowest (highest) volatilities. The statistics in the columns labeled Mean and Std. Dev. are measured in monthly percentage terms and apply to total, not excess, returns. Size reports the average log market capitalization for firms within the portfolio and B/M reports the average book-to-market ratio. The row "5 − 1" refers to the difference in monthly returns between portfolio 5 and portfolio 1. The Alpha columns report alpha with respect to the CAPM or FF3. Robust Newey and West (1987) t-statistics based on three lags are reported in square brackets. In Panel A, the sample period is July 1963 to December 2000, which is the Ang et al. (2006) sample period. In Panels B and C, respectively, the sample periods are January 2001 to December 2016 and August 1926 to June 1963.

Interpretation: The results in Panel A closely match those of Ang et al. (2006). Over their original sample period, and the two out-of-sample periods, CAPM and FF3 alphas are significantly lower for the portfolio of stocks in the top idiosyncratic volatility quintile than in the bottom quintile.

Panel A: Ang et al. (2006) sample (1963 to 2000)

Rank	Mean	Std. Dev	% Mkt Share	Size	B/M	CAPM Alpha	FF3 Alpha
1	1.05	3.83	55.90	5.38	0.95	0.10	0.04
						[1.67]	[1.00]
2	1.17	4.78	25.70	4.88	0.91	0.10	0.09
						[2.00]	[1.53]
3	1.21	5.88	11.40	4.19	0.93	0.04	0.07
						[0.39]	[0.90]
4	0.85	7.15	5.00	3.51	1.00	−0.41	−0.36
						[−2.51]	[−3.60]
5	−0.03	8.16	1.90	2.67	1.22	−1.29	−1.33
						[−5.07]	[−7.65]
5 − 1	−1.08					−1.40	−1.37
	[−3.18]					[−4.58]	[−6.93]

Panel B: 2001 to 2016

Rank	Mean	Std. Dev	% Mkt Share	Size	B/M	CAPM Alpha	FF3 Alpha
1	0.63	3.59	57.70	7.47	0.74	0.14	0.15
						[2.20]	[2.76]
2	0.58	5.04	23.80	6.69	0.76	−0.06	−0.10
						[−0.88]	[−1.53]
3	0.60	6.29	11.20	5.99	0.80	−0.16	−0.21
						[−1.25]	[−1.94]
4	0.47	8.21	5.40	5.29	0.91	−0.44	−0.48
						[−1.60]	[−2.16]
5	0.46	9.70	1.90	4.29	1.17	−0.55	−0.66
						[−1.65]	[−2.40]
5 − 1	−0.17					−0.69	−0.81
	[−0.30]					[−1.82]	[−2.61]

Table 5: Continued

Panel C: 1926 to 1963							
Rank	Mean	Std. Dev	% Mkt Share	Size	B/M	CAPM Alpha	FF3 Alpha
1	1.02	5.82	59.80	4.26	1.09	0.15	0.16
						[4.10]	[5.88]
2	1.02	7.75	19.60	3.49	1.37	−0.09	−0.11
						[−1.55]	[−2.07]
3	1.06	8.88	11.00	2.97	1.76	−0.19	−0.23
						[−2.14]	[−3.44]
4	1.00	9.25	6.50	2.41	2.49	−0.25	−0.32
						[−1.66]	[−3.80]
5	0.72	9.85	3.10	1.52	6.47	−0.49	−0.58
						[−2.00]	[−3.31]
5 − 1	−0.30					−0.63	−0.74
	[−0.92]					[−2.38]	[−4.00]

Table 6: (Replication of size-related elements of Table VII of Ang et al., 2006) Fama-French three-factor alphas of portfolios sorted on idiosyncratic volatility relative to the Fama-French three-factor model

Description: This table reports average returns and FF3 alphas for strategies rebalanced monthly based on idiosyncratic volatility relative to the FF3 model, which is estimated the same way as in Table 5. In the row labeled "NYSE Stocks Only," we sort stocks into quintile portfolios based on their idiosyncratic volatility using only NYSE stocks. In the rows labeled "Size Quintiles," we first sort stocks each month into five quintiles on the basis of market capitalization. Then, within each size quintile, we sort stocks into five portfolios sorted by idiosyncratic volatility. In the row labeled "Controlling for Size", for each idiosyncratic volatility ranking, we average the five portfolios with the same ranking across the five size quintiles. Hence, they represent idiosyncratic volatility-quintile portfolios controlling for size. All portfolios are value weighted. Robust Newey and West (1987) t-statistics based on three lags are in square brackets below the corresponding alphas. In Panel A, the sample period is July 1963 to December 2000. In Panels B and C, respectively, the sample periods are January 2001 to December 2016 and August 1926 to June 1963.

Interpretation: The results in Panel A closely match those of Ang et al. (2006). Over their original sample period, and the two out-of-sample periods, controlling for size does not eliminate the spread in alphas between the portfolio of stocks in the top idiosyncratic volatility quintile and those in the bottom quintile.

Panel A: Ang et al. (2006) sample, 1963 to 2000

		FF3 Alpha						Mean
		Ranking on Idiosyncratic Volatility						
		1 Low	2	3	4	5 High	5 − 1	5 − 1
NYSE Stocks Only		0.07	0.03	−0.02	−0.06	−0.64	−0.71	−0.32
		[1.48]	[0.42]	[−0.21]	[−0.63]	[−5.74]	[−5.30]	[−1.42]
Size Quintiles	Small 1	0.29	0.35	0.23	−0.17	−1.10	−1.38	−0.98
		[1.65]	[2.00]	[1.26]	[−0.78]	[−3.69]	[−4.59]	[−2.52]
	2	0.33	0.32	−0.02	−0.63	−1.88	−2.21	−1.89
		[2.52]	[2.62]	[−0.22]	[−4.88]	[−8.44]	[−8.25]	[−5.49]
	3	0.18	0.23	0.06	−0.19	−1.49	−1.67	−1.52
		[1.75]	[2.55]	[0.76]	[−2.28]	[−9.81]	[−7.70]	[−4.86]
	4	0.05	0.24	0.24	0.01	−0.79	−0.84	−0.83
		[0.57]	[2.77]	[3.28]	[0.16]	[−6.05]	[−4.33]	[−2.69]
	Large 5	0.09	0.04	0.05	0.12	−0.19	−0.28	−0.20
		[1.61]	[0.84]	[0.82]	[1.58]	[−1.50]	[−1.76]	[−0.77]
Controlling for Size		0.19	0.24	0.11	−0.17	−1.09	−1.28	−1.08
		[2.10]	[2.96]	[1.58]	[−2.20]	[−7.65]	[−6.63]	[−3.66]

Table 6: Continued

		Panel B: 2001 to 2016						
		FF3 Alpha						Mean
		Ranking on Idiosyncratic Volatility						
		1 Low	2	3	4	5 High	5 − 1	5 − 1
NYSE Stocks Only		0.17	0.01	−0.13	−0.20	−0.44	−0.61	−0.03
		[2.25]	[0.06]	[−1.62]	[−1.35]	[−1.61]	[−1.90]	[−0.06]
Size Quintiles	Small 1	0.87	0.75	0.46	−0.20	−1.80	−2.67	−2.18
		[3.46]	[2.32]	[1.12]	[−0.40]	[−3.75]	[−6.24]	[−3.77]
	2	0.79	0.58	0.37	−0.03	−1.24	−2.03	−1.40
		[5.03]	[2.86]	[1.75]	[−0.11]	[−3.89]	[−6.20]	[−2.55]
	3	0.52	0.30	0.14	−0.10	−0.70	−1.22	−0.68
		[4.23]	[2.30]	[1.12]	[−0.68]	[−2.85]	[−4.10]	[−1.31]
	4	0.32	0.17	0.02	−0.14	−0.56	−0.88	−0.42
		[3.70]	[1.99]	[0.22]	[−1.41]	[−2.49]	[−3.22]	[−0.90]
	Large 5	0.15	0.16	−0.03	−0.21	−0.41	−0.56	−0.18
		[2.08]	[2.17]	[−0.42]	[−2.13]	[−2.02]	[−2.22]	[−0.41]
Controlling for Size		0.53	0.39	0.19	−0.13	−0.94	−1.47	−0.97
		[5.60]	[3.32]	[1.43]	[−0.76]	[−4.33]	[−6.10]	[−2.13]

		Panel C: 1926 to 1963						
		FF3 Alpha						Mean
		Ranking on Idiosyncratic Volatility						
		1 Low	2	3	4	5 High	5 − 1	5 − 1
NYSE Stocks Only		0.16	−0.11	−0.22	−0.31	−0.56	−0.71	−0.27
		[5.87]	[−1.99]	[−3.36]	[−3.75]	[−3.19]	[−3.89]	[−0.83]
Size Quintiles	Small 1	0.56	0.27	0.04	−0.07	−0.15	−0.71	−0.30
		[3.84]	[1.74]	[0.26]	[−0.39]	[−0.54]	[−2.17]	[−0.73]
	2	0.45	0.33	0.07	−0.22	−1.01	−1.46	−1.04
		[4.62]	[3.52]	[0.60]	[−1.82]	[−6.76]	[−7.67]	[−3.73]
	3	0.34	0.23	0.03	−0.23	−0.63	−0.97	−0.43
		[4.19]	[2.65]	[0.35]	[−2.52]	[−5.56]	[−6.76]	[−1.70]
	4	0.27	0.11	0.04	−0.08	−0.40	−0.68	−0.20
		[3.01]	[1.26]	[0.47]	[−0.84]	[−3.55]	[−4.25]	[−0.80]
	Large 5	0.20	0.16	−0.05	−0.15	−0.46	−0.66	−0.26
		[3.59]	[2.56]	[−0.80]	[−1.95]	[−4.87]	[−5.24]	[−1.20]
Controlling for Size		0.36	0.22	0.02	−0.15	−0.53	−0.90	−0.45
		[6.54]	[4.03]	[0.41]	[−2.21]	[−5.96]	[−7.55]	[−1.98]

Table 7: Fama-Macbeth regressions of returns on idiosyncratic volatility and $\beta_{\Delta VIX}$

Description: This table presents average regression slopes and their t-statistics from monthly cross-sectional regressions of returns on $\beta_{\Delta VIX}$, idiosyncratic volatility relative to the FF3 model (IV^{FF3}), and control variables. Control variables include: the natural logarithm of the book-to-market ratio (Log(B/M)), market capitalization (Log(ME)), the prior 12-month excluding prior-month return ($r_{12,2}$), the prior-month return ($r_{1,1}$), operating profitability, and investment. Accounting variables come from the most recent fiscal year end that is at least six-months prior and market capitalization comes from the end of the most recent prior June. Panel A presents results over the February 1986 to December 2016 sample for which $\beta_{\Delta VIX}$ exists. Panel B presents results for the July 1963 to December 2016 period. Columns labelled "Excluding Microcaps" exclude stocks whose market capitalization is below the 20th NYSE percentile. Variables in each regression are trimmed at the 1st and 99th percentiles based on all explanatory variables so the sample is constant across columns. t-statistics are below estimates in brackets.

Interpretation: Even when excluding microcaps, $\beta_{\Delta VIX}$ and IV^{FF3} negatively predict the cross-section of stock returns controlling for each other and other predictors.

Panel A: 1986 to 2016

	All stocks	All stocks	All stocks	Excluding microcaps	Excluding microcaps	Excluding microcaps
Log(ME)	−0.04	−0.10	−0.10	−0.04	−0.08	−0.08
	[−0.86]	[−2.55]	[−2.80]	[−0.70]	[−1.55]	[−1.90]
Log(B/M)	0.29	0.26	0.25	0.12	0.06	0.14
	[3.26]	[3.18]	[3.99]	[1.19]	[0.73]	[1.97]
$\beta_{\Delta VIX}$	−10.00	−8.37	−8.44	−16.55	−15.76	−15.55
	[−2.25]	[−2.03]	[−2.11]	[−2.32]	[−2.51]	[−2.74]
IV^{FF3}		−17.85	−10.46		−17.02	−11.33
		[−3.68]	[−2.73]		[−2.34]	[−1.91]
Operating Profitability			3.74			2.12
			[7.58]			[3.27]
Investment			−0.67			−0.32
			[−8.04]			[−3.59]
$r_{12,2}$			0.31			0.30
			[1.59]			[1.40]
$r_{1,1}$			−3.47			−1.87
			[−7.39]			[−3.52]
Constant	1.56	2.26	1.99	1.50	1.95	1.85
	[3.80]	[8.12]	[7.38]	[2.85]	[4.58]	[4.55]
R^2	0.02	0.03	0.04	0.03	0.04	0.07

Panel B: 1963 to 2016

	All stocks	All stocks	Excluding microcaps	Excluding microcaps
Log(ME)	−0.12	−0.11	−0.10	−0.09
	[−3.54]	[−3.33]	[−2.45]	[−2.44]
Log(B/M)	0.30	0.43	0.13	0.32
	[4.10]	[7.35]	[1.62]	[4.76]
IV^{FF3}	−23.68	−9.99	−22.62	−12.06
	[−6.00]	[−2.99]	[−3.76]	[−2.42]
Operating Profitability		10.37		7.84
		[13.04]		[8.67]
Investment		−0.75		−0.49
		[−8.08]		[−3.45]
$r_{12,2}$		0.54		0.58
		[3.41]		[3.22]
$r_{1,1}$		−5.28		−3.26
		[−12.66]		[−7.12]
Constant	2.43	1.82	2.17	1.74
	[9.67]	[7.57]	[6.21]	[5.23]
R^2	0.03	0.05	0.04	0.07

Table 8: Alphas with respect to recent asset-pricing models of long-short portfolios sorted on idiosyncratic volatility relative to the Fama-French three-factor model

Description: This table presents alphas and factor loadings with respect to different asset-pricing models, whose factors are defined by the row headings. The left-hand-side return is that of the high-minus-low-idiosyncratic volatility-quintile ("$5-1$") portfolio defined in Table 5. FF5 denotes the Fama and French (2015) five-factor model. FF6 denotes the FF5 model augmented with MOM. HXZ4 denotes the Hou et al. (2015) four-factor model. SY4 denotes the Stambaugh and Yuan (2017) four-factor model. BS6 denotes the Barillas and Shanken (2018) six-factor model. Newey and West (1987) t-statistics based on three lags are in brackets. In Panel A, the sample period is January 1967 to December 2016. In Panel B, the sample period is January 1967 to December 2000, which overlaps with the Ang et al. (2006) sample. In Panel C, the sample period is January 2001 to December 2016.

Interpretation: The Stambaugh and Yuan (2017) and Barillas and Shanken (2018) models explain the abnormal returns of the $5-1$ portfolio over the extended sample period. However, the alphas of the portfolio are significant for every model during the period that overlaps with the Ang et al. (2006) sample, January 1967 to December 2000.

	Mean	CAPM	FF3	FF5	FF6	HXZ4	SY4	BS6
	Panel A: 1967 to 2016							
Alpha	−0.87	−1.26	−1.30	−0.65	−0.46	−0.36	−0.33	−0.21
	[−2.88]	[−5.05]	[−7.08]	[−3.88]	[−2.66]	[−1.91]	[−1.45]	[−1.01]
MKT		0.75	0.43	0.26	0.22	0.32	0.09	0.30
		[8.36]	[5.53]	[4.72]	[4.80]	[5.56]	[1.41]	[5.66]
SMB			1.37					
			[17.74]					
$SMB5$				1.03	1.04			1.00
				[14.34]	[16.13]			[12.77]
$SMB(SY)$							1.02	
							[7.89]	
ME						0.96		
						[10.73]		
HML			−0.24	−0.08	−0.23			
			[−1.66]	[−0.61]	[−2.44]			
$HML(m)$								−0.45
								[−3.51]
MOM					−0.27			−0.19
					[−3.31]			[−1.77]
RMW				−1.34	−1.27			
				[−9.88]	[−11.95]			
ROE						−1.11		−1.17
						[−8.16]		[−8.95]
CMA				−0.73	−0.60			
				[−4.23]	[−4.51]			
IA						−0.89		−0.40
						[−5.67]		[−3.06]
$MGMT$							−1.04	
							[−8.86]	
$PERF$							−0.59	
							[−6.16]	
Adj-R^2	0.00	0.23	0.59	0.73	0.76	0.69	0.64	0.71
	Panel B: 1967 to 2000							
Alpha	−1.20	−1.53	−1.47	−1.00	−0.99	−0.72	−0.90	−0.72
	[−3.34]	[−4.78]	[−6.92]	[−5.01]	[−4.55]	[−2.67]	[−2.86]	[−2.63]
	Panel C: 2001 to 2016							
Alpha	−0.17	−0.69	−0.81	0.12	0.02	−0.02	0.33	−0.08
	[−0.30]	[−1.83]	[−2.64]	[0.40]	[0.08]	[−0.08]	[1.02]	[−0.31]

Table 9: Alphas of long-short portfolios based on idiosyncratic volatility relative to recent asset-pricing models

Description: At the end of each month, we sort stocks into value-weighted quintile portfolios based on sorts on idiosyncratic volatility estimated using daily data during the month relative to the model defined by the panel headings. This table presents average returns and alphas of the resulting $5-1$ portfolio with respect to the models defined by the column headings. The models used to evaluate returns include the model specified by the panel heading in columns labeled "Own Model," along with the SY4 and BS6 in the remaining columns. The latter two models are chosen because they perform the best in pricing portfolios sorted on IV^{FF3} in Table 8. In each panel, we estimate alphas over three sample periods specified above the column headings. Newey and West (1987) t-statistics based on three lags are in brackets.

Interpretation: No $5-1$ idiosyncratic volatility portfolio earns a significant BS6 alpha over the extended sample period. However, the alphas of the $5-1$ portfolios are significant for every model during the period that overlaps with the Ang et al. (2006) sample, February 1967 to December 2000.

1967 to 2016				1967 to 2000				2001 to 2016			
Mean	Own Model Alpha	SY4 Alpha	BS6 Alpha	Mean	Own Model Alpha	SY4 Alpha	BS6 Alpha	Mean	Own Model Alpha	SY4 Alpha	BS6 Alpha
Panel A: FF5 Model											
−0.90	−0.69	−0.39	−0.28	−1.17	−1.00	−0.88	−0.72	−0.32	−0.07	0.14	−0.28
[−3.08]	[−4.35]	[−1.73]	[−1.37]	[−3.32]	[−5.80]	[−2.96]	[−2.96]	[−0.59]	[−0.23]	[0.48]	[−1.10]
Panel B: HXZ4 Model											
−0.93	−0.45	−0.41	−0.33	−1.20	−0.74	−0.85	−0.70	−0.37	−0.29	0.06	−0.40
[−3.14]	[−2.31]	[−1.80]	[−1.50]	[−3.40]	[−2.88]	[−2.78]	[−2.69]	[−0.66]	[−1.02]	[0.20]	[−1.31]
Panel C: SY4 Model											
−0.91		−0.39	−0.26	−1.24		−0.93	−0.77	−0.19		0.27	−0.11
[−3.04]		[−1.72]	[−1.29]	[−3.50]		[−2.91]	[−2.83]	[−0.35]		[0.87]	[−0.44]
Panel D: BS6 Model											
−0.88		−0.39	−0.32	−1.18		−0.87	−0.74	−0.22		0.20	−0.26
[−3.08]		[−1.71]	[−1.48]	[−3.39]		[−2.89]	[−2.86]	[−0.45]		[0.65]	[−0.97]

Table 10: Alphas of portfolios sorted on idiosyncratic volatility and the mispricing measure of Stambaugh et al. (2015)

Description: At the end of each month, we independently sort stocks into quintiles based on sorts on idiosyncratic volatility estimated using daily data during the month, as well as the mispricing measure of Stambaugh et al. (2015) that is updated by Stambaugh and Yuan (2017). The idiosyncratic volatility is estimated relative to the model defined by the panel headings. We form 25 value-weighted portfolios based on the intersection of these two quintile sorts. This table presents alphas with respect to the model corresponding to the panel for each of the 25 IV/mispricing portfolios. The row "$5-1$" refers to the difference in monthly returns between those in the quintile-5 and quintile-1 idiosyncratic volatility portfolios in the column. The column $5-1$ refers to the difference in monthly returns between those in the quintile-5 ("Most Overpriced") and quintile-1 ("Most Underpriced") mispricing portfolios in the row. Asymmetry is the sum of the two alphas in the $5-1$ rows from the "Most Underpriced" and "Most Overpriced" columns. In all panels, the sample period is February 1967 to December 2016. White (1980) heteroskedasticity-robust standard errors are in brackets.

Interpretation: Using FF3, the alpha of the $5-1$ idiosyncratic volatility portfolio of the "Most Overpriced" stocks is significantly higher, in absolute value, than the alpha of the $5-1$ idiosyncratic volatility portfolio of the "Most Underpriced" stocks, as the "Asymmetry" estimate shows, which is consistent with the findings of Stambaugh et al. (2015). However, such asymmetry does not exist for idiosyncratic volatilities estimated relative to the SY4 and BS6 models.

Panel A: FF3

	Most Underpriced	2	3	4	Most Overpriced	5 − 1
Low IV	0.16	0.17	0.06	−0.09	−0.26	−0.41
	[2.33]	[2.61]	[0.72]	[−1.07]	[−2.09]	[−2.80]
2	0.38	0.12	−0.16	−0.17	−0.54	−0.91
	[4.90]	[1.58]	[−1.85]	[−2.03]	[−4.50]	[−6.32]
3	0.44	0.00	−0.06	−0.16	−0.59	−1.03
	[4.70]	[0.01]	[−0.56]	[−1.47]	[−4.23]	[−5.95]
4	0.48	0.14	−0.11	−0.38	−0.93	−1.41
	[3.92]	[1.15]	[−0.94]	[−2.99]	[−7.10]	[−7.74]
High IV	0.41	−0.11	−0.15	−0.82	−1.78	−2.19
	[2.59]	[−0.65]	[−0.93]	[−5.42]	[−11.17]	[−10.15]
5 − 1	0.25	−0.28	−0.21	−0.72	−1.52	−1.78
	[1.41]	[−1.54]	[−1.06]	[−3.98]	[−7.89]	[−7.15]
Asymmetry						−1.27
						[−4.59]

Panel B: SY4

	Most Underpriced	2	3	4	Most Overpriced	5 − 1
Low IV	−0.22	−0.05	0.01	0.00	0.04	0.26
	[−3.23]	[−0.64]	[0.09]	[0.01]	[0.34]	[1.84]
2	0.05	0.00	−0.11	−0.05	−0.08	−0.13
	[0.57]	[0.04]	[−1.13]	[−0.48]	[−0.65]	[−0.89]
3	0.12	0.16	0.09	0.05	−0.07	−0.19
	[1.20]	[1.58]	[0.68]	[0.48]	[−0.49]	[−1.20]
4	0.26	0.08	0.24	−0.01	−0.23	−0.50
	[1.99]	[0.60]	[1.90]	[−0.09]	[−1.86]	[−2.79]
High IV	0.42	−0.06	0.05	−0.33	−0.92	−1.34
	[2.31]	[−0.36]	[0.27]	[−1.85]	[−5.82]	[−6.07]
5 − 1	0.63	−0.01	0.04	−0.33	−0.96	−1.60
	[3.21]	[−0.06]	[0.17]	[−1.50]	[−4.28]	[−5.76]
Asymmetry						−0.33
						[−1.03]

Table 10: Continued

Panel C: BS6						
	Most Underpriced	2	3	4	Most Overpriced	5 − 1
Low IV	−0.11	0.01	−0.10	−0.16	−0.29	−0.19
	[−1.48]	[0.10]	[−1.14]	[−1.55]	[−2.35]	[−1.25]
2	0.07	−0.07	−0.36	−0.27	−0.44	−0.51
	[0.81]	[−0.72]	[−3.49]	[−2.76]	[−3.74]	[−3.82]
3	0.26	0.01	0.02	−0.04	−0.46	−0.72
	[2.44]	[0.08]	[0.16]	[−0.34]	[−3.33]	[−4.07]
4	0.42	0.11	0.21	−0.12	−0.33	−0.74
	[3.34]	[0.85]	[1.46]	[−0.81]	[−2.15]	[−3.57]
High IV	0.43	0.22	0.09	−0.28	−1.01	−1.44
	[2.31]	[1.08]	[0.47]	[−1.63]	[−6.14]	[−6.74]
5 − 1	0.53	0.21	0.19	−0.13	−0.72	−1.25
	[2.49]	[0.99]	[0.88]	[−0.62]	[−3.46]	[−4.84]
Asymmetry						−0.19
						[−0.56]

Appendix: Replication of additional Ang et al. (2006) results

This appendix contains in- and out-of-sample replications of the following main tables of Ang et al. (2006) that are not present in the main body of the paper:

- Table III
- Table IV
- Table V
- Table VII (parts not replicated in main body)
- Table IX
- Table X

Table A1: Ang et al. (2006) Table III: Characteristic controls for portfolios sorted on $\beta_{\Delta VIX}$

The table reports the means and standard deviations of the excess returns on the $\beta_{\Delta VIX}$ quintile portfolios characteristic matched by size and book-to-market ratios. Each month, each stock is matched with one of the Fama and French (1993) 25 size and book-to-market portfolios according to its size and book-to-market characteristics. The table reports value-weighted simple returns in excess of the characteristic-matched returns. The columns labeled "Excluding Small, Growth Firms" exclude the Fama-French portfolio containing the smallest stocks and the firms with the lowest book-to-market ratios. The row "5 − 1" refers to the difference in monthly returns between portfolio 5 and portfolio 1. Robust Newey-West (1987) t-statistics are reported in square brackets. In Panel A (B), the sample period is from February 1986 to December 2000 (January 2001 to December 2016).

Panel A: 1986 to 2000				
	All Firms		Excluding Small, Growth Firms	
Rank	Mean	Std. Dev.	Mean	Std. Dev.
1	0.28	1.83	0.23	1.80
2	0.05	1.05	0.04	1.04
3	0.07	0.98	0.12	1.03
4	−0.09	0.95	−0.17	0.95
5	−0.57	2.80	−0.50	2.74
5 − 1	−0.85		−0.74	
	[3.50]		[3.49]	

Panel B: 2001 to 2016				
	All Firms		Excluding Small, Growth Firms	
Rank	Mean	Std. Dev.	Mean	Std. Dev.
1	0.03	2.42	0.06	2.45
2	0.05	1.17	0.05	1.08
3	−0.02	0.91	−0.03	0.92
4	−0.01	1.13	−0.02	1.20
5	−0.28	3.11	−0.26	3.07
5 − 1	−0.31		−0.33	
	[3.86]		[3.87]	

Table A2: (Ang et al. (2006) Table IV) Portfolios sorted on $\beta_{\Delta VIX}$ controlling for liquidity, volume, and momentum

In Panels A and B, we first sort stocks into five quintiles based on their historical liquidity beta, following Pástor and Stambaugh (2003). Then, within each quintile, we sort stocks based on their $\beta_{\Delta VIX}$ loadings into five portfolios. All portfolios are rebalanced monthly and are value weighted. The five portfolios sorted on $\beta_{\Delta VIX}$ are then averaged over each of the five liquidity beta portfolios. Hence, they are $\beta_{\Delta VIX}$ quintile portfolios controlling for liquidity. In Panels C and D (E and F), the same approach is used except we control for average dollar trading volume over the past month (past 12-month returns). The statistics in the columns labeled Mean and Std. Dev. are measured in monthly percentage terms and apply to total, not excess, returns. The table also reports CAPM and FF3 alphas. The row "5 − 1" refers to the difference in monthly returns between portfolio 5 and portfolio 1. The pre-formation betas refer to the value-weighted $\beta_{\Delta VIX}$ within each quintile portfolio at the start of the month. We report the pre-formation $\beta_{\Delta VIX}$ averaged across the whole sample. The last column reports ex post β_{FVIX} factor loadings over the whole sample, where $FVIX$ is the factor mimicking aggregate-volatility risk. To correspond with the Fama and French (1993) alphas, we compute the ex post betas by running a four-factor regression with the three Fama and French (1993) factors together with the $FVIX$ factor, following the regression in equation (6). Robust Newey and West (1987) t-statistics are reported in square brackets. In Panels A, C, and E (B, D, and F) the sample period is from February 1986 to December 2000 (January 2001 to December 2016).

Rank	Mean	Std. Dev.	CAPM Alpha	FF3 Alpha	Pre-formation $\beta_{\Delta VIX}$ Loading	Post-formation β_{FVIX} Loading
Panel A: Controlling for Liquidity, 1986 to 2000						
1	1.57	5.56	0.17	0.14	−1.27	−4.33
			[1.04]	[0.94]		[−1.78]
2	1.43	4.60	0.17	0.10	−0.42	−7.61
			[1.62]	[1.09]		[−3.98]
3	1.39	4.59	0.12	0.03	0.03	−1.57
			[1.27]	[0.43]		[−1.13]
4	1.34	4.86	0.02	0.00	0.48	1.42
			[0.25]	[0.02]		[0.81]
5	0.73	5.95	−0.69	−0.52	1.37	13.42
			[−3.59]	[−2.81]		[5.10]
5 − 1	−0.84		−0.86	−0.66		
	[−3.53]		[−3.30]	[−2.45]		

Table A2: Continued

Rank	Mean	Std. Dev.	CAPM Alpha	FF3 Alpha	Pre-formation $\beta_{\Delta VIX}$ Loading	Post-formation β_{FVIX} Loading
Panel B: Controlling for Liquidity, 2001 to 2016						
1	0.66	5.52	−0.01	−0.05	−1.03	7.29
			[−0.05]	[−0.34]		[1.10]
2	0.66	4.30	0.09	0.08	−0.32	−3.42
			[1.50]	[1.40]		[−0.84]
3	0.66	4.31	0.09	0.06	0.05	7.12
			[1.27]	[0.95]		[1.61]
4	0.68	4.95	0.04	0.00	0.44	−2.50
			[0.53]	[−0.05]		[−0.65]
5	0.61	6.82	−0.19	−0.29	1.21	21.69
			[−1.19]	[−1.96]		[1.67]
5 − 1	−0.05		−0.18	−0.25		
	[−0.24]		[−0.89]	[−1.20]		
Panel C: Controlling for Volume, 1986 to 2000						
1	1.00	5.26	−0.27	−0.27	−1.70	−7.21
			[−1.10]	[−2.11]		[−2.34]
2	1.09	4.11	−0.02	−0.16	−0.48	−9.00
			[−0.12]	[−1.38]		[−4.94]
3	1.04	3.81	−0.03	−0.17	0.03	−7.06
			[−0.15]	[−1.54]		[−5.12]
4	0.91	4.23	−0.24	−0.32	0.55	−2.52
			[−1.21]	[−2.85]		[−1.46]
5	0.22	5.56	−1.08	−0.91	1.81	5.49
			[−3.74]	[−5.53]		[2.18]
5 − 1	−0.78		−0.81	−0.64		
	[−4.49]		[−4.03]	[−3.82]		
Panel D: Controlling for Volume, 2001 to 2016						
1	0.81	5.86	0.12	−0.06	−1.49	1.74
			[0.64]	[−0.51]		[0.26]
2	0.96	4.47	0.39	0.22	−0.36	1.06
			[2.82]	[2.90]		[0.34]
3	1.01	4.29	0.47	0.29	0.05	1.04
			[3.16]	[3.72]		[0.33]
4	0.93	4.71	0.34	0.15	0.47	4.44
			[2.40]	[2.02]		[1.17]
5	0.80	6.33	0.07	−0.17	1.48	19.61
			[0.34]	[−1.42]		[2.08]
5 − 1	−0.01		−0.05	−0.10		
	[−0.06]		[−0.42]	[−0.83]		

Table A2: Continued

Rank	Mean	Std. Dev.	CAPM Alpha	FF3 Alpha	Pre-formation $\beta_{\Delta VIX}$ Loading	Post-formation β_{FVIX} Loading
Panel E: Controlling for Past 12-Month Returns, 1986 to 2000						
1	1.17	5.54	−0.22	−0.28	−1.42	−1.09
			[−1.27]	[−1.95]		[−0.42]
2	1.20	4.74	−0.06	−0.19	−0.46	−1.05
			[−0.47]	[−1.61]		[−0.44]
3	1.16	4.70	−0.11	−0.22	0.03	3.70
			[−0.83]	[−1.91]		[1.39]
4	1.09	4.86	−0.20	−0.30	0.53	8.72
			[−1.43]	[−2.25]		[3.42]
5	0.39	5.79	−1.02	−0.88	1.51	11.71
			[−5.01]	[−4.76]		[4.07]
5 − 1	−0.78		−0.80	−0.60		
	[−4.12]		[−3.69]	[−2.96]		
Panel F: Controlling for Past 12-Month Returns, 2001 to 2016						
1	0.61	6.27	−0.12	−0.16	−1.11	−2.52
			[−0.69]	[−1.01]		[−0.23]
2	0.67	5.20	0.02	−0.02	−0.35	5.20
			[0.16]	[−0.16]		[0.71]
3	0.64	5.03	0.00	−0.03	0.06	7.56
			[−0.05]	[−0.38]		[0.90]
4	0.66	5.61	−0.03	−0.12	0.47	11.45
			[−0.26]	[−1.05]		[0.96]
5	0.48	6.82	−0.32	−0.45	1.28	21.09
			[−1.92]	[−2.98]		[1.37]
5 − 1	−0.14		−0.19	−0.29		
	[−0.58]		[−0.81]	[−1.25]		

Table A3: (Ang et al. (2006) Table V) Estimating the price of volatility risk

Panels A and C report the Fama and MacBeth (1973) factor premiums on 25 portfolios sorted first on β_{MKT} and then on $\beta_{\Delta VIX}$. MKT is the excess return on the market portfolio, $FVIX$ is the mimicking factor for aggregate volatility innovations, SMB and HML are the Fama and French (1993) size and value factors, MOM is the momentum factor constructed by Kenneth French, and LIQ is the aggregate liquidity measure from Pástor and Stambaugh (2003). In Panels B and D, we report ex post factor loadings on $FVIX$, from the specification in the first column (FF3 model plus $FVIX$). Shanken (1992) t-statistics that account for the errors-in-variables for the first-stage estimation in the factor loadings are reported in square brackets. In Panels A and B (C and D) the sample period is from February 1986 to December 2000 (January 2001 to December 2016).

Panel A: Fama-MacBeth (1973) Factor Premiums, 1986 to 2000

Constant	0.53	0.47	0.61
	[0.80]	[0.69]	[0.91]
MKT	0.39	0.46	0.27
	[0.54]	[0.63]	[0.37]
FVIX	−2.03	−2.40	−1.60
	[−1.45]	[−1.70]	[−1.15]
SMB	−0.74	−0.64	−0.75
	[−2.07]	[−1.79]	[−2.07]
HML	−0.59	−0.44	−0.38
	[−1.35]	[−0.99]	[−0.86]
MOM		1.01	0.69
		[1.33]	[0.89]
LIQ			−0.01
			[−0.81]
R^2	0.60	0.61	0.64

Panel B: Ex Post Factor Loadings on FVIX, 1986 to 2000

Pre-ranking	Pre-ranking $\beta_{\Delta VIX}$				
β_{MKT}	1 Low	2	3	4	5 High
Low 1	−0.04	−0.09	−0.05	−0.08	−0.04
	[−0.55]	[−1.78]	[−0.83]	[−1.77]	[−0.59]
2	−0.11	−0.13	−0.10	0.00	0.00
	[−2.09]	[−3.51]	[−2.84]	[−0.05]	[−0.05]
3	−0.14	−0.04	0.00	0.00	0.01
	[−3.33]	[−0.99]	[0.12]	[0.12]	[0.24]
4	−0.02	−0.03	−0.03	0.09	0.22
	[−0.36]	[−0.83]	[−0.96]	[3.05]	[4.43]
High 5	0.06	0.00	0.13	0.22	0.15
	[0.79]	[0.05]	[2.37]	[4.08]	[1.97]

Table A3: Continued

Panel C: Panel A: Fama-MacBeth (1973) Factor Premiums, 2001 to 2016			
Constant	1.00	1.04	1.05
	[2.65]	[2.87]	[2.79]
MKT	−0.41	−0.45	−0.45
	[−0.83]	[−0.91]	[−0.90]
FVIX	0.61	0.63	0.63
	[0.94]	[1.00]	[1.02]
SMB	−0.07	−0.12	−0.12
	[−0.14]	[−0.22]	[−0.23]
HML	−0.02	−0.02	−0.02
	[−0.03]	[−0.04]	[−0.04]
MOM		0.00	−0.01
		[0.00]	[−0.02]
LIQ			0.00
			[0.05]
R^2	0.54	0.54	0.54

Panel D: Ex Post Factor Loadings on FVIX, 2001 to 2016					
Pre-ranking	Pre-ranking $\beta_{\Delta VIX}$				
β_{MKT}	1 Low	2	3	4	5 High
Low 1	0.10	0.10	−0.06	0.02	0.03
	[0.71]	[0.85]	[−0.61]	[0.16]	[0.21]
2	0.03	−0.12	−0.06	−0.15	0.11
	[0.34]	[−1.75]	[−0.84]	[−1.92]	[1.22]
3	0.08	−0.07	0.02	0.00	0.08
	[0.93]	[−1.06]	[0.41]	[0.05]	[0.97]
4	0.10	0.07	0.17	0.02	0.31
	[1.07]	[0.93]	[2.18]	[0.28]	[2.58]
High 5	−0.10	−0.06	0.12	0.10	0.34
	[−0.63]	[−0.44]	[1.12]	[0.78]	[1.67]

Table A4: (Ang et al. (2006) Table VII: Non-size-related parts)

The table reports Fama and French (1993) alphas, with robust Newey and West (1987) t-statistics in square brackets, for idiosyncratic volatility-quintile portfolios that control for various characteristics. The column 5 − 1 refers to the difference in FF3 alphas between portfolio 5 and portfolio 1. We use daily data over the previous month and rebalance monthly. To form the strategies, each month, we first sort stocks based on the first characteristic (book-to-market, leverage, liquidity, volume, turnover, bid-ask spreads, or dispersion of analysts' forecasts) and then, within each quintile we sort stocks based on idiosyncratic volatility relative to the FF3 model. The five idiosyncratic volatility portfolios are then averaged over each of the five characteristic portfolios. Hence, they represent idiosyncratic volatility quintile portfolios controlling for the characteristic. Liquidity represents the Pástor and Stambaugh (2003) historical liquidity beta, leverage is defined as the ratio of total book value of assets to book value of equity, volume represents average dollar volume over the previous month, turnover represents volume divided by the total number of shares outstanding over the past month, and the bid-ask spread control represents the average daily bid-ask spread over the previous month. The coskewness measure is computed following Harvey and Siddique (2000) and the dispersion of analysts' forecasts is computed by Diether, Malloy, and Scherbina (2002). In Panel A, the sample period is July 1963 to December 2000 for all controls with the exceptions of liquidity (February 1968 to December 2000), the dispersion of analysts' forecasts (February 1983 to December 2000), and the bid-ask spread (January 1992 to December 2000). All portfolios are value weighted.

Panel A: 1963 to 2000						
	Ranking on Idiosyncratic Volatility					
	1 Low	2	3	4	5 High	5 − 1
Controlling for BM	0.00	0.03	0.04	−0.28	−1.02	−1.02
	[0.03]	[0.46]	[0.33]	[−2.82]	[−7.56]	[−6.37]
Controlling for Leverage	0.07	0.12	0.08	−0.34	−1.12	−1.20
	[1.63]	[2.07]	[0.99]	[−3.34]	[−7.61]	[−7.23]
Controlling for Liquidity	0.12	0.07	−0.06	−0.19	−1.14	−1.26
	[2.67]	[1.11]	[−0.60]	[−1.97]	[−9.39]	[−8.81]
Controlling for Volume	−0.04	0.01	−0.09	−0.38	−1.23	−1.19
	[−0.85]	[0.13]	[−1.42]	[−4.70]	[−8.81]	[−6.29]
Controlling for Turnover	0.15	0.05	−0.07	−0.53	−1.33	−1.48
	[2.85]	[0.95]	[−0.99]	[−6.29]	[−9.25]	[−9.45]
Controlling for BidAskSpread	0.09	−0.29	−0.45	−0.86	−1.56	−1.64
	[0.28]	[−1.44]	[−2.66]	[−3.40]	[−5.02]	[−4.51]
Controlling for Coskewness	0.12	0.10	0.11	−0.24	−1.06	−1.18
	[2.35]	[1.62]	[1.49]	[−2.70]	[−8.93]	[−8.15]
Controlling for Dispersion	−0.05	0.04	0.11	−0.09	−0.58	−0.53
	[−0.76]	[0.33]	[0.77]	[−0.80]	[−2.52]	[−2.14]

Table A4: Continued

Panel B: 2001 to 2016						
	Ranking on Idiosyncratic Volatility					
	1 Low	2	3	4	5 High	5 − 1
Controlling for BM	−0.38	−0.49	−0.56	−0.72	−0.84	−0.46
	[1.82]	[−0.15]	[−0.73]	[−1.21]	[−1.42]	[−1.58]
Controlling for Leverage	−0.40	−0.57	−0.61	−0.78	−0.98	−0.58
	[1.49]	[−1.21]	[−1.48]	[−1.97]	[−1.94]	[−2.01]
Controlling for Liquidity	−0.33	−0.50	−0.67	−0.76	−0.91	−0.58
	[2.42]	[−0.37]	[−2.36]	[−1.71]	[−1.70]	[−2.00]
Controlling for Volume	0.44	0.24	0.09	−0.11	−0.77	−1.21
	[6.12]	[2.96]	[1.04]	[−0.89]	[−3.55]	[−4.76]
Controlling for Turnover	0.15	0.00	−0.07	−0.21	−0.76	−0.90
	[2.40]	[0.16]	[−0.89]	[−1.66]	[−3.41]	[−3.66]
Controlling for BidAskSpread	0.27	0.19	−0.18	−0.25	−0.91	−1.17
	[4.12]	[2.26]	[−1.24]	[−1.30]	[−4.08]	[−4.85]
Controlling for Coskewness	0.15	−0.08	−0.12	−0.22	−0.51	−0.66
	[2.75]	[−1.04]	[−0.92]	[−1.20]	[−1.74]	[−2.07]
Controlling for Dispersion	−0.03	−0.01	−0.34	−0.34	−0.42	−0.38
	[−0.63]	[−0.14]	[−2.88]	[−2.11]	[−2.29]	[−1.88]

Table A5: (Ang et al. (2006) Table IX) The idiosyncratic volatility effect controlling for aggregate volatility risk

We control for exposure to aggregate volatility using the $\beta_{\Delta VIX}$ loading at the beginning of the month, computed using daily data over the previous month following equation (3). We first sort all stocks on the basis of $\beta_{\Delta VIX}$ into quintiles. Then, within each$\beta_{\Delta VIX}$ quintile, we sort stocks into five portfolios sorted by idiosyncratic volatility, relative to the FF3 model. In Panels A and C, we report FF3 alphas of these portfolios. We average the five idiosyncratic volatility portfolios over each of the five $\beta_{\Delta VIX}$ portfolios. Hence, these portfolios represent idiosyncratic volatility quintile portfolios controlling for exposure to aggregate-volatility risk. The column $5-1$ refers to the difference in FF3 alphas between portfolio 5 and portfolio 1. In Panels B and D, we report ex post $FVIX$ factor loadings from a regression of each of the 25 $\beta_{\Delta VIX}$/idiosyncratic volatility portfolios onto the Fama and French (1993) model augmented with $FVIX$ as in equation (6). Robust Newey and West (1987) t-statistics using three lags are reported in square brackets. All portfolios are value weighted. In Panels A and B (C and D) The sample period is from February 1986 to December 2000 (January 2001 to December 2016).

Panel A: FF3 Alphas, 1986 to 2000

	Ranking on Idiosyncratic Volatility					
	1 Low	2	3	4	5 High	$5-1$
Controlling for Exposure to	0.06	−0.05	−0.18	−0.39	−1.23	−1.29
Aggregate-Volatility Risk	[0.86]	[−0.63]	[−1.49]	[−2.36]	[−4.47]	[−4.02]

Panel B: FVIX Factor Loadings, 1986 to 2000

		Ranking on Idiosyncratic Volatility				
		1 Low	2	3	4	5 High
$\beta_{\Delta VIX}$ Quintiles	Low 1	−13.03	−2.96	2.11	14.74	10.52
		[−4.21]	[−0.66]	[0.45]	[1.80]	[1.07]
	2	−5.51	−7.01	0.19	−7.31	1.95
		[−2.33]	[−2.56]	[0.07]	[−1.66]	[0.16]
	3	−4.13	−2.67	−0.75	−0.28	10.46
		[−1.63]	[−1.02]	[−0.22]	[−0.10]	[1.96]
	4	5.32	7.88	5.12	3.33	17.03
		[2.07]	[3.23]	[1.71]	[0.75]	[3.62]
	High 5	14.77	5.92	19.87	11.68	3.92
		[4.21]	[1.39]	[3.78]	[1.80]	[0.42]

Table A5: Continued

Panel C: FF3 Alphas, 2001 to 2016						
	Ranking on Idiosyncratic Volatility					
	1 Low	2	3	4	5 High	5 − 1
Controlling for Exposure to Aggregate-Volatility Risk	0.04	0.01	−0.23	−0.41	−0.62	−0.66
	[0.77]	[0.10]	[−2.03]	[−2.67]	[−2.58]	[−2.52]

Panel D: FVIX Factor Loadings, 2001 to 2016						
		Ranking on Idiosyncratic Volatility				
		1 Low	2	3	4	5 High
$\beta_{\Delta VIX}$ Quintiles	Low 1	1.47	12.99	17.83	−15.75	8.38
		[0.19]	[0.83]	[0.81]	[−0.69]	[0.24]
	2	−2.95	−8.18	0.92	11.81	26.22
		[−0.52]	[−1.03]	[0.10]	[0.86]	[1.09]
	3	2.59	1.40	14.44	11.71	10.59
		[0.45]	[0.22]	[1.51]	[0.86]	[0.62]
	4	−1.64	4.86	2.61	17.22	32.57
		[−0.23]	[0.86]	[0.29]	[0.63]	[1.24]
	High 5	3.96	16.04	49.00	64.14	−14.12
		[0.44]	[1.40]	[1.49]	[1.38]	[−0.56]

Table A6: (Ang et al. (2006) Table X) Quintile portfolios of idiosyncratic volatility for $L/M/N$ strategies

The table reports Fama and French (1993) three-factor alphas, with robust Newey and West (1987) t-statistics using three lags in square brackets. The column $5-1$ refers to the difference in FF3 alphas between portfolio 5 and portfolio 1. We rank stocks into quintile portfolios of idiosyncratic volatility, relative to FF3, using $L/M/N$ strategies, which are defined as follows. At month t, we compute idiosyncratic volatilities from the regression (7) using daily data over an L month period from months $t-L-M$ to month $t-M$. At time t, we construct value-weighted portfolios based on these idiosyncratic volatilities and hold these portfolios for N months. In Panel A (B) The sample period is July 1963 to December 2000 (January 2001 to December 2016).

Panel A: 1963 to 2000						
	Ranking on Idiosyncratic Volatility					
Strategy	1 Low	2	3	4	5 High	$5-1$
1/1/1	0.04	0.00	0.05	−0.18	−0.89	−0.93
	[1.23]	[0.08]	[0.65]	[−1.85]	[−5.04]	[−4.72]
1/1/12	0.04	0.01	−0.01	−0.10	−0.57	−0.61
	[1.06]	[0.36]	[−0.12]	[−1.12]	[−4.22]	[−3.83]
12/1/1	0.03	0.03	−0.04	−0.31	−1.04	−1.07
	[0.96]	[0.59]	[−0.41]	[−2.33]	[−5.31]	[−4.97]
12/1/12	0.03	0.05	0.01	−0.20	−0.67	−0.70
	[0.99]	[0.90]	[0.09]	[−1.69]	[−3.57]	[−3.36]
Panel B: 2001 to 2016						
	Ranking on Idiosyncratic Volatility					
Strategy	1 Low	2	3	4	5 High	$5-1$
1/1/1	0.11	0.09	−0.16	−0.54	−1.16	−1.27
	[2.29]	[1.44]	[−1.27]	[−2.74]	[−4.48]	[−4.57]
1/1/12	0.10	0.00	−0.14	−0.32	−0.60	−0.71
	[2.33]	[−0.03]	[−1.58]	[−2.21]	[−3.01]	[−3.07]
12/1/1	0.10	−0.02	−0.21	−0.66	−0.85	−0.95
	[2.07]	[−0.27]	[−1.62]	[−3.04]	[−2.38]	[−2.48]
12/1/12	0.09	−0.02	−0.15	−0.49	−0.79	−0.88
	[1.84]	[−0.21]	[−1.04]	[−2.58]	[−2.98]	[−2.98]