

Understanding the Performance of Components in Betting Against Beta

Xing Han *

First version: 30 June 2017 This version: 7 November 2019

Abstract

Betting against beta (BAB) can be seen as the combination of three investable component portfolios: Two cross-sectional components exploiting the beta anomaly attributable to stock selection and rank weighting scheme, and one time-series component with a *dynamic* net-long position due to “beta-parity”. Virtually all superior performance of BAB stems from the time-series component. The two cross-sectional components only provide hedging benefits in market downturns. The time-series component has modest portfolio turnover. Betting against correlation (BAC) yields similar findings, except that the two cross-sectional components in BAC outperform on a risk-adjusted basis. However, this effect arises purely from the positive association between firm size and stock correlation. Excluding micro-cap stocks, the performance of BAC shrinks more than that of BAB. Overall, only the time-series component remains as the robust source for the profits of the BA-type strategies.

JEL Classification: G11, G12

Keywords: Beta Anomaly, Return Decomposition, Betting Against Correlation, Asset Pricing

* Corresponding author: Department of Accountancy and Finance, University of Otago, 60 Clyde Street, Dunedin 9016, New Zealand. Tel.: +64-(0)3 479 9066. Fax: +64 (0)3 479 8171. E-mail: xing.han@otago.ac.nz (X. Han).

The work benefits from the suggestions and comments from Youwei Li, Lukas Menkhoff, Zheyao Pan, Ivo Welch (the Editor), Jing Zhao and seminar participants at Central University of Finance and Economics (Beijing) and University of Auckland (Auckland). The author is grateful to Kenneth French, Robert Stambaugh, Jeffrey Wurgler, and Lu Zhang for making their research data available.

The author is aware of the concurrent work by Robert Novy-Marx and Mihail Velikov, but executes the empirical analysis independently and without being influenced by theirs. All remaining errors are the author’s responsibility.

1. Introduction

The low-beta anomaly refers to the phenomenon in finance that securities with lower-than-average market beta outperform securities with higher-than-average market beta on a risk-adjusted basis ([Friend & Blume 1970](#); [Haugen & Heins 1975](#); [Baker et al. 2011](#)). This well-established empirical fact, however, goes counter to the central prediction of the [Sharpe \(1964\)](#) and [Lintner \(1965\)](#) capital asset pricing model (*i.e.*, risk-return trade-off). In view of this, [Frazzini and Pedersen \(2014\)](#) proposed a market-neutral betting against beta (BAB) strategy, which takes a levered long position on low-beta stocks and a deleveraged short position on high-beta counterparts. It has also spurred debates among economists: Various alternative explanations have been put forward in the literature, which includes, among others, leverage aversion, lottery demand, margin and arbitrage constraints, speculation and divergence of opinion, benchmark as limits to arbitrage, sentiment, overconfidence, time-varying of risk exposure, and latent factor ([Black 1972](#); [Baker et al. 2011](#); [Frazzini & Pedersen 2014](#); [Antoniou et al. 2015](#); [Schneider et al. 2015](#); [Cederburg & O'Doherty 2016](#); [Hong & Sraer 2016](#); [Bali et al. 2017](#); [Liu et al. 2018](#)).

To disentangle the mechanisms at work and to quantify the leverage-aversion channel, [Asness et al. \(2018\)](#) decompose stock beta into a correlation and a volatility component, respectively. They argue that only the volatility component is related to the behavioural mechanism(s), and therefore, their newest betting against correlation (BAC) factor is particularly suited to differentiate between the leverage-aversion explanation ([Asness et al. 2012](#)) and the behavioural explanations ([Baker et al. 2011](#); [Antoniou et al. 2015](#); [Hong & Sraer 2016](#); [Bali et al. 2017](#); [Liu et al. 2018](#)). Empirically, [Asness et al. \(2018\)](#) document that BAC, which is highly sensitive to adverse changes in margin/leverage constraints, generates higher risk-adjusted returns than its BAB counterpart.

Despite the empirical success of the above BA-type strategies, several legitimate concerns remain. First, the exact portfolio formation procedures used in [Frazzini and Pedersen \(2014\)](#) to generate the BA-type factors seem ad hoc. It involves selecting stocks with certain attributes, adopting a rank-based portfolio weighting scheme, and applying the beta-parity approach (*i.e.*, leverage/deleverage) to ensure its market-neutral property. It is not clear which of these “optimization” procedures lead to the superior performance. Second, the portfolio formation procedures maximize the return spreads for these BA-type factors without consideration of portfolio turnover and transaction costs. It is not clear whether the superior performance of the BA-type strategies would survive transaction costs.

My study provides a critical assessment of the BA-type strategies, and their underlying mechanisms. I start the empirical analysis by replicating the popular BAB and BAC strategies in the literature (see

figure 1). The replication confirms that the BA-type strategies deliver superior performance from July 1963 to December 2016. BAC produces relatively larger (risk-adjusted) returns than its BAB counterpart. The BAB and BAC factors also do not span each other. They offer non-overlapping price-related information.

My paper derives a non-parametric performance decomposition to dissect the monthly (risk-adjusted) returns of BAB and BAC. The BA-type strategies can be viewed as the sum of three investable component portfolios: Two cross-sectional components attributed to stock selection and rank weighting scheme that are designed to explore the low-beta (low-correlation) anomaly, and one time-series component with a time-varying net-long position due to the beta-parity approach. The time-varying net-long position of the time-series component has a positive market exposure, and its performance resembles a market timing strategy. First, virtually all superior average return performance of BAB is driven by its time-series component, which has a strong positive market exposure. Second, the time variation of the two cross-sectional components (capturing the pure low-beta anomaly) seems to be fully captured by the conventional factors (such as the market, size, profitability, and momentum factors). They generate little, if any, (risk-adjusted) return. However, the two cross-sectional components provide hedging benefits in market downturns (*i.e.*, NBER recession). In that sense, the two cross-sectional components serve as return stabilizers for the BAB strategy. Third, all three component portfolios contribute to the superior average return performance of BAC. The two cross-sectional components (due to stock selection and rank-based weighting scheme) contribute more than half of the risk-adjusted returns to BAC. This implies that stock correlation is a stronger return determinant than its beta counterpart in the cross section.

The decomposition approach also helps assess whether the components of BAB (BAC) can cover their transaction costs. The two cross-sectional components of BAB (BAC) have relatively high portfolio turnover of 183% and 189% (151% and 151%) per annum. In contrast, the time-series component of BAB (BAC) has low annualized portfolio turnover of 76% (48%). The time-series component could withstand implausibly large transaction costs before losing its (risk-adjusted) returns.¹ Therefore, the time-series component remains the most robust source for the profits of the BA-type strategies, both before and after taking transaction costs into account.

My paper also investigates the low-correlation anomaly (*i.e.*, securities with lower-than-average correlation have higher returns than securities with higher-than-average correlation), because these

¹ A back-of-the-envelope calculation of the break-even transaction costs based on the average (risk-adjusted) returns versus the portfolio turnover is consolidated in **Tables A3** and **A4** in the appendix.

two cross-sectional components of BAC contribute more than half of its risk-adjusted returns. First, I find evidence that stock correlation is a consistently strong negative return determinant at the firm level, although beta remains unpriced in the cross section. Second, I examine the firm characteristics of the low- and high-correlation stocks. Stock correlation and firm size have a pairwise correlation coefficient of 0.74. This suggests that low-correlation stocks tend to be small-cap firms, while high-correlation stocks are large-cap counterparts. This is contrary to the reasoning of the leverage-aversion theory ([Frazzini & Pedersen 2014](#)) that high-correlation stocks are (perceived) riskier than their low-correlation counterparts. Third, results from the [Hou and Loh \(2016\)](#) “horse race” point toward the same direction: The effect of stock correlation is fully captured by firm size. Therefore, the superior performance of BAC represents the higher compensation for betting *on*, rather than *against*, small firms (*i.e.*, size-related risk). Excluding the smallest firms (*i.e.*, below-median market capitalization stocks) in the investment universe, the outperformance of BAC shrinks more than its BAB counterpart.

Finally, I provide new evidence on the time-series component in the BA-type strategies. To ensure an *ex ante* market beta of zero, BAB (BAC) mechanically builds in a *time-varying* net-long position in the time-series component. However, the time-varying net-long position, by itself, contains little market timing ability. The estimated timing premium is less than 5 basis points (bps) per month. Thus, it is not the main source of the outperformance of the beta-parity component. Instead, using a tradable factor which tracks the changes in the survey-based sentiment measure, I find that investor sentiment can explain roughly 40% of the risk-adjusted return of the beta-parity component (and BAB). As a result, the (remaining) non-sentiment related return in BAB drops to approximately 20 bps per month, and it is insignificant from zero. The sentiment impact on BA-type strategies is robust under a battery of robustness checks.

The central message of my paper is that a *pure* (zero-cost) low-minus-high-beta portfolio provides some hedging benefits, but no superior average return performance. The time-series component (*i.e.*, beta-parity component) is the only robust source for the superior performance of the BA-type strategies, both before and after adjusting for risk exposures, and most of it is related to the familiar sentiment factor.

The structure of my paper is as follows. Section 2 describes the sample data and variable construction. Section 3 generates the baseline results regarding the BAB and the BAC factors. Section 4 provides our benchmark-based performance attribution framework and evaluates the risk-adjusted performance of the components of the BA-type portfolio, both before and after transaction costs. Section 5 provides a critical reassessment of the low-correlation effect using firm-level data. Section 6 presents further

analysis on the time-series component, the most robust source of the profits for BAB and BAC. Section 7 discusses the implications of the findings and concludes.

2. Data and Variable Construction

2.1. Data Sources

The stock market data and financial variables are retrieved from the CRSP/Compustat merged database, which includes all common stocks (share codes 10 and 11) that are traded on the NYSE, AMEX, and NASDAQ exchanges. The sample period spans from July 1963 to December 2016. The [Fama and French \(2015\)](#) five factors (*i.e.*, the market factor (RMRF), the size factor (SMB), the value factor (HML), the profitability factor (RMW), and the investment factor (CMA)), the momentum factor (MOM), and the short-term reversal factor (STREV) are downloaded from the Ken French Data Library. The Michigan consumer confidence index is retrieved from Datastream, the TED spread and NBER recession indicator are from the webpage of the Federal Reserve Bank of St. Louis, and the CBOE VIX index is from WRDS.²

2.2. Variable Construction

To benchmark our study against [Frazzini and Pedersen \(2014\)](#) and the “betting against correlation” strategy ([Asness *et al.* 2018](#)), I follow [Frazzini and Pedersen \(2014\)](#) to derive market beta, and its correlation and volatility components.

$$\hat{\beta}_i^{TS} = \hat{\rho}_i \times \frac{\hat{\sigma}_i}{\hat{\sigma}_m}. \quad [2.1]$$

The market beta is constructed as the product of the 60-month return correlation (with the market portfolio) and the 12-month market-adjusted volatility. [Frazzini and Pedersen \(2014\)](#) propose a past one-year rolling window of daily returns to calculate the standard deviation for the volatility component and a past five-year horizon of daily returns for the correlation component. They require at least six months (*i.e.*, 120 trading days) of non-missing data to estimate volatilities and at least three years (*i.e.*, 750 trading days) of non-missing return data for correlations.³ The market-adjusted

² To ensure a longer sample period, I use the CBOE S&P 100 volatility index (now known as VXO), which starts from 1 January 1986.

³ Note the idea to separate the estimation of correlation and volatility is not new in financial econometrics, as the two processes have different dynamics. The separation have the potential advantages such as the optimal use of available

volatility is calculated using one-day log-returns, while the correlation is constructed from overlapping three-day log-returns, $r_{i,t}^{3d} = \sum_{k=0}^2 \ln(1 + r_{t+k}^i)$, to control for nonsynchronous trading. [Frazzini and Pedersen \(2014\)](#) then shrinks the time series estimate of beta $\hat{\beta}_i^{TS}$ toward the cross-sectional mean β^{XS} .

$$\hat{\beta}_i = w_i \hat{\beta}_i^{TS} + (1 - w_i) \hat{\beta}^{XS} = 0.6 \times \hat{\beta}_i^{TS} + 0.4 \times 1. \quad [2.2]$$

The constructed beta, correlation, and volatility measures are used in the empirical tests on the BAB and BAC strategies in the next section.

In addition to the above variables of interest, we also calculate a number of firm characteristics and conventional risk measures, such as the log of the market capitalization (lnME), the log of the book-to-market equity ratio (lnBTM), the ratio of operational profits and book equity (OP), the growth rate of total assets (INV), the intermediate-term return momentum (RET^{MOM}), the prior-month return (RET^{STREV}), the lottery-feature (MAX5), systematic skewness (SSKEW), idiosyncratic skewness (ISKEW), idiosyncratic volatility (IVOL), Amihud illiquidity ratio (ILLIQ), and the turnover ratio (TURN). Details of the variable construction procedure is documented in **A.2. Variable Definition** in the appendix.

3. Betting Against Beta versus Betting Against Correlation

3.1. Portfolio Construction of BAB and BAC

[Frazzini and Pedersen \(2014\)](#) propose a market-neutral BAB strategy, which takes a leveraged long position in low-beta stocks and a deleveraged short position in high-beta stocks to “capitalize” on the low-risk anomaly. Based on similar portfolio construction procedures, [Asness et al. \(2018\)](#) utilize the correlation component of beta to generate the market-neutral BAC strategy in later work. In general, the BA-type factors are constructed in three separate steps:

- First, the stock selection procedure. At the beginning of each month, all available stocks are ranked by their *ex ante* attribute, either beta or correlation. Stocks with attributes below (above) the cross-sectional median are assigned to the low- (high-) attribute portfolio.

information and enhanced precision of the estimators (See [Bollerslev \(1990\)](#) and [Engle \(2002\)](#) in the application of the multivariate GARCH modelling). Despite the potential advantages, in a recent study [Welch \(2019\)](#) points out that the [Frazzini and Pedersen \(2014\)](#) beta performs relatively poorly in predicting future market beta (calculated from non-overlapping time-series data) among alternative beta measures.

- Second, the rank-based weighting scheme. The portfolio weights are determined by their rankings of the sorting variable (*i.e.*, the stock attribute): Relatively lower- (higher-) ranked stocks in the low- (high-) attribute portfolio are given higher portfolio weights. Analytically, the rank-based weighting scheme for the low- (high-) risk portfolio is expressed as follows:

$$\mathbf{w}_{L(H)}^{RW} = k(\mathbf{z} - \bar{\mathbf{z}})^{-(+)}, \quad [3.1]$$

where $k = 2(\mathbf{1}'_n |\mathbf{z} - \bar{\mathbf{z}}|)^{-1}$ is the normalizing factor, \mathbf{z} is the $n \times 1$ vector of the rankings in the sorting variable (*i.e.*, beta or correlation), $\bar{\mathbf{z}}$ is the $n \times 1$ vector with each element equal to the cross-sectional mean of the rankings, and $x^{-(+)}$ denotes the negative (positive) elements of the $n \times 1$ vector \mathbf{x} .

- Third, the beta-parity approach. In fact, this is the beta version of the risk-parity approach proposed in [Asness *et al.* \(2012\)](#), which helps avoid that the total portfolio risk is dominated by more volatile assets within the portfolio. Both the low-attribute portfolio (*i.e.*, the long-leg) and the high-attribute portfolio (*i.e.*, the short-leg) are rescaled to produce an *ex ante* unit portfolio beta at the portfolio formation. That is, the long leg (short leg) is scaled up (down) by leveraging (deleveraging) its position. In this way, the BA-type portfolio becomes a market-neutral, levered long-and-short portfolio which has an *ex ante* beta of zero.

$$BAB = \frac{1}{\beta_L} R_{LOW}^{RW} - \frac{1}{\beta_H} R_{HIGH}^{RW}, \quad [3.2]$$

where the return of the long leg and that of the short leg are calculated as follows: $R_{LOW}^{RW} = \mathbf{r}' \mathbf{w}_L^{RW}$, $R_{HIGH}^{RW} = \mathbf{r}' \mathbf{w}_H^{RW}$, $\beta_L = \boldsymbol{\beta}' \mathbf{w}_L^{RW}$, $\beta_H = \boldsymbol{\beta}' \mathbf{w}_H^{RW}$, and \mathbf{r} is the vector of the excess returns. For conciseness purposes, the time subscript in the above expression has been dropped, despite the fact that the portfolio betas and returns change period-by-period. Note leveraging the low-beta portfolio and deleveraging the high-beta portfolio produce an overall net investment of $\$ \left(\frac{1}{\beta_L} - \frac{1}{\beta_H} \right)$ in the risky stocks (*i.e.*, a net-long position). The net-long position is paired with a dollar investment of $\$ \left(\frac{1}{\beta_L} - \frac{1}{\beta_H} \right)$ in the risk-free asset to generate the excess return of BAB (See **Subsection 4.2** on the zero-investment requirement).

3.2. Portfolio Performance of BAB and BAC

Figure 1 plots the (cumulative) portfolio values of the BAB and BAC strategies and the value-weighted market from July 1963 to December 2016, assuming an initial investment of \$1 at the start

of the sample period. Both BAB and BAC grow steadily in value and outperform the market portfolio over the sample period.⁴ Moreover, as a defensive, market-neutral strategy, BAB and BAC experience relatively less drawdown than the market portfolio in market downturns (*i.e.*, NBER recessions).

Table 1 shows that the BAB (BAC) strategy delivered an impressive average excess return of 0.88 (0.99) percent per month, with a standard deviation of 3.40 (4.59) percent. The annualized Sharpe ratios of BAB (BAC) is 0.90 (0.75), offered a better risk-return trade-off than the market portfolio (*i.e.*, Sharpe ratio = 0.50).

[Insert Table 1 here]

While earlier studies tend to rely on the CAPM model or the Fama-French three-factor model as the benchmark model to evaluate the low-beta anomaly ([Frazzini & Pedersen 2014](#); [Schneider *et al.* 2015](#)), competing asset pricing models have evolved over the years. They have stronger power in explaining the asset returns in the cross section. [Barillas and Shanken \(2018\)](#) recommend a six-factor asset pricing model which incorporates the momentum factor with the market, size, value, profitability, and investment factors ([Fama & French 2018](#); [Li 2018](#)). [Asness *et al.* \(2018\)](#) augment the Fama-French five-factor model with the short-term reversal factor. I use a Fama-French five-factor model augmented with both the momentum and short-term reversal factors, denoted as FF7.

The next few columns in **Panel A of Table 1** report the risk-adjusted return (*i.e.*, alpha) of BAB and BAC under alternative factor models. In general, the risk-adjusted return of BAB gets smaller in magnitude the more risk factors that are included in the model. For example, the CAPM alpha of the BAB strategy amounts to 0.92 percent per month with a *t*-statistic of 4.30. The risk-adjusted performance reduces to 0.74, 0.46, and 0.34 percent per month, when evaluated with the Fama-French three-factor model (FF3), the Fama-French five-factor model (FF5), and the Fama-French six-factor model (FF6), respectively. The superior performance of BAB is partially due to the exposures to existing risk factors (such as profitability, investment, and momentum). In comparison, the risk-adjusted returns of BAC remain quite stable at around 70 bps per month under the alternative factor models.

⁴ One slight technical difference is that [Asness *et al.* \(2018\)](#) construct their BAC factor by averaging across the five volatility-sorted quintile BAC portfolios. In unreported analysis, I replicate their volatility-controlled BAC factor, and find that the two alternative BAC factors are highly correlated, but their volatility-controlled BAC is completely spanned by my BAC factor.

Panel B of Table 1 confirms findings in [Asness et al. \(2018\)](#) that BAC performs better than BAB in terms of risk-adjusted returns: BAC retains an alpha of 68 basis points (bps) per month with a t -statistic of 3.80 (significant at the 1% level), which is larger than the alpha of 34 bps earned by BAB.

Although BAB and BAC are constructed similarly, they exhibit notable differences in terms of risk exposures. BAC is highly exposed to the size factor (SMB) with a coefficient of 0.8 ($t = 10$), but seems unrelated to all other factors, including the market factor (RMRF), the profitability factor (RMW), the investment factor (CMA), the momentum factor (MOM), and the short-term reversal factor (STREV). Part of the strong performance of BAC is compensated by the size premium. In contrast, BAB does not load on the size factor. It loads heavily on the momentum factor. In addition, the BAB factor also significantly loads on the profitability (RMW) and the investment (CMA) factor.

3.3. Time-series Spanning Test

Panel C of Table 1 reports the results of the time-series spanning tests. When the excess returns of the BAB strategy are regressed on the excess returns of the BAC strategy, the intercept term of the regression is 0.63 percent with a t -statistic of 3.27, which is significant at the 1% level. BAB is not spanned by its BAC counterpart, if the BAC strategy captures only the leverage-related channel of the low-beta anomaly.

BAC is not spanned by BAB, either. The intercept term of the regression is 0.59 percent with a t -statistic of 2.46. Overall, the spanning test suggests that both BAB and BAC provide unique price-related information that is not subsumed by each other.

4. Decomposition of the Betting Against Beta and the Betting Against Correlation Strategies

4.1. A Decomposition Framework

I now introduce the key contribution of the paper: A non-parametric partition of the original BAB (BAC) portfolio into three stand-alone investable portfolios. They quantify the contributions attributed to three components:

1. To quantify the contribution due to stock selection, I rely on the zero-cost low-minus-high-beta portfolio (denoted as $BAB^{Selection}$) with an equal-weighting scheme in both the long leg and short leg, respectively. The adoption of the “neutral” equal-weighting scheme is motivated by

the fact that the equally-weighted portfolio weights are not correlated with either the rankings of the sorting variable (*i.e.*, beta) or the stock returns in the cross section. Therefore, the portfolio return of $BAB^{Selection}$ represents the pure stock selection skills that are not influenced by the “aggressive” rank-based weighting scheme or the “conservative” market-cap-based weighting scheme. Analytically, the stock selection component is constructed as follows:

$$BAB^{Selection} = R_{LOW}^{EW} - R_{HIGH}^{EW}, \quad [4.1]$$

where $R_{LOW}^{EW} = \mathbf{r}'\mathbf{w}_L^{EW}$ and $R_{HIGH}^{EW} = \mathbf{r}'\mathbf{w}_H^{EW}$ are defined similarly as in the original BAB strategy, except using an equal-weighting portfolio scheme. Note, by construction, the $BAB^{Selection}$ portfolio would have a negative market exposure, as the portfolio beta of the long leg is less than that of the short leg.

2. To quantify the contribution due to rank-based weighting scheme, I rely on the zero-cost rank-weighted BAB portfolio, denoted by BAB^{RW} , which has the same composite stocks in the long (short) leg as for the $BAB^{Selection}$ portfolio, but uses the rank-based portfolio weights rather than equally-weighted portfolio weights. As it stands, it represents the combined contribution due to both the stock selection and the rank-based weighting scheme. Therefore, the *pure* contribution due to the rank-based weighting scheme is captured by the additional returns earned by the rank-based portfolio (BAB^{RW}) over the equally-weighted portfolio ($BAB^{Selection}$). Analytically, the rank weight component, denoted as BAB^{Rank} , is constructed as follows:

$$BAB^{Rank} \equiv BAB^{RW} - BAB^{Selection} \quad [4.2]$$

Intuitively, the rank weight component portfolio can be treated as a zero-cost long-and-short portfolio which goes long BAB^{RW} and short $BAB^{Selection}$ at the same time. By construction, the rank weight component portfolio also has a negative market exposure (*i.e.*, CAPM beta) as the portfolio beta of BAB^{RW} is smaller than that of $BAB^{Selection}$ due to usage of rank-based weighting scheme.⁵

⁵ A more precise view on the rank weight portfolio is that it captures the sum of the outperformance of ultra-low beta stocks over moderate-low beta stocks and the outperformance of moderate-high beta stocks over ultra-high beta stocks. In principle, among all the below-median beta stocks in the long leg of BAB, the rank-based weighting scheme dictates that it will put more portfolio weights on stocks with relatively low betas than those with relatively high betas. The same applies to all the above-median beta stocks in the short leg. In other words, it bets on ultra-low beta stocks and against moderate-low beta stocks in the below-median beta group, and it bets on moderate-high beta stocks and against ultra-high beta stocks in the above-median beta group (see **Table A1** in the appendix).

3. The contribution due to the beta-parity approach could be measured by evaluating the additional return earned by the original BAB portfolio over the (unlevered) rank-weighted low-minus-high-beta portfolio (BAB^{RW}). Analytically, the beta-parity component, denoted as BAB^{Parity} , is constructed as follows:

$$BAB^{Parity} \equiv BAB - BAB^{RW}. \quad [4.3]$$

The beta-parity component, BAB^{Parity} , also has a time-varying net-long position of $\$ \left(\frac{1}{\beta_L} - \frac{1}{\beta_H} \right)$ in the risky stocks, a property carried over from the original BAB portfolio. In fact, it has non-negative portfolio weights in all composite stocks (see **A.1** in the appendix for the mathematical proof). Thus, the beta-parity component has a positive market exposure (*i.e.*, CAPM beta), so that, combined with the other two components, the overall BAB portfolio is *ex ante* market-neutral. To calculate its excess return, the net-long position is paired with a dollar investment of $\$ \left(\frac{1}{\beta_L} - \frac{1}{\beta_H} \right)$ in the risk-free asset (See **Subsection 4.2** on the zero-investment requirement).

Overall, the excess return of the original BAB strategy can be decomposed into three investable component portfolios in the following manner:

$$BAB = \underbrace{BAB^{Selection}}_{\substack{\text{contribution} \\ \text{due to} \\ \text{stock selection}}} + \underbrace{BAB^{Rank}}_{\substack{\text{contribution} \\ \text{due to} \\ \text{weighting scheme}}} + \underbrace{BAB^{Parity}}_{\substack{\text{contribution} \\ \text{due to} \\ \text{beta parity}}}, \quad [4.4]$$

which attributes the total (excess) return of BAB to the stock selection, the rank weighting scheme, and the beta-parity approach, respectively. A full mathematical proof of the above decomposition framework is available in **A.1** in the appendix, complemented by one numerical example with eight hypothetical stocks to illustrate the portfolio weights in the three components (**Table A1** in the appendix).

The above non-parametric decomposition framework has several merits.

First, it helps break down the complicated BA-type portfolios into easy-to-understand investment strategies. In principle, the original BAB portfolio blends in both the cross-sectional strategy and the time-series strategy. Both the stock selection component and the rank weight component represent a cross-sectional strategy aimed at exploring the *pure* low-beta (low-correlation) anomaly, while the

This idea, however, builds on the pre-assumption that there exists a monotonic pattern between expected returns and the stock attribute such as beta. Should the beta-return relation be “flattened” or non-monotonic such as an empirically U-shaped or inversely U-shaped pattern as warned in [Patton and Timmermann \(2010\)](#), the rank weight component would not add any value for the overall BAB strategy.

beta-parity component resembles a market timing strategy, which makes a time-varying bet on the stock market (*i.e.*, a dynamic net-long position).

Second, the decomposition approach allows for an exact replication of the BA-type portfolio: The sum of the (excess) returns of the three component portfolios equals the (excess) return of the original BAB. In other words, we could quantify their relative return contributions, and pin down the source(s) of the superior performance of the original BA-type strategy. The three component portfolios also serve as new test assets, and we could quantify their risk-adjusted performance (as we did for BAB and BAC).

Third, from a practical perspective, the decomposition generates three new investable component portfolios out of the original BA-type portfolio. Each of the component portfolio is a stand-alone investment vehicle, which could be combined with existing trading strategy (*i.e.*, size, value, and etc.) to enhance portfolio performance.⁶ Besides, we could also assess whether the stand-alone component portfolios are feasible to implement or not, by assessing their portfolio turnover and (estimated) transaction costs.

4.2. A Note on the Excess Returns of the Beta-Parity Component, BAB, and BAC

The [Fama and French \(1993\)](#) time-series test on the regression intercept (*i.e.*, alpha) requires the testing asset to be a zero investment (*i.e.*, a self-financing portfolio). This self-financing status is achieved by betting an equal (dollar) amount on the long side and the short side. For example, to assess whether a conventional long-only decile portfolio generates a positive CAPM alpha, the zero-investment requirement dictates the use of the excess return of the portfolio. That is, going long \$1 of the decile portfolio and short \$1 of the risk-free asset.

Equations [3.2], [4.3], and [4.4] indicate that the beta-parity component in BAB (BAC) and the BA-type portfolios all have a net-long position of $\$ \left(\frac{1}{\beta_L} - \frac{1}{\beta_H} \right)$ invested in the risky assets (*i.e.*, the long side). To ensure their risk-adjusted performance is evaluated on a zero-investment basis, they are paired with a dollar investment of $\$ \left(\frac{1}{\beta_L} - \frac{1}{\beta_H} \right)$ in the risk-free asset (*i.e.*, the short side). Throughout the article, the excess returns of the beta-parity component, BAB, and BAC are calculated by netting

⁶ Unlike the original leveraged long-and-short BA-type portfolio which is crafted for hedge funds, the net-long time-series component is more implementable and cost-effective for the long-only investors, such as mutual funds, pension funds and retail investors. In unreported analysis, I consider a simple asset allocation exercise and find that the inclusion of the time-series component into the existing factor universe (*i.e.*, market, size, value, profitability, investment, momentum, and short-term reversal) would increase the optimal Sharpe ratio from 1.40 to 1.50, enhancing the investment opportunity for a mean-variance investor.

off $\left(\frac{1}{\beta_L} - \frac{1}{\beta_H}\right) \times RF_t$ from their respective portfolio returns. This ensures that they all satisfy the zero-investment requirement.

4.3. Performance Evaluation of the Component Portfolios

Panel A of Table 2 lists the 5 worst performing months of the overall US stock market (measured by the market excess return, RMRF) during the entire sample period, covering the 1973 Oil Crisis, the Black Monday in October 1987, the 1998 Ruble Crisis, and the 2008 Global Financial Crisis (GFC). Although the BAB and BAC factors are, by construction, market-neutral strategies, their overall performances are still worse during market crashes. The individual component portfolios yield more insights: In general, both the stock-selection component and the rank-weight component of the BAB factor generate positive (excess) returns during market crashes, reinforcing the value of a defensive strategy in adverse market conditions: low-beta stocks suffer relatively less than high-beta stocks during market downturns, hedging in adverse market conditions. In contrast, the beta-parity component of BAB suffers dramatically during market crashes. It has a strong positive exposure to the overall market (*i.e.*, CAPM beta). The three components of the BAC factor behave in a similar manner. The stock selection component and the rank-weight component of BAC have lower variance during these extreme periods. The beta parity component of BAC suffers greatly again.

Panel B of Table 2 lists the 5 best performing months of the overall US stock market. The BAB factor tends to earn moderate returns during bull markets. Both its stock-selection component and rank-weight component experience negative returns, which is as expected because low-beta stocks would earn relatively less than high-beta stocks when market risk premium is large. In contrast, the beta-parity component gains substantially with its positive market exposure. The behaviour of the components of the BAC factor, however, is less clear. For example, unlike its counterparts of BAB, the stock-selection component and rank-weight component of BAC experience large positive returns in January 1975, inconsistent with the fact that it has a negative portfolio beta by construction.⁷

[Insert Table 2 here]

⁷ In fact, the strong performance of the stock-selection component and rank-weight component of BAC is consistent with the fact that stock correlation is highly positively correlated with firm size in the cross section. See **Section 5** on addressing the size issue.

Panel A of Figure 2 visualizes the performance of the three components of BAB, assuming an equal amount of \$1 is invested at the start of the sample period.⁸ The superior performance of BAB was solely driven by its beta-parity component portfolio, which outperformed the market portfolio over this sample period. In comparison, both the stock-selection component and the rank-weight component do not provide any monetary gains for the investor over the entire 54-year period. It should be noted, however, the two zero-cost components (with negative loadings on the market factor) do provide hedging benefits to the overall BAB strategy, because they perform relatively well in adverse market conditions (*i.e.*, NBER recession) when the beta-parity component tends to crash. **Panel B of Figure 2** plots the portfolio values of the three components of BAC, together with the market portfolio. When treated alone, all three components of BAC seem to provide monetary gains for the investor, which is different from those of BAB. Again, the performance of the two zero-cost components (the stock-selection component and the rank-weight component) perform relatively stable in adverse market conditions.

[Insert Table 3 here]

Table 3 provides more depth on the decomposition for BAB. In the case of BAB (**Panel A**), the risk-adjusted returns of the two zero-cost components are indistinguishable from zero (*i.e.*, 10 and 4 bps per month). The beta-parity component remains the major driver of BAB, which delivers an alpha of 20 bps per month, significant at the 1% level. However, a different pattern of the risk-adjusted performance emerges for BAC (**Panel B**): The stock-selection component, the rank-weight component, and the beta-parity component contribute 31, 12, and 25 bps per month, respectively, to the overall BAC strategy. The associated Newey-West *t*-statistics for the three components are all statistically significantly at the 1% level. Almost 64 percent of the alpha is generated by the two zero-cost components (*i.e.*, the cross-sectional effect). Note our findings on the risk-adjusted performance of the three components in BAB and BAC are robust under alternative asset pricing models. **Table A2** in the appendix provides the robustness checks using the Investment q-factor model (denoted as HXZ4) and the Mispricing four-factor model (denoted as M4), respectively.⁹

The performance decomposition generates more insights when evaluating the risk exposures of the three components:

⁸ Strictly speaking, the graph depicts the “incremental” portfolio value, as I exclude the effect of the risk-free rate in the calculation of the portfolio value. However, adding back the risk-free rate would not change the key results regarding the relative performance of the three components.

⁹ I acknowledge Lu Zhang and Robert Stambaugh for sharing the data used in their competing asset pricing models.

- First, in general, the two zero-cost components behave very similarly, as they are aimed to “capitalize” on the low-beta (low-correlation) anomaly. In the case of BAB (**Panel A**), its stock-selection component and rank-weight component have negative exposures to the market and size factors, but load positively on the profitability, investment, and momentum factors. In the case of BAC (**Panel B**), its stock-selection component and rank-weight component also have negative market exposure. But unlike their counterparts in BAB, they have a strong positive exposure to the size factor. The two zero-cost components of BAC also have strong positive exposure to the momentum factor, but do not load much on the profitability and investment factor.
- Second, the respective beta-parity components of BAB and BAC behave quite similarly, as they both load strongly on the market factor and the size factor, indicating much of their time-variation is driven by the market risk premium and size premium.

Overall, the performance decomposition generates new insights regarding the popular BAB and BAC strategies. The success of the BAB strategy is mainly due to the beta-parity component, a market timing strategy with time-varying net-long position. In contrast, the outperformance of BAC is contributed by all three components. In fact, the two cross-sectional components (the stock selection component and the rank weight component) of BAC contribute more than half of its risk-adjusted performance (evaluated by the augmented seven-factor model). The significant alphas generated by the two cross-sectional components of BAC implies that the stock correlation is a much stronger return determinant (than market beta) in the cross section, which is deferred to later sections (*i.e.*, **Section 5**) for formal evaluation.

Note the decomposition approach also helps assess whether the components of BAB (BAC) can cover their transaction costs, as the above BA-type strategies are designed to maximize the pre-transaction portfolio returns without any consideration of practical implementation. In **Appendix A.3**, I explore the portfolio turnover and transaction costs aspects of the BA-type strategies and their component portfolios (see **Table A3** in the appendix). The two cross-sectional components of BAB (BAC) has relatively high portfolio turnover of 183% and 189% (151% and 151%) per annum. In contrast, the time-series component of BAB (BAC) has a low annualized turnover ratio of 76% (48%). The time-series component could withstand implausibly large transaction costs before losing its (risk-adjusted) returns. Collectively, the beta-parity component remains the most robust source of the profits for the two BA-type strategies, both before and after taking transaction costs into account.

5. Understanding the Cross-sectional Components of BAB and BAC

5.1. Firm-level Evidence

The portfolio decomposition evidence in **Section 4.2** reveals that unlike their counterparts of BAB, the two cross-sectional components of BAC (*i.e.*, the stock-selection component and rank-weight component) generate superior risk-adjusted performance, implying that the low-correlation anomaly is stronger than the low-beta anomaly in the cross section.

To evaluate the above notion and also to test the pricing power of stock correlation at the firm level, we perform the [Fama and MacBeth \(1973\)](#) cross-sectional regression over the entire sample period from July 1963 to December 2016 (*i.e.*, 642 months). In each month, the cross-sectional excess returns (over the risk-free rate, RF) are regressed on the *ex ante* stock correlation as defined in [Frazzini and Pedersen \(2014\)](#). The slope coefficients are then averaged over the entire sample periods.

$$Ret_i - RF = 1.39 - 1.53 \times \rho_i + \varepsilon_i \quad [5.1]$$

[4.77] [-3.91]

The base-line Fama-MacBeth regression result confirms a strong *low-correlation* effect at the firm level. The slope coefficient has a negative value of -1.53 , which is significant at the 1% level, indicated by the Newey-West t -statistics (in brackets). The strongly negative slope coefficient implies an *inverse* return-correlation relation in the cross section. That is, the higher the return correlation of a stock, the lower its expected return.

In comparison, stock beta is “unpriced” at the firm level, which is consistent with prior work that typically documents a “flattened” security market line in the US ([Fama & French 1992, 2006](#); [Blitz & Vidojevic 2017](#)).

$$Ret_i - RF = 1.08 - 0.11 \times \beta_i + \varepsilon_i \quad [5.2]$$

[4.58] [-0.35]

To purge the possible behavioural mechanisms that might be embedded in stock correlation, we then control for return volatility (VOL) or idiosyncratic volatility (IVOL) as defined in [Ang et al. \(2009\)](#), and re-run the Fama-MacBeth cross-sectional regression to test the low-correlation effect.

$$Ret_i - RF = 0.96 - 0.96 \times \rho_i + 0.09 \times VOL_i + \varepsilon_i \quad [5.3]$$

[3.97] [-2.46] [1.19]

$$Ret_i - RF = 1.31 - 1.27 \times \rho_i - 3.78 \times IVOL_i + \varepsilon_i \quad [5.4]$$

[6.30] [-3.52] [-0.63]

In both cases, we find that the negative relation between correlation and stock returns gets a bit weaker, but remains strong.

Overall, the firm-level evidence lends strong support to our findings in **Section 4.2** that both stock selection and rank-based weighting scheme add values to the BAC strategy, because low-correlation stocks earn higher returns than high-correlation stocks in the cross section.

5.2. An Assessment of the Low-correlation Anomaly

To better understand the low-correlation anomaly and the new BAC factor, I start by estimating the pairwise correlation coefficients of stock correlation with other firm characteristics to understand the features of the high- (low-) correlation stocks.

Panel A of Table 4 presents the time-series average of the period-by-period correlation coefficients between correlation and one particular firm characteristic. A few salient features are worth clarifying. First, and the most remarkable, is that the stock correlation measure is highly correlated with firm size (lnME) with a pairwise correlation coefficient of 0.74, which suggests that the pricing power of stock correlation might stem from the size effect. Second, stock correlation is negatively correlated with the lottery-demand measure (*i.e.*, ISKEW and MAX5) and the idiosyncratic risk measure (IVOL). It is also negatively correlated with the Amihud illiquidity ratio (ILLIQ), implying low-correlation stocks are more illiquid than their high-correlation counterparts. Third, correlation does not seem to be highly associated with profitability, investment, return momentum, and return reversal proxies (*i.e.*, OP, INV, RET^{MOM}, RET^{STREV}), a fact that is consistent with BAC's risk exposure in **Table 1**.

The estimated pairwise correlation coefficients pose challenges to the premise in [Asness et al. \(2018\)](#) that BAC supports the leverage aversion theory.¹⁰ In general, low-correlation stocks tend to be small-sized and illiquid firms with high idiosyncratic volatility and idiosyncratic skewness, and possibly with high lottery demand, which looks much riskier than their high-correlation counterparts. Despite the puzzling empirical pattern, it should be noted from the outset that the pricing power of the test variable (*i.e.*, correlation) might stem from the part that is orthogonal to the particular firm characteristic such as size ([Hou & Loh 2016](#)). Therefore, to pin down the driver of the low-correlation anomaly and also to rule out the possibility that the pricing power stems from the residual component that is orthogonal

¹⁰ Reasoning that the BAC factor supports the low-risk anomaly, however, assumes that high-correlation stocks have characteristics that are (perceived) riskier than their low-correlation counterparts. For example, within the framework of the leverage-aversion theory, leverage-constrained investors make sub-optimal portfolio choices by oversubscribing riskier stocks, bidding up their price, and thus leading to the low-risk anomaly. Therefore, high-correlation stocks should be riskier, or at least have stock characteristics that seem riskier, than their low-correlation counterparts. This pre-assumption of the leverage-aversion theory is not supported by the firm-level data.

to size (or other firm characteristics), I perform the novel [Hou and Loh \(2016\)](#) decomposition, an extensive “horse race”, to evaluate the possible explanations of the low-correlation effect.¹¹

The [Hou and Loh \(2016\)](#) decomposition exercise proceeds in three stages.

Stage 1: The univariate Fama-MacBeth cross-sectional regression.

For each period, the excess returns of stocks are regressed on the lagged stock correlations (*i.e.*, **Equation [5.1]** in **Subsection 5.1**). The time series of the slope coefficient, $\hat{\gamma}_t$, capturing the negative correlation-return relation, is retained for the final stage.

Stage 2: The orthogonalization procedure on the lagged correlation.

A cross-sectional orthogonalization regression is performed for each month, which regresses the stock correlation on one candidate variable Z (*i.e.*, one firm characteristic). The orthogonalization procedure partitions the lagged stock correlation $\rho_{i,t-1}$ into two orthogonal parts: The explained part due to the candidate variable Z (denoted as $\rho(Z)_{i,t-1}^{Explained}$), and the unexplained part orthogonal to the candidate variable Z (denoted as $\rho_{i,t-1}^{Unexplained}$). That is, $\rho_{i,t-1} = \rho(Z)_{i,t-1}^{Explained} + \rho_{i,t-1}^{Unexplained}$. The time series of the two orthogonal parts is also retained for the final stage.

Stage 3: Decomposition of the time-series average of the slope coefficient $\bar{\gamma}_t$.

In the final stage, the time series of $\hat{\gamma}_t$ (from stage 1) is decomposed into two components based on the two orthogonal parts of $\rho_{i,t-1}$ (from stage 2) and the properties of covariance (*i.e.*,

$$\underbrace{\frac{Cov(Ret_{i,t}-RF_t, \rho_{i,t-1})}{Var(\rho_{i,t-1})}}_{\hat{\gamma}_t} = \underbrace{\frac{Cov(Ret_{i,t}-RF_t, \rho(Z)_{i,t-1}^{Explained})}{Var(\rho_{i,t-1})}}_{\hat{\gamma}_t^{Explained}} + \underbrace{\frac{Cov(Ret_{i,t}-RF_t, \rho_{i,t-1}^{Unexplained})}{Var(\rho_{i,t-1})}}_{\hat{\gamma}_t^{Unexplained}}.$$

Thus, the time-series average of the slope coefficient $\bar{\gamma}_t$ has an explained component due to the candidate variable Z , denoted as $\overline{\hat{\gamma}_t^{Explained}}$, and a remaining unexplained component, denoted as $\overline{\hat{\gamma}_t^{Unexplained}}$. The final-stage decomposition could be concisely expressed as follows:

$$\text{In levels:} \quad \underbrace{\bar{\gamma}_t}_{\text{Total}} = \underbrace{\overline{\hat{\gamma}_t^{Explained}}}_{\text{Explained Coefficient}} + \underbrace{\overline{\hat{\gamma}_t^{Unexplained}}}_{\text{Unexplained Coefficient}}, \quad [5.5]$$

¹¹ It should be noted that the [Hou and Loh \(2016\)](#) “horse race” is performed at the firm level, which differs from concurrent work on the low-beta anomaly that focuses mainly on portfolio-level evidence. Portfolio analysis does have some drawbacks. First, aggregation to the portfolio level would inflate the correlation between firm characteristics and the test variable. Second, according to [Ang et al. \(2008\)](#) the aggregation to the portfolio level might “overlook” important cross-sectional information and does not necessarily increase the precision of the coefficient estimates.

or in relative terms:
$$\underbrace{100\%}_{\text{Total}} = \underbrace{\frac{\overline{\hat{\gamma}(Z)_t^{\text{Explained}}}}{\overline{\hat{\gamma}_t}}}_{\text{Explained Proportion}} + \underbrace{\frac{\overline{\hat{\gamma}_t^{\text{Unexplained}}}}{\overline{\hat{\gamma}_t}}}_{\text{Unexplained Proportion}}. \quad [5.6]$$

The merits of the [Hou and Loh \(2016\)](#) decomposition exercise are: First, the explained and the unexplained coefficients (*i.e.*, $\overline{\hat{\gamma}(Z)_t^{\text{Explained}}}$ and $\overline{\hat{\gamma}_t^{\text{Unexplained}}}$) sum up to the time-series average of the slope coefficient $\overline{\hat{\gamma}_t}$, making it easy to quantify the pure contribution of the candidate variable Z in explaining the negative correlation-return relation. Second, it could be repeated for a number of candidate variables, which provides a “horse race” to objectively compare the ability of each candidate variable in explaining the negative correlation-return relation.

Panel B of Table 4 presents the “horse race”. To save space, only the final-stage results are reported.¹² Consistent with the patterns in **Panel A** of the table, firm size (lnME) remains the most powerful candidate variable in explaining the low-correlation effect (Column 1 in Panel B). The slope coefficient on the explained part (due to size) is highly negative. Moreover, the proportion explained by firm size amounts to 132 percent, with a t -statistic of 4.92 (significant at the 1% level). Therefore, it seems that the low-correlation effect is completely driven by the part that is explained by firm size.¹³ The slope coefficient on the orthogonalized part turns out to be positive with a value of 0.49, suggesting that the orthogonalized part of the correlation measure is no longer a negative return determinant.

The [Amihud \(2002\)](#) illiquidity ratio, denoted as ILLIQ, is another plausible variable in explaining the low-correlation effect, as 73 percent of the low-correlation effect is captured by the illiquidity measure, which is highly significant at the 1% level. Moving across the table, none of the remaining ten candidate variables could explain a sizeable portion of the low-correlation effect, as their explained portions are all below 32% or less.

[Insert Table 4 here]

Overall, after the massive firm-level “horse race”, we are able to generate two key insights regarding the low-correlation phenomenon:

First, there is no evidence in support of the notion that stock correlation is a valid firm characteristic that resembles the riskiness of a firm. In fact, the firm-level evidence points to the opposite direction:

¹² Stage 1 results uniformly reconfirm the low-correlation anomaly (*i.e.*, negative correlation-return relation in the cross section), which are statistically significant at the 1% level.

¹³ It should be noted that the fraction explained by the candidate variable is not bounded from 0 to 100%. The decomposition procedure only requires that explained and unexplained fractions add up to 100% in total.

Low-correlation stocks tend to be highly risky stocks with small market capitalization, high trading costs, and large (idiosyncratic) volatility or lottery demand.

Second, the fact that low- (high-) correlation stocks are small-cap and illiquid (large-cap and liquid) firms also explains why the low-minus-high-correlation strategy is particularly sensitive to adverse changes in funding/leverage constraints ([Asness et al. 2018](#)). In principle, the returns of small firms are particularly sensitive to changes in the funding constraints of the market ([Brunnermeier & Pedersen 2009](#); [Næs et al. 2011](#)). Therefore, instead of treating the outperformance of a *pure* low-minus-high-correlation strategy as the success of the low-risk anomaly, a more plausible, alternative interpretation is that it behaves more like a high-risk strategy, which actively bets on size-related risk. As a result, the success of the BAC factor reflects partially the rational compensation for holding small and illiquid firms.

5.3. Addressing the Size Concern

A legitimate concern for the size effect on the BA-type strategies arises naturally from a number of observations. First, BAB (BAC) is implemented on all available stocks in the CRSP/Compustat universe, which contains numerous micro-cap stocks. Second, there exists a striking positive association between stock correlation and firm size (see **Section 5.2**).

To address the size concern, I perform alternative implementation for BAB and BAC by restricting the investment universe to firms with above-median value of the (lagged) market capitalization. This drops 50% of the sample firms. On average the above-medium market-cap firms account for nearly 98% of the total market capitalization in the US.

[Insert Table 6 here]

Panel A of Table 6 presents the portfolio performance for the alternative BAB and its three components that are formed by firms with above-medium market capitalizations. Similar to their full-sample counterparts in **Table 3**, the two cross-sectional components contribute little to the total return of BAB (*i.e.*, only 6 and 3 bps per month). They still have large negative market exposure by construction (*i.e.*, low-minus-high beta portfolio). Moreover, the two cross-sectional components load heavily and positively on the value, profitability, investment, and momentum factors. One major difference from **Table 3** lies in the beta-parity component, as its excess return shrinks to 43 bps per month (with the *t*-statistics of 3.41) after excluding the smallest firms in the sample. Again, the beta-

parity component loads highly positively on the market factor, consistent with its net-long position. As expected, the excess return of the alternative BAB shrinks to 0.51 percent per month. Overall, the alternative BAB loads highly positively on the value, profitability, and investment factors, and has a seven-factor alpha indifferent from zero (*i.e.*, -4 bps per months). This indicates that the alternative size-controlled BAB factor earns its return by tilting mainly towards the (positive) exposures to the value, profitability, and investment factors.

A similar pattern emerges when we focus on the alternative BAC strategy and its three components (**Panel B**). Compared to the figures in **Table 3**, the stock selection component and the rank-weighted component no longer earn any significant excess returns (or risk-adjusted returns). As a result, the time-series component (*i.e.*, beta-parity component) becomes the only source of the profit earned by the alternative BAC strategy. The excess return of the beta-parity component amounts to 24 bps per month, which is significant at the 5% level. The alternative BAC factor earns an average excess return of 30 bps per month, which is less pronounced than its BAB counterpart (*i.e.*, 51 bps in **Panel A**). Therefore, once we exclude the smallest stocks from our sample, BAC no longer outperform BAB in the sample period. The alternative BAC factor still loads highly positively on the size factor (with the *t*-statistics of 7.46), which is expected as the strong correlation-size relation also holds in the subsample of firms with above-median market capitalization. Therefore, sorting on stock correlation generates an implicit sort on firm size, which leads to the tilt towards size premium. Another major difference between **Table 3** and **Table 6** lies in the risk-adjusted performance: Once we focus on the relatively large firms, the alternative BAC (BAB) and its beta-parity component no longer generate any significant alphas under the augmented seven-factor model.

Table A4 in the appendix presents the portfolio turnover and breakeven transaction costs for the alternative versions of BAB, BAC, and their three components. In general, the portfolio turnover of BAB, BAC, and their components in the above-medium market-cap group is similar to their full-sample counterparts. More importantly, the breakeven transaction costs for the beta-parity component of the alternative BAB (BAC) strategy formed by these relatively large firms could still withstand a reasonably large transaction cost that are associated in trading these (relatively liquid) stocks, reinforcing that it is the most robust source for the profits of the BA-type strategies, both before and after taking transaction costs into account.

Overall, the performance of the BAC strategy is much more sensitive to the size effect than that of the BAB strategy, which is consistent with our findings in **Section 5.2** that the low-correlation anomaly is mainly driven by firm size. Therefore, once excluding the small-sized stocks, the excess return of BAC

shrinks more than that of BAB. Dropping the small-sized firms also makes the risk-adjusted returns of both BAB and BAC disappear.

6. Understanding the Time-series Component of BAB and BAC

In this section, I rely on a mostly-theory-agnostic analysis to better understand the time variation of the beta-parity component in BAB (BAC). The beta-parity component is the only robust source for the profits of the BA-type strategies, both before and after taking transaction costs into account. However, little is known about what underlying mechanism(s) might drive the time variation of the beta-parity component.

6.1. An Assessment of the Market Time Ability

To ensure an *ex ante* market beta of zero, BAB (BAC) mechanically builds in a time-varying net-long position in the risky assets (*i.e.*, the beta-parity component). This *time-varying* net-long position somehow resembles a market timing strategy that bets dynamically on the stock market (see **Figure 3** for the time series of the time-varying net-long position). Therefore, it is possible that the superior average return performance in the beta-parity component is driven by its market timing ability.

In principle, the excess return of the beta-parity component can be expressed as follows:

$$BAB_t^{Parity} = NetLong_t \times R_t^{Parity}, \quad [6.1]$$

where $NetLong_t \equiv \left(\frac{1}{\beta_L} - \frac{1}{\beta_H}\right)_t$ is the net-long position, and R_t^{Parity} denotes the excess return of the *unscaled* underlying portfolio.

Taking expectation on both sides, and utilize the properties of covariance, the expected excess return of the beta-parity component can be further decomposed into two components:

$$E(BAB_t^{Parity}) = \underbrace{E(NetLong_t) \times E(R_t^{Parity})}_{Scaled\ Premium} + \underbrace{cov(NetLong_t, R_t^{Parity})}_{Timing\ Premium}. \quad [6.2]$$

For the second term, the timing premium, to be a major source of the excess return of the beta-parity component, the net-long position needs to fulfil two conditions. First, it needs to be highly correlated with the underlying asset returns. Second, it needs to have relatively large variation over time (*i.e.*, standard deviation).

However, neither of the two conditions are fulfilled. On average, the monthly contribution due to the market timing effect is less than 5 bps. This is not surprising for two reasons. First, the per-period net-long position is determined by the beta spread between low-beta stocks and high-beta stocks. The beta-spread, as a distribution feature, evolves very slowly, making the net-long position a highly persistent process with relatively small time variation (*i.e.*, standard deviation amounts to 0.16). Second, the persistence of the net-long position also makes it highly insensitive to the dynamic asset return process (*i.e.*, close-to-zero correlation with asset returns).

Table 2 also provides evidence on the (no-)timing ability. On average, the net-long position is larger in bear markets than in bull markets. If the time-varying net-long position is able to time the market, its value should be relatively small (large) in bear (bull) markets. **Figure 3** echoes the finding of the (no-)timing ability: In most cases, the net-long position remains stable during the NBER recessions. Overall, the time-varying net-long position in BAB (BAC) governs the magnitude of the excess return of the beta-parity component, but it contains little, if any, market timing ability, and thus is not the main source that drives the outperformance of BAB (BAC) over time.

6.2. Further Analysis on the Beta-Parity Component

In the framework of [Frazzini and Pedersen \(2014\)](#), the time variation of BAB is assumed to be solely driven by funding liquidity constraints. In principle, the beta-parity component and BAB could be induced by other rational and behavioural mechanisms as well. For example, the return of the beta-parity component could be due to the shifts in the aggregated volatility ([Ang et al. 2006](#)). Moreover, investor sentiment (and more broadly, the behavioural mechanism) represents another legitimate mechanism for BAB ([Baker et al. 2011](#); [Antoniou et al. 2015](#); [Hong & Sraer 2016](#); [Bali et al. 2017](#); [Liu et al. 2018](#)). To evaluate the (possible) impacts due to volatility and sentiment, I follow the two-pass empirical procedures in [Ang et al. \(2006\)](#) to generate the traded volatility risk factor (denoted as FVIX) and the traded sentiment factor (denoted as FSENT), which track the innovations of the VIX index and the US sentiment index over time, respectively: In the first stage, stocks are sorted into quintile portfolios based on past return sensitivities to changes of the VIX index (the sentiment index). In the second stage, the changes in the VIX index (sentiment index) are regressed on the excess returns of the quintile portfolios. The fitted part of the regression is the traded volatility (sentiment) factor (See **Appendix A.2** for details of the factor construction approach). Following [Qiu and Welch \(2004\)](#) and [Lemmon and Portniaguina \(2006\)](#), I rely on the Michigan consumer confidence index, a survey-based measure, as the proxy of investor sentiment. The use of survey-based sentiment measure helps avoid

the legitimate concern that the documented explanatory power might simply be the result of a “fad” in prices being washed away, as the market-based sentiment measure is typically constructed by a number of financial measures ([Baker & Wurgler 2006, 2007](#)). The time-series test is then performed by regressing the beta-parity component in BAB on the two traded factors, while controlling the seven conventional factors.

[Insert Table 5 here]

Panel A of Table 5 presents the outputs of the time-series regression with the beta-parity component in BAB as the dependent variable. The sample period is restricted to 1986 – 2016 due to the data availability of FSENT and FVIX factors. During this more recent sample period, the beta-parity component generates a seven-factor risk-adjusted return of 30 bps (**Column 1**), which is similar to that of the full sample estimates in **Table 3**. Interestingly, once we include the traded sentiment factor (FSENT), a salient feature emerges: The beta-parity component loads strongly on investor sentiment with a slope coefficient of 0.46 ($t = 7.51$) after controlling the other seven factors (**Column 2**). The risk-adjusted return drops from 30 bps to 18 bps (*i.e.*, a 40% drop), and it also becomes less statistically significant. Moreover, the inclusion of the sentiment factor also increases the model fit (*i.e.*, adjusted R-square). In contrast, the inclusion of the traded aggregated volatility factor (FVIX) does not help explain the alpha of beta-parity component (**Column 3**). The sentiment impact on the beta-parity component remains intact as we include both the sentiment factor and the volatility factor (**Column 4**). **Panel B of Table 5** replicates the factor regressions with BAB as the dependent variable. The same empirical pattern emerges: After accounting for the seven factors, BAB still loads highly positively on the traded sentiment factor with a coefficient of 0.53 ($t = 3.87$). Moreover, the risk-adjusted return also decreases by approximately 40% (*i.e.*, drops from 37 bps to 23 bps), once we include the traded sentiment factor. In unreported analysis, I find the strong sentiment impact on the BA-type strategies survive a battery of robustness checks.¹⁴

¹⁴ In unreported analysis, I also perform a battery of robustness checks. First, I find very similar results for the beta-parity component in BAC. That is, the factor loadings on FSENT are 0.37 and 1.08 for the beta-parity component in BAC and BAC (both significant at 1% level), respectively. Second, using the non-traded factor of investor sentiment, I find similar empirical patterns, as the slope coefficient on the shifts in the sentiment index is highly positive and statistically significant. This strong positive exposure to investor sentiment is robust when adding other additional control variables such as the VIX index and TED spread. Third, I perform the conventional [Goyal and Welch \(2008\)](#) predictive regression using the lagged FSENT factor as the predicting variable. The predictive regression indicates that the lagged FSENT also positively drives the beta-parity component in BAB, though the magnitude is smaller (*i.e.*, coefficient amounts to 0.14 with a t-statistic of 6.94). Overall, the sentiment impact on BAB and BAC is very robust in the sample period.

Overall, the factor analysis shows that investor sentiment plays a non-negligible role in explaining the time variation of BAB. It accounts for 40% or more of the risk-adjusted returns in BAB and its time-series component.

7. Conclusion

This paper reassesses the popular betting against beta strategy. I show that the (risk-adjusted) return of BAB (BAC) is the sum of three investable component portfolios: Two cross-sectional components attributed to stock selection and rank-based weighting scheme, and one time-series component with a dynamic net-long position due to “beta-parity”.

The cross-sectional components and time-series component exhibit different empirical properties: First, all superior average return performance of BAB stems from the time-series component, both before and after adjusting for risk. Second, much of the time variation of the two cross-sectional components (exploring the *pure* low-beta anomaly) is explained by the positive exposures to the conventional risk factors (*i.e.*, profitability, investment, and momentum). Thus, they do not yield any strong risk-adjusted returns. However, the two cross-sectional components do lower the market beta of BAB (*i.e.*, hedging benefits). Third, the time-series component is a low-turnover strategy, while the two cross-sectional components require more frequent portfolio turnover. Therefore, among the three stand-alone components, the time-series component remains the most robust source for the profits of the BA-type strategies, both before and after taking transaction costs into account.

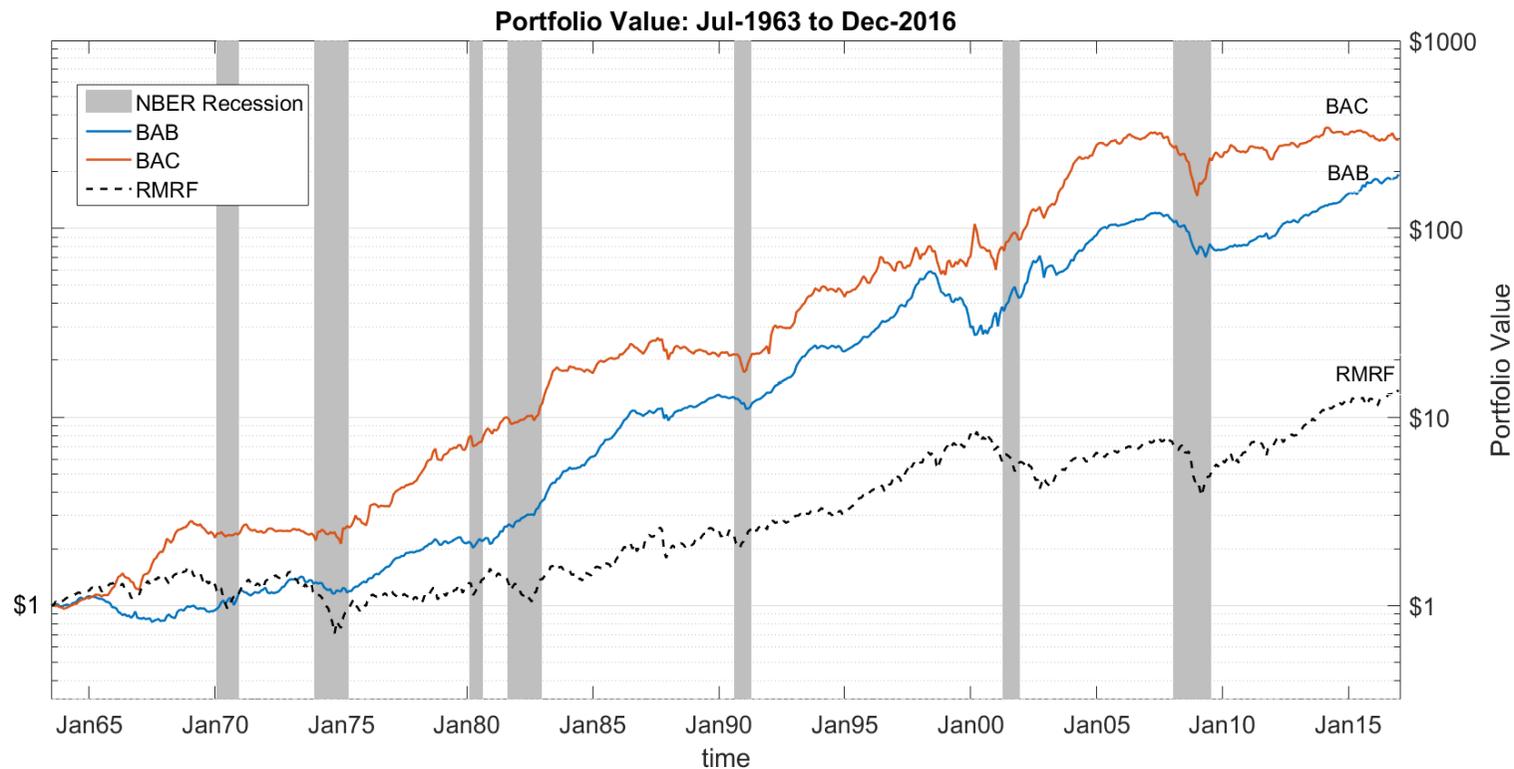
These empirical regularities also hold for the recently proposed BAC strategy ([Asness et al. 2018](#)), except that the two cross-sectional components contribute more of the risk-adjusted mean returns to BAC. There is a strong low-correlation anomaly at the firm level. However, the strong low-correlation anomaly (*i.e.*, low-correlation stocks outperform high-correlation stocks) arises from the strong positive association between firm size and stock correlation in the cross section. Therefore, after excluding the small-sized firms (*i.e.*, the below-median market-cap stocks), which are 2% of the total market capitalization, the performance of BAC disappears.

Finally, the mechanically built-in time-varying net-long position, by itself, is not the main source of the outperformance of the time-series component. It provides little market timing ability (*i.e.*, less than 5 bps per month). Investor sentiment has an important impact on the time-series component in BAB: The risk-adjusted return of the time-series component drops from 30 bps to 18 bps after including a

traded sentiment factor. Similarly, the unexplained return of BAB drops to only around 20 bps per month, and is no longer statistically significant.

My paper is related to [Novy-Marx and Velikov \(2018\)](#), which is both contemporaneous and independent. Both papers break down the BAB strategy into “pieces” to better understand its underlying dynamics. However, we follow different routes: My paper explores the BA-type strategies through the lens of a non-parametric decomposition, which highlights the source (*i.e.*, beta-parity component) and possible mechanism (*i.e.*, investor sentiment) that drive the outperformance of BAB (BAC) over time. [Novy-Marx and Velikov \(2018\)](#) has a more econometric flavour: They document that the [Frazzini and Pedersen \(2014\)](#) beta measure has a built-in time-series bias that are related to the time variation of market volatility. This built-in bias mechanically predicts the “beta compression” phenomenon, even though it should not be treated as evidence in favour of the underlying theory in [Frazzini and Pedersen \(2014\)](#). Besides, they also point out that the rank weighting scheme and the hedging approach lead the BAB portfolio to tilt more towards the equal-weighting scheme than the value-weighting scheme. This biases up the performance of BAB, if transaction costs are not taken into account.

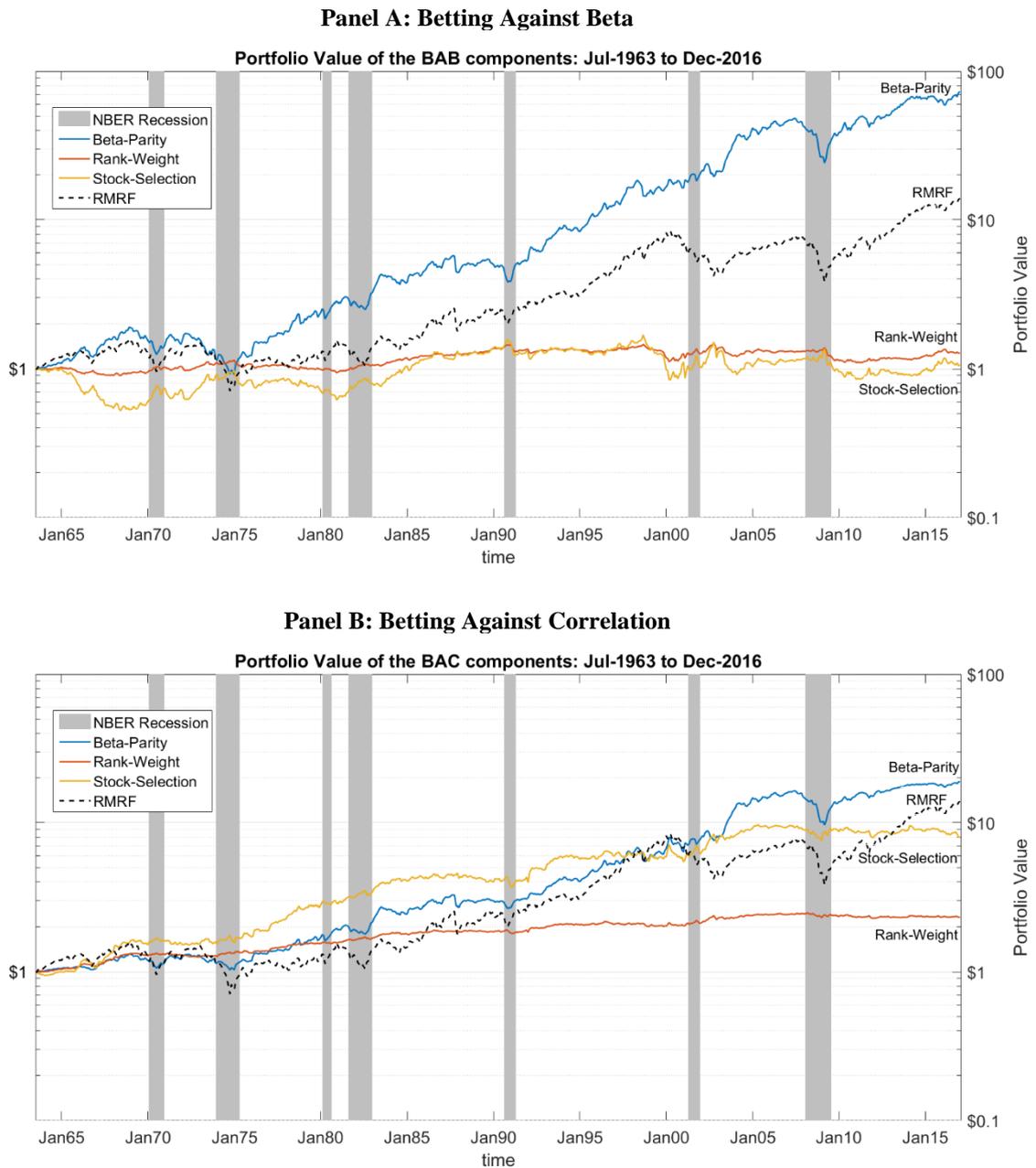
Figure 1. Betting Against Beta and Betting Against Correlation



Description: The figure plots the portfolio values of the betting against beta (BAB) strategy, the betting against correlation (BAC) strategy, and the market portfolio (RMRF), accumulated over the sample period between July 1963 and December 2016. An equal amount of \$1 for both strategies at the beginning of the sample period.

Interpretation: Both BAB and BAC outperform the market strategy (RMRF) in the sample period. BAB and BAC behave similarly over time, but BAC slightly outperform BAB in the sample period.

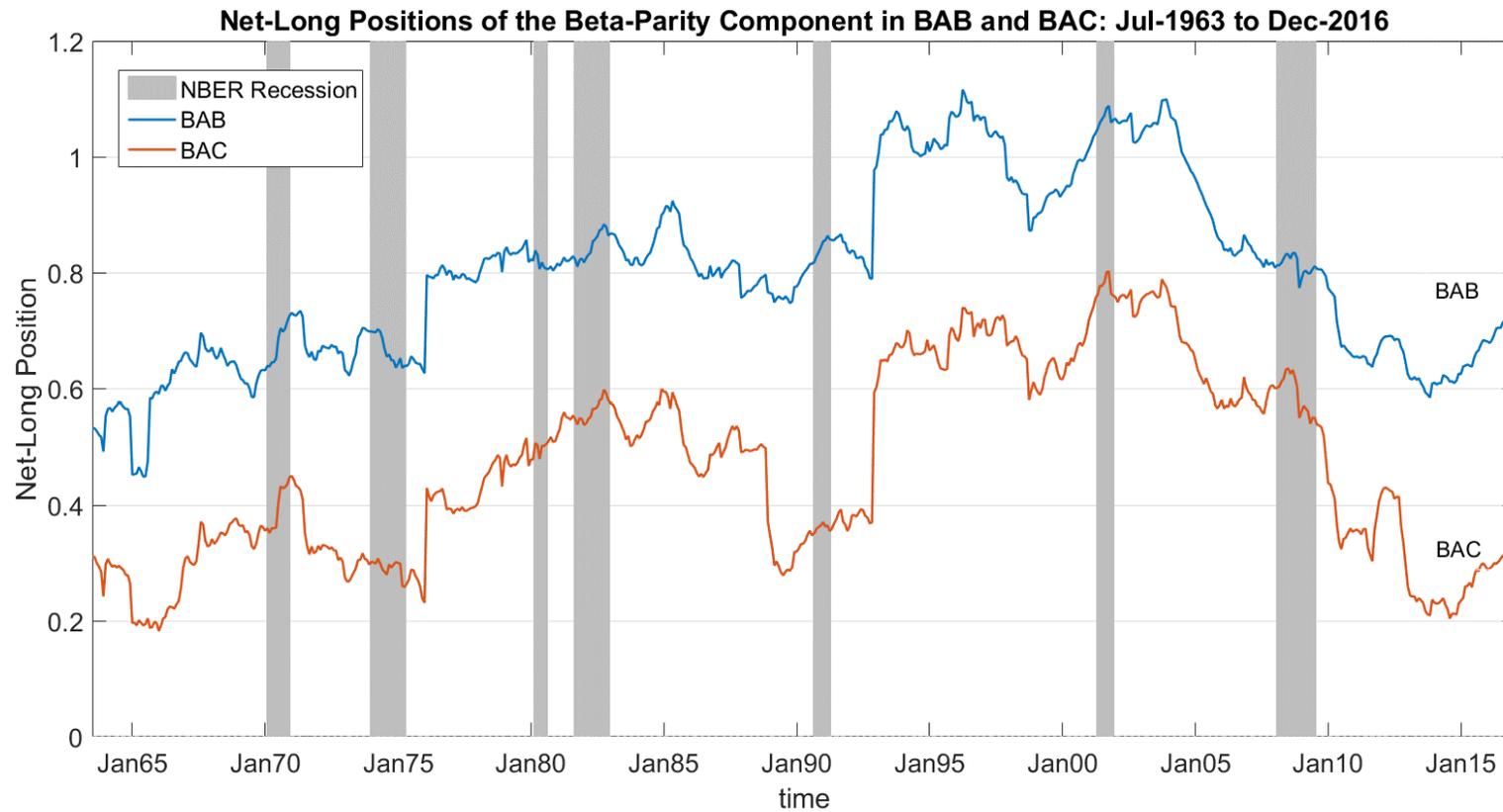
Figure 2. Components of the BAB and BAC Strategies



Description: Panel A (B) plots the portfolio values of the market portfolio (RMRF), the three component portfolios of BAB (BAC): the stock-selection component, the rank-weight component, and the beta-parity component. An equal amount of \$1 for all portfolios at the beginning of the sample period. The sample period runs from July 1963 to December 2016.

Interpretation: The beta-parity component is the dominant component in driving the BAB (BAC) performance in the sample period. For BAC, the beta-parity component and stock-selection component matter.

Figure 3. Net-Long Positions of the Beta-Parity Component in BAB and BAC



Description: The time-varying net-long positions of the beta-parity component in BAB and BAC, respectively. The sample period runs from July 1963 to December 2016.

Interpretation: The time-varying net-long position is highly persistent. It provides little, if any, market timing ability: During NBER recessions (*i.e.*, downside market), the value of the net-long position does not necessarily decrease.

Table 1. Betting Against Beta versus Betting Against Correlation Strategies (Monthly, quoted in percent)

Description: Panel A reports, for the betting against beta (BAB) and betting against correlation (BAC) strategies, the mean and standard deviation of the excess returns, the risk-adjusted returns under the CAPM model, the Fama-French three-factor model (FF3), the Fama-French five-factor model (FF5), and the Fama-French six-factor model (FF6). Panel B reports the alpha and factor loadings under the augmented seven-factor model (FF7). RMRF, SMB, HML, RMW, CMA, MOM, and STREV denotes the market, size, value, profitability, investment, momentum, and short-term reversal factors, respectively. Panel C reports the time-series spanning tests between BAB and BAC. Newey-West adjusted *t*-statistics are reported in brackets. *Adj. R*² is the adjusted R-square, and Obs. is the number of observations. The sample period is between July 1963 and December 2016. ***, **, and * denotes the statistical significance at the 1%, 5%, and 10% level, respectively.

Interpretation: Panel A: Both BAB and BAC produce significant mean excess returns and risk-adjusted returns under CAPM, FF3, FF5, and FF6 models. Panel B: First, BAB and BAC still produce significant risk-adjusted returns under the augmented seven-factor model (FF7). Second, the alpha of BAC is larger than that of BAB. Third, BAB and BAC have different risk exposures. BAB loads strongly on the market, value, profitability, investment, and momentum factors, while BAC loads strongly on the size factor. Panel C: BAB and BAC do not span each other. They contain non-overlapping price-related information.

Panel A: Return and Risk-Adjusted Returns under CAPM, FF3, FF5, and FF6 model							
	Mean	STD	CAPM Alpha	FF3 Alpha	FF5 Alpha	FF6 Alpha	Obs.
BAB	0.88***	3.40	0.92***	0.74***	0.46***	0.34**	642
[<i>t</i> -stat.]	[4.43]		[4.30]	[4.07]	[2.67]	[1.96]	
BAC	0.99***	4.59	0.93***	0.70***	0.75***	0.71***	642
[<i>t</i> -stat.]	[4.77]		[4.48]	[4.58]	[4.71]	[3.98]	

Panel B: Augmented Seven-factor Model (FF7)

	Alpha	RMRF	SMB	HML	RMW	CMA	MOM	STREV	<i>Adj. R</i> ²	Obs.
BAB	0.34*	0.11**	0.05	0.32**	0.50***	0.37**	0.17***	0.01	0.27	642
[<i>t</i> -stat.]	[1.88]	[2.14]	[0.78]	[2.57]	[4.93]	[2.41]	[2.80]	[0.22]		
BAC	0.68***	-0.01	0.81***	0.24*	-0.18	0.06	0.06	0.06	0.33	642
[<i>t</i> -stat.]	[3.80]	[-0.21]	[10.36]	[1.83]	[-1.61]	[0.35]	[0.97]	[0.79]		

Panel C: Time-series Spanning Test

	Alpha	BAB	BAC	<i>Adj. R</i> ²	Obs.
BAB	0.63***		0.25***	0.12	642
[<i>t</i> -stat.]	[3.27]		[3.11]		
BAC	0.59**	0.46***		0.12	642
[<i>t</i> -stat.]	[2.46]	[3.03]			

Table 2. Portfolio Performance during the Top 5 Best and Worst Months of the Overall Market

Description: Panels A (B) of the table list the top 5 worst (best) performing months of the overall market, measured by the excess return of the value-weighted market portfolio, RMRF. The corresponding excess returns in BAB, BAC, and their respective component portfolios: the stock-selection component (Selection), the rank-weight component (Rank), and the beta-parity component (Parity) are also reported. Position is the dollar amount of the net-long position of the beta-parity component and BAB (BAC). Average reports the mean of the 5 observations in these extreme periods. The sample period is between July 1963 and December 2016.

Interpretation: During bull (bear) markets, the beta-parity component has large gains (losses). The stock-selection and rank-weight components provide hedging benefits as they do relatively well (badly) in bear (bull) markets.

Rank	Month	RMRF	BAB					BAC				
			Selection	Rank	Parity	BAB	Position	Selection	Rank	Parity	BAC	Position
Panel A: Top 5 worst performing months of the overall market, July 1963 to December 2016												
1	1987:10	-23.2%	6.3%	2.4%	-20.7%	-12.1%	\$0.81	-0.2%	0.7%	-14.3%	-13.7%	\$0.53
2	2008:10	-17.2%	6.4%	1.7%	-15.0%	-6.9%	\$0.82	2.3%	1.2%	-11.1%	-7.6%	\$0.60
3	1998:08	-16.1%	6.8%	2.4%	-14.3%	-5.1%	\$0.94	0.5%	0.2%	-12.0%	-11.4%	\$0.65
4	1980:03	-12.9%	5.3%	2.0%	-12.7%	-5.3%	\$0.83	-3.2%	-0.7%	-8.2%	-12.2%	\$0.50
5	1973:11	-12.7%	6.9%	2.4%	-13.2%	-3.9%	\$0.70	2.3%	0.6%	-5.5%	-2.6%	\$0.30
	Average	-16.4%	6.3%	2.2%	-15.2%	-6.7%	\$0.82	0.3%	0.4%	-10.2%	-9.5%	\$0.52
Panel B: Top 5 best performing months of the overall market, July 1963 to December 2016												
1	1974:10	16.1%	-1.7%	-1.1%	6.5%	3.7%	\$0.65	-7.4%	-1.9%	3.7%	-5.6%	\$0.30
2	1975:01	13.7%	-9.2%	-4.9%	18.7%	4.6%	\$0.65	9.5%	3.2%	8.4%	21.1%	\$0.30
3	1987:01	12.5%	-4.4%	-1.8%	8.5%	2.4%	\$0.81	-0.7%	-1.0%	5.8%	4.1%	\$0.51
4	1976:01	12.2%	-6.8%	-3.1%	14.3%	4.4%	\$0.80	1.8%	0.6%	7.8%	10.2%	\$0.43
5	2011:10	11.4%	-10.0%	-3.2%	6.9%	-6.3%	\$0.66	-7.8%	-2.9%	3.7%	-7.0%	\$0.38
	Average	13.1%	-6.4%	-2.8%	11.0%	1.7%	\$0.71	-0.9%	-0.4%	5.9%	4.5%	\$0.38

Table 3. Performance Evaluation of the Components of the BAB and BAC Strategies (Monthly, quoted in percent)

Description: The table reports the mean excess returns, annualized Sharpe ratios, and the risk-adjusted returns and the factor loadings under the augmented seven-factor model (FF7) for the three components of BAB (in Panel A) and BAC (in Panel B): the stock-selection component, the rank-weight component, and the beta-parity component, denoted as Selection, Rank, and Parity, respectively. Newey-West adjusted t -statistics are reported in brackets. $Adj. R^2$ is the adjusted R-square, and Obs. is the number of observations. The sample period is between July 1963 and December 2016. ***, **, and * denotes the statistical significance at the 1%, 5%, and 10% level, respectively.

Interpretation: Panel A: The beta-parity component is the main contributor to the outperformance of BAB. The excess returns and the risk-adjusted returns of the stock-selection and rank-weight components are indifferent from zero. Panel B: All three components contribute to the outperformance of BAC, both before and after adjusting for risk. All three components load strongly and positively on the size factor.

Panel A: BAB and its Component Portfolios												
	Excess return	Sharpe	Alpha	RMRF	SMB	HML	RMW	CMA	MOM	STREV	$Adj. R^2$	Obs.
Selection	0.08	0.07	0.10	-0.39***	-0.40***	0.07	0.30***	0.22**	0.18***	-0.04	0.74	642
[t -stat.]	[0.53]		[0.85]	[-10.59]	[-8.37]	[0.99]	[5.08]	[2.13]	[4.25]	[-0.85]		
Rank	0.05	0.11	0.04	-0.16***	-0.15***	0.05*	0.14***	0.06	0.08***	-0.00	0.70	642
[t -stat.]	[0.82]		[0.74]	[-10.41]	[-7.42]	[1.74]	[5.17]	[1.43]	[4.19]	[-0.12]		
Parity	0.75***	0.65	0.20***	0.66***	0.60***	0.20***	0.06	0.09*	-0.09***	0.06**	0.90	642
[t -stat.]	[4.05]		[3.08]	[35.66]	[17.56]	[4.48]	[1.40]	[1.95]	[-3.63]	[2.04]		
BAB	0.88***	0.90	0.34*	0.11**	0.05	0.32**	0.50***	0.37**	0.17***	0.01	0.27	642
[t -stat.]	[4.43]		[1.88]	[2.14]	[0.78]	[2.57]	[4.93]	[2.41]	[2.80]	[0.22]		

Panel B: BAC and its Component Portfolios

	Excess return	Sharpe	Alpha	RMRF	SMB	HML	RMW	CMA	MOM	STREV	<i>Adj. R</i> ²	Obs.
Selection	0.36***	0.46	0.31***	-0.28***	0.34***	0.15*	-0.06	0.02	0.09**	0.02	0.31	642
[<i>t</i> -stat.]	[3.82]		[3.00]	[-8.54]	[6.23]	[1.67]	[-0.82]	[0.16]	[2.46]	[0.61]		
Rank	0.14***	0.49	0.12***	-0.12***	0.14***	0.07***	-0.01	-0.02	0.04***	-0.00	0.42	642
[<i>t</i> -stat.]	[3.93]		[3.56]	[-10.11]	[7.42]	[2.84]	[-0.25]	[-0.54]	[3.15]	[-0.25]		
Parity	0.49***	0.64	0.25***	0.38***	0.33***	0.02	-0.12***	0.06	-0.06**	0.04	0.77	642
[<i>t</i> -stat.]	[3.85]		[3.47]	[16.20]	[10.63]	[0.55]	[-3.33]	[1.11]	[-2.26]	[1.09]		
BAC	0.99***	0.75	0.68***	-0.01	0.81***	0.24*	-0.18	0.06	0.06	0.06	0.33	642
[<i>t</i> -stat.]	[4.77]		[3.80]	[-0.21]	[10.36]	[1.83]	[-1.61]	[0.35]	[0.97]	[0.79]		

Table 4. Horse Race: Decomposing the Low-Correlation Anomaly

Description: Panel A reports the time-series average of the period-by-period pairwise correlation coefficients between correlation and each candidate variable. All candidate variables are defined in **Appendix A.2**. Panel B reports the final-stage of the firm-level [Hou and Loh \(2016\)](#) decomposition (see **Section 5.2**). **Explained** is the component of the slope coefficient explained by the candidate variable. **Unexplained** is the remaining component of the slope coefficient unrelated to the candidate variable. **Total** is the sum of the explained and unexplained components. The relative proportion of the explained and unexplained parts is also reported, together with their *t*-statistics in brackets. The sample period is between July 1963 and December 2016. ***, **, and * denotes the statistical significance at the 1%, 5%, and 10% level, respectively.

The final-stage decomposition could be concisely expressed as follows:

The slope coefficient decomposed into two parts:

$$\underbrace{\widehat{\gamma}_t}_{\text{Total}} = \underbrace{\widehat{\gamma}(Z)_t^{\text{Explained}}}_{\text{Explained Coefficient}} + \underbrace{\widehat{\gamma}_t^{\text{Unexplained}}}_{\text{Unexplained Coefficient}}$$

The relative proportions of the two components:

$$\underbrace{100\%}_{\text{Total}} = \underbrace{\frac{\widehat{\gamma}(Z)_t^{\text{Explained}}}{\widehat{\gamma}_t}}_{\text{Explained Proportion}} + \underbrace{\frac{\widehat{\gamma}_t^{\text{Unexplained}}}{\widehat{\gamma}_t}}_{\text{Unexplained Proportion}}$$

Interpretation: The negative correlation-return relation is completely subsumed by firm size (lnME) in the “horse race”. The illiquidity ratio (ILLIQ) also explains a lion’s share of the low-correlation effect (*i.e.*, 73%).

Panel A: Correlation Coefficients												
	lnME	lnBTM	OP	INV	RET ^{MOM}	RET ^{STREV}	SSKEW	ISKEW	IVOL	MAX5	ILLIQ	TURN
Correlation Coef.	0.74	-0.20	0.13	0.04	-0.02	-0.01	0.14	-0.20	-0.34	-0.25	-0.36	0.21

Panel B: Final-Stage in the Horse Race

Explained: Coef. = $\overline{\hat{\gamma}(Z)_t^{Explained}}$, and Proportion = $\frac{\overline{\hat{\gamma}(Z)_t^{Explained}}}{\overline{\hat{\gamma}_t}}$

	lnME	lnBTM	OP	INV	RET ^{MOM}	RET ^{STREV}	SSKEW	ISSKEW	IVOL	MAX5	ILLIQ	TURN
Coef.	-2.02	-0.51	0.02	-0.13	0.04	-0.19	-0.12	-0.21	-0.37	-0.07	-1.09	-0.45
Proportion	132%***	32%***	-1%	8%***	-3%	13%**	8%**	14%***	24%**	5%	73%***	30%***
[t-stat.]	[4.93]	[3.66]	[-0.67]	[3.70]	[-0.60]	[2.25]	[2.15]	[3.48]	[2.03]	[0.50]	[4.60]	[2.87]

Unexplained: Coef. = $\overline{\hat{\gamma}_t^{Unexplained}}$, and Proportion = $\frac{\overline{\hat{\gamma}_t^{Unexplained}}}{\overline{\hat{\gamma}_t}}$

	lnME	lnBTM	OP	INV	RET ^{MOM}	RET ^{STREV}	SSKEW	ISSKEW	IVOL	MAX5	ILLIQ	TURN
Coef.	0.49	-1.07	-1.65	-1.50	-1.57	-1.34	-1.41	-1.32	-1.16	-1.46	-0.41	-1.05
Proportion	-32%	68%***	101%***	92%***	103%***	87%***	92%***	86%***	76%***	95%***	27%*	70%***
[t-stat.]	[-1.20]	[7.72]	[58.83]	[43.15]	[23.37]	[15.57]	[24.91]	[21.90]	[6.39]	[10.36]	[1.74]	[6.65]

Total: Coef. = $\overline{\hat{\gamma}(Z)_t^{Explained}} + \overline{\hat{\gamma}_t^{Unexplained}} = \overline{\hat{\gamma}_t}$, and Proportion = $\frac{\overline{\hat{\gamma}(Z)_t^{Explained}}}{\overline{\hat{\gamma}_t}} + \frac{\overline{\hat{\gamma}_t^{Unexplained}}}{\overline{\hat{\gamma}_t}} = 100\%$

	lnME	lnBTM	OP	INV	RET ^{MOM}	RET ^{STREV}	SSKEW	ISSKEW	IVOL	MAX5	ILLIQ	TURN
Coef.	-1.53	-1.57	-1.63	-1.63	-1.53	-1.53	-1.53	-1.53	-1.53	-1.53	-1.50	-1.50
Proportion	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%

Table 5. Regressions on the Traded Sentiment and Aggregate Volatility Factors (Monthly, quoted in percent)

Description: The table reports the factor regressions with the traded sentiment factor (FSENT) and the traded aggregated volatility factor (FVIX). The dependent variable is the beta-parity component of BAB (in Panel A) and BAB (in Panel B). The control variables are the seven traded factors: market (RMRF), size (SMB), value (HML), profitability (RMW), investment (CMA), momentum (MOM), and short-term reversal (STREV). Newey-West adjusted *t*-statistics are reported in brackets. *Adj. R*² is the adjusted R-square, and Obs. is the number of observations. The sample period runs from February 1986 to December 2016 (*i.e.*, 371 months). ***, **, and * denotes the statistical significance at the 1%, 5%, and 10% level, respectively.

Interpretation: Both the beta-parity component in BAB and BAB itself load strongly on the (traded) sentiment factor. Once accounting for the (traded) sentiment factor, the risk-adjusted return of the beta-parity component and BAB itself drops by approximately 40%.

	Panel A: $y = \text{Beta-Parity Component in BAB}$				Panel B: $y = \text{BAB}$			
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
Const.	0.30***	0.18**	0.29***	0.16**	0.37	0.23	0.33	0.19
[<i>t</i> -stat.]	[3.07]	[2.41]	[3.03]	[2.25]	[1.44]	[0.90]	[1.35]	[0.79]
FSENT		0.46***		0.46***		0.53***		0.53***
[<i>t</i> -stat.]		[7.51]		[7.52]		[3.87]		[3.75]
FVIX			-0.04	-0.05			-0.17	-0.18
[<i>t</i> -stat.]			[-0.35]	[-0.73]			[-0.57]	[-0.71]
Control 7 Factors	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
<i>Adj. R</i> ²	0.85	0.89	0.85	0.89	0.37	0.42	0.37	0.42
Obs.	371	371	371	371	371	371	371	371

Table 6. Performance Evaluation: Top 50% Market-cap Firms (98% of Total Market Capitalization)

Description: The table reports the mean excess returns, annualized Sharpe ratios, the risk-adjusted returns, and the factor loadings under the augmented seven-factor model (FF7) for the stock-selection component, the rank-weight component, and the beta-parity component of the alternative BAB (in Panel A) and BAC (in Panel B), formed by stocks with market capitalization above the median value. Newey-West adjusted t -statistics are reported in brackets. $Adj. R^2$ is the adjusted R-square, and Obs. is the number of observations. The sample period is between July 1963 and December 2016. ***, **, and * denotes the statistical significance at the 1%, 5%, and 10% level, respectively.

Interpretation: Excluding the micro-cap firms, the alphas of BAB and BAC are completely gone (as compared to **Table 3**).

Panel A: BAB and its Component Portfolios												
	Excess return	Sharpe	Alpha	RMRF	SMB	HML	RMW	CMA	MOM	STREV	$Adj. R^2$	Obs.
Selection	0.06	0.05	-0.01	-0.39***	-0.26***	0.21***	0.38***	0.22**	0.11**	0.01	0.72	642
[t -stat.]	[0.37]		[-0.05]	[-11.51]	[-6.55]	[2.82]	[6.09]	[2.43]	[2.37]	[0.27]		
Rank	0.03	0.06	-0.01	-0.15***	-0.12***	0.11***	0.17***	0.08*	0.07***	-0.01	0.71	642
[t -stat.]	[0.44]		[-0.21]	[-10.54]	[-6.37]	[3.36]	[6.02]	[1.93]	[3.45]	[-0.47]		
Parity	0.43***	0.47	-0.02	0.61***	0.33***	0.10***	0.08**	0.08**	-0.05***	0.03***	0.94	642
[t -stat.]	[3.41]		[-0.50]	[45.76]	[12.99]	[3.30]	[2.35]	[2.48]	[-4.09]	[2.72]		
BAB	0.51***	0.50	-0.04	0.07	-0.05	0.42***	0.62***	0.37***	0.13*	0.04	0.40	642
[t -stat.]	[2.69]		[-0.22]	[1.32]	[-0.71]	[3.34]	[6.13]	[2.78]	[1.89]	[0.48]		

Panel B: BAC and its Component Portfolios

	Excess return	Sharpe	Alpha	RMRF	SMB	HML	RMW	CMA	MOM	STREV	<i>Adj. R</i> ²	Obs.
Selection	0.04	0.06	-0.06	-0.24***	0.23***	0.12*	-0.02	0.07	0.17***	-0.01	0.46	642
[<i>t</i> -stat.]	[0.48]		[-0.73]	[-9.90]	[5.32]	[1.77]	[-0.25]	[0.77]	[4.95]	[-0.22]		
Rank	0.03	0.11	-0.02	-0.09***	0.11***	0.04*	0.01	0.04	0.05***	-0.00	0.44	642
[<i>t</i> -stat.]	[0.87]		[-0.73]	[-9.46]	[7.98]	[1.93]	[0.41]	[1.22]	[4.62]	[-0.06]		
Parity	0.24***	0.41	0.00	0.37***	0.19***	0.01	-0.00	0.05*	-0.03***	0.02	0.85	642
[<i>t</i> -stat.]	[2.94]		[0.05]	[20.91]	[8.20]	[0.60]	[-0.25]	[1.81]	[-3.53]	[1.16]		
BAC	0.30**	0.36	-0.08	0.05	0.53***	0.17*	-0.01	0.16	0.19***	0.01	0.40	642
[<i>t</i> -stat.]	[2.55]		[-0.67]	[1.35]	[7.46]	[1.72]	[-0.12]	[1.26]	[4.19]	[0.23]		

Appendix

A.1. Proof of the Return Decomposition

The excess return of an arbitrary portfolio can be written concisely in matrix terms as $R_p = \mathbf{r}'\mathbf{w}$, with the linear constraint that $\mathbf{1}'\mathbf{w} = c$. Here \mathbf{r} denotes the column of the excess returns of the composite stocks, \mathbf{w} denotes the column of the portfolio weights, and c is a constant. When $c = 0$, it is a zero-cost portfolio. When $c = 1$, it is the typical (unlevered) long-only portfolio. Actually, as long as $c > 0$, the portfolio represents an overall net-long position of the composite stocks.

In matrix terms, the excess return of $BAB^{Selection}$ portfolio can be written as

$$BAB^{Selection} = \mathbf{r}'\mathbf{w}^{EW},$$

where $\mathbf{w}^{EW} = \mathbf{w}_L^{EW} - \mathbf{w}_H^{EW}$. Note $\mathbf{1}'\mathbf{w}^{EW} = 0$, as it is a long-and-short, zero-cost portfolio with equal weighting scheme.

The excess return of the zero-cost, rank-weighted low-minus-high beta portfolio, BAB^{RW} , can be written as

$$BAB^{RW} = \mathbf{r}'\mathbf{w}^{RW},$$

where $\mathbf{w}^{RW} = \mathbf{w}_L^{RW} - \mathbf{w}_H^{RW}$. Similarly, $\mathbf{1}'\mathbf{w}^{RW} = 0$.

The excess return of the betting-against-beta portfolio can be written as

$$BAB = \mathbf{r}'\mathbf{w}^{BAR},$$

where $\mathbf{w}^{BAR} = \frac{1}{\beta_L}\mathbf{w}_L^{RW} - \frac{1}{\beta_H}\mathbf{w}_H^{RW}$. Note BAB leverages the low-beta stocks and deleverages the high-beta stocks, producing a net-long position in stocks. This can be shown as follows:

$$position = \mathbf{1}'\mathbf{w}^{BAR} = \frac{1}{\beta_L}\mathbf{1}'\mathbf{w}_L^{RW} - \frac{1}{\beta_H}\mathbf{1}'\mathbf{w}_H^{RW} = \frac{1}{\beta_L} - \frac{1}{\beta_H} > 0, \quad [A.1]$$

where the last equality utilized the fact: $\mathbf{1}'\mathbf{w}_L^{RW} = \mathbf{1}'\mathbf{w}_H^{RW} = 1$. Note the excess returns of BAB is calculated by netting off $\left(\frac{1}{\beta_L} - \frac{1}{\beta_H}\right) \times RF_t$ to ensure the zero-investment requirement.

Based on the above equations, the decomposition of the excess return of the betting-against-beta portfolio can be rewritten concisely in matrix form:

$$\begin{aligned} BAB &= BAB^{Parity} + BAB^{Rank} + BAB^{Selection} & [A.2] \\ &= (BAB - BAB^{RW}) + (BAB^{RW} - BAB^{Selection}) + (BAB^{Selection}) \\ &= \underbrace{\mathbf{r}'(\mathbf{w}^{BAR} - \mathbf{w}^{RW})}_{\substack{\text{contribution} \\ \text{due to} \\ \text{beta parity}}} + \underbrace{\mathbf{r}'(\mathbf{w}^{RW} - \mathbf{w}^{EW})}_{\substack{\text{contribution} \\ \text{due to} \\ \text{weighting scheme}}} + \underbrace{\mathbf{r}'\mathbf{w}^{EW}}_{\substack{\text{contribution} \\ \text{due to} \\ \text{stock selection}}} \end{aligned}$$

where $BAB^{Parity} \equiv BAB - BAB^{RW}$, and $BAB^{Rank} \equiv BAB^{RW} - BAB^{Selection}$.

With a bit of algebraic manipulation, it can be shown that the excess return of the beta-parity component, denoted as BAB^{Parity} , is

$$BAB^{Parity} = \mathbf{r}'(\mathbf{w}^{BAB} - \mathbf{w}^{RW}) = \mathbf{r}' \left[\left(\frac{1}{\beta_L} - 1 \right) \mathbf{w}_L^{RW} \right] + \mathbf{r}' \left[\left(1 - \frac{1}{\beta_H} \right) \mathbf{w}_H^{RW} \right]. \quad [A.3]$$

The last equality utilizes the definitions of $\mathbf{w}^{BAR} = \frac{1}{\beta_L} \mathbf{w}_L^{RW} - \frac{1}{\beta_H} \mathbf{w}_H^{RW}$ and $\mathbf{w}^{RW} = \mathbf{w}_L^{RW} - \mathbf{w}_H^{RW}$. For the last equality, both terms in the squared brackets have non-negative weights, indicating a net-long position in stocks (*i.e.*, a positive market exposure/beta).

A.2. Variable Definitions

Notation	Definition
ME and lnME	The market capitalization and the natural logarithm of the market capitalization of a stock, defined as the (natural logarithm of) firm's total market capitalization measured at the end of June in year t .
BTM and lnBTM	The book-to-market ratio and the natural logarithm of the book-to-market ratio, defined as the (natural logarithm of) firm's book-to-market equity measured at the fiscal year ending in $t - 1$.
OP	Operational profitability, defined as the ratio of operational profits and book equity measured at the fiscal year ending in $t - 1$, which follows from Fama and French (2017) .
INV	Asset investments, defined as the growth rate of total assets for the fiscal year ending in $t - 1$, which follows from Fama and French (2017) .
RET ^{MOM}	Intermediate-term return momentum, defined as the cumulative returns over the past 12-month rolling window, skipping the most recent month according to Fama and French (2012) .
SSKEW	Systematic skewness (also known as coskewness), defined as in Harvey and Siddique (2000) , is calculated as the slope coefficient on the squared market terms in the following regression.

$$R_i - RF = \alpha_i + \beta_i RMRF + \gamma_i RMRF^2 + \varepsilon_i$$

The above regression is performed using daily observations over the past 12-month rolling window. The estimation procedure is repeated each month to obtain the *ex ante* SSKEW measure for each month.

ISKEW Idiosyncratic skewness, defined as the skewness of the daily residual terms obtained from the same regression used to calculate the (monthly) SSKEW measure.

IVOL Idiosyncratic volatility, defined similarly as in [Ang et al. \(2006\)](#), which is the standard deviation of the residuals from the following regression.

$$R_i - RF = \alpha_i + \beta_i^{RMRF} RMRF + \beta_i^{SMB} SMB + \beta_i^{HML} HML + \varepsilon_i$$

The *ex ante* IVOL measure is constructed using the above Fama-French three-factor model using daily observations over the prior month, which requires at least 10 observations to run the regression.

MAX5 The lottery demand measure, defined as the average of the largest five daily returns in the prior month ([Bali et al. 2011](#); [Bali et al. 2017](#)).

ILLIQ Amihud illiquidity ratio, defined as the 12-month rolling average of the ratio of absolute return and the dollar trading volume ([Amihud 2002](#)).

RET^{STREV} Short-term return reversal, defined as the one-month stock returns in the prior month ([Jegadeesh & Titman 1993](#)).

TURN Turnover ratio, defined as the average daily turnover ratio over the past 12-month rolling window. A minimum number of 100 daily observations is required in order to compute the statistics.

FVIX The traded aggregated volatility factor, defined as the factor-mimicking portfolio which tracks the daily changes in the VIX index. To ensure a longer sample period, I adopt the old CBOE VIX index on S&P 100, which starts from 1 January 1986.

Following [Ang et al. \(2006\)](#), the traded factor is constructed by regressing the daily changes in VIX index on the daily excess returns of the basis assets (*i.e.*, the quintile portfolios sorted on past return sensitivities to VIX changes) with the full sample (to ensure precision):

$$\Delta VIX_t = \alpha + \sum_{i=1}^5 \gamma_i (RET_{i,t}^{VIX} - RF_t) + \varepsilon_t,$$

where $RET_{i,t}^{VIX}$ is the i -th quintile portfolio sorted on past return sensitivities to VIX changes. The traded factor is the fitted part of the above regression less the intercept term. The daily FVIX factor is then cumulated to monthly level to generate the monthly time series.

FSENT

The traded sentiment factor, defined as the factor-mimicking portfolio which tracks the monthly changes in the survey-based US sentiment index (*i.e.*, the Michigan consumer confidence index).

The traded sentiment factor is constructed (in a similar manner as the FVIX factor) by regressing the monthly changes in the consumer confidence index on the excess returns to the basis assets (*i.e.*, the quintile portfolios sorted on past return sensitivities to changes of the consumer confidence) with the full sample (to ensure precision).

$$\Delta SENT_t = \alpha + \sum_{i=1}^5 \gamma_i (RET_{i,t}^{SENT} - RF_t) + \varepsilon_t,$$

where $RET_{i,t}^{SENT}$ is the i -th quintile portfolio sorted on past return sensitivities to changes of the consumer confidence index. The traded factor is the fitted part of the above regression less the intercept term.¹⁶

A.3. Portfolio Turnover of BAB, BAC, and their Component Portfolios

This subsection addresses the legitimate concern that whether the BA-type strategies (including their component portfolios) could survive reasonable transaction costs.

Panel A of Table A3 presents the *annualized* portfolio turnover for BAB, BAC, and their stand-alone component portfolios over the sample period. Following the classification of [Novy-Marx and Velikov](#)

¹⁶ Note the coefficient γ_i serves effectively as the “weight” in the mimicking factor portfolio. Therefore, I rescale the γ_i coefficient by a factor of $|\sum \gamma_i|$, so that these weights add up to one. This helps conform the traded factor to a conventional portfolio (*i.e.*, bet \$1 on the sentiment-related risky assets and \$1 on the risk-free rate).

(2015), the BAB (BAC) strategy is a mid-turnover strategy as its annualized portfolio turnover is in between one and five times per year. The two zero-cost component portfolios of BAB (BAC), the stock selection and the rank weight components are also mid-turnover strategies. In contrast, the beta-parity component of BAB (BAC) is a low-turnover strategy with annualized portfolio turnover of 76% (48%), less than one time on a yearly basis. In general, the component portfolios of BAC have lower portfolio turnover than the counterparts of BAB.

Panel B of Table A3 provides a simple, back-of-the-envelope calculation of the transaction costs involved in implementing these investment strategies. We report the breakeven transaction costs that would eliminate the average excess returns and the risk-adjusted returns of BAB, BAC, and their component portfolios. Focusing on BAB and its three components, it seems that the beta-parity component is the most implementable strategy in practice, as its cut-off costs are 559, 339, 314, 393, 355 bps under the alternative factor models. For perspective, [Korajczyk and Sadka \(2004\)](#) estimate that the effective spread ranges from 0.16 to 141 bps with a mean of 5.59 bps for the US stocks over the period 1967 – 1999. Given the high portfolio turnover and close-to-zero (risk-adjusted) returns of the stock selection and rank weight components (see **Panel A**), their cut-off costs do not seem to withstand the requirements for practical implementation. The breakeven transaction costs for the overall BAB strategy are still economically meaningful when evaluated by the CAPM and FF3. However, when evaluated by FF5, FF6, and FF7 factor models, it could only withstand 87, 45, and 41 bps instead.

Similar pattern also holds for BAC and its components. The portfolio turnover of BAC and its components is less than that of their counterparts in BAB. For example, the breakeven transaction costs, which wipe out the FF7 alphas, for the three component portfolios in BAC are 180, 74, 616 bps, respectively. The cut-off costs for the overall BAC strategy ranges from 385 to 524 bps for the alphas under alternative model specification, indicating it remains beneficial to implement the BAC strategy when transaction costs are taken into account.

When interpreting the evidence with **Sections 4** collectively, it becomes clear that the time-series component (*i.e.*, the beta-parity component) is the most robust source for the profits of the BA-type strategies, both before and after taking transaction costs into account.

Table A1. A Numerical Example of the Portfolio Weights with Eight Stocks

Description: The table depicts the portfolio weights in a hypothetical investment universe with eight stocks of different market betas (ranging from 0.65 to 1.35). It illustrates the beta ranking of the stocks, and the portfolio weights in BAB and its three components: the stock selection component (Selection), the rank weight component (Rank), and the beta-parity component (Parity).

Interpretation: The stock selection component (Selection) and the rank weight component (Rank) are zero-cost investments with a total sum of portfolio weights of zero. The beta-parity component (Parity) has a net-long position. The BAB portfolio also has a net-long position.

Stock	Beta	Beta Ranking	Selection Weights	Rank Weights	Parity Weights	BAB Weights
1	0.65	1	0.25	0.19	0.16	0.59
2	0.75	2	0.25	0.06	0.11	0.42
3	0.85	3	0.25	-0.06	0.07	0.25
4	0.95	4	0.25	-0.19	0.02	0.08
5	1.05	5	-0.25	0.19	0.01	-0.05
6	1.15	6	-0.25	0.06	0.04	-0.15
7	1.25	7	-0.25	-0.06	0.06	-0.25
8	1.35	8	-0.25	-0.19	0.09	-0.35
Sum			0.00	0.00	0.56	0.56

Table A2. Performance Attribution of BAB and BAC under Alternative Asset Pricing Models

Description: The table reports the mean excess returns, Sharpe ratio, and the risk-adjusted returns of the stock-selection component, the rank-weight component, and the beta-parity component in BAB (Panel A) and in BAC (Panel B). HXZ4 Alpha and M4 Alpha are the intercept terms estimated by the regression of the Investment q-factor model (HXZ4) and the Mispricing four-factor model (M4), respectively. Newey-West adjusted *t*-statistics are reported in brackets. The sample period is between July 1963 and December 2016. ***, **, and * denotes the statistical significance at the 1%, 5%, and 10% level, respectively.

Interpretation: The beta-parity component remains the sole driver of the outperformance of BAB, while all three components contributes to BAC on a risk-adjusted basis.

	Panel A: Decomposition of BAB				Panel B: Decomposition of BAC			
	Selection	Rank	Parity	BAB	Selection	Rank	Parity	BAC
Excess Return	0.08	0.05	0.75***	0.88***	0.36***	0.14***	0.49***	0.99***
[<i>t</i> -stat.]	[0.53]	[0.82]	[4.05]	[4.43]	[3.82]	[3.93]	[3.85]	[4.77]
Sharpe	0.07	0.11	0.65	0.90	0.46	0.49	0.64	0.75
Proportion	9.02%	5.69%	85.29%		36.32%	13.74%	49.93%	
HXZ4 Alpha	0.06	0.02	0.28***	0.36*	0.44***	0.15***	0.32***	0.91***
[<i>t</i> -stat.]	[0.41]	[0.34]	[2.94]	[1.71]	[4.22]	[4.13]	[3.53]	[4.85]
Proportion	16.67%	5.55%	77.78%		48.35%	16.49%	35.16%	
M4 Alpha	0.09	0.04	0.26***	0.39*	0.32***	0.12***	0.28***	0.72***
[<i>t</i> -stat.]	[0.65]	[0.66]	[3.41]	[1.91]	[2.83]	[3.06]	[3.45]	[3.63]
Proportion	22.69%	9.84%	67.48%		44.94%	16.08%	38.98%	

Table A3. Portfolio Turnover and Breakeven Transaction Costs

Description: Panel A reports the annualized portfolio turnover of the stock-selection component (Selection), the rank-weight component (Rank), and the beta-parity component (Parity) in the BAB and BAC strategies. For a long-and-short portfolio, the turnover is summed over the long and short sides. Panel B reports the breakeven transaction costs that would zero out the average excess returns and the risk-adjusted returns (*i.e.*, alphas) under the CAPM model, the Fama-French three-factor model (FF3), the Fama-French five-factor model (FF5), the Fama-French six-factor model (FF6), and the augmented seven-factor model (FF7). - indicates that the breakeven transaction cost is either below the threshold of 10 basis points (bps), or undefined as the pre-cost average (risk-adjusted) return is negative. The sample period is from July 1963 to December 2016.

Interpretation: The beta-parity component in BAB (or BAC) remains the most cost-effective component with low portfolio turnover.

	Selection	Rank	Parity	BAB	Selection	Rank	Parity	BAC
Panel A: Annualized Portfolio Turnover: 196307 - 201612								
	182.58%	189.19%	76.45%	206.70%	151.46%	151.40%	47.93%	175.11%
Panel B: Break-even Transaction Costs (in bps): 196307 - 201612								
Excess Return	11.14	-	1,065.64	285.61	238.31	86.43	1,001.44	558.29
CAPM Alpha	130.48	36.16	559.48	300.25	294.38	117.65	620.81	524.62
FF3 Alpha	129.59	38.78	339.15	215.56	224.61	86.50	524.18	384.52
FF5 Alpha	26.02	-	313.93	86.77	225.01	86.05	591.25	403.77
FF6 Alpha	-	-	393.34	44.79	188.06	72.49	652.72	389.25
FF7 Alpha	-	-	355.09	41.24	180.26	73.55	615.74	372.26

Table A4. Portfolio Turnover and the Breakeven Transaction Costs for the Betting Against Beta and Betting Against Correlations Strategies

Description: Panel A reports the annualized portfolio turnover of the stock-selection component (Selection), the rank-weight component (Rank), and the beta-parity component (Parity) in the alternative BAB and BAC strategies. The alternative BAB and BAC strategies are formed by stocks with market capitalization above the median value in the cross section. For a long-and-short portfolio, the turnover is averaged over the long and short sides. Panel B reports the breakeven transaction costs that would zero out the average excess returns and the risk-adjusted returns (*i.e.*, alphas) under the augmented seven-factor model (FF7). - indicates that the breakeven transaction cost is either below the threshold of 10 basis points (bps), or undefined as the pre-cost average (risk-adjusted) return is negative. The sample period is from July 1963 to December 2016.

Interpretation: The beta-parity component in BAB (or BAC) remains the most cost-effective component with low portfolio turnover.

	Selection	Rank	Parity	BAB	Selection	Rank	Parity	BAC
Panel A: Annualized Portfolio Turnover: 196307 - 201612								
All Firms	182.58%	189.19%	76.45%	206.70%	151.46%	151.40%	47.93%	175.11%
Top 50%	187.21%	202.97%	67.39%	206.97%	163.84%	169.63%	44.40%	185.95%
Bottom 50%	255.04%	260.47%	117.62%	296.07%	239.87%	246.78%	92.75%	283.59%
Panel B: Break-even Transaction Costs (in bps): 196307 - 201612								
<u>Excess Return</u>								
All Firms	11.14	-	1,065.64	285.61	238.31	86.43	1,001.44	558.29
Top 50%	57.44	-	608.57	171.75	-	-	437.27	75.94
Bottom 50%	-	-	1,157.60	256.32	116.28	12.20	1,107.52	434.55
<u>FF7 Alpha</u>								
All Firms	-	-	355.09	41.24	180.26	73.55	615.74	372.26
Top 50%	-	-	-	-	-	-	-	-
Bottom 50%	-	-	528.52	55.56	113.41	28.00	647.78	307.24

Reference

- Amihud, Y., 2002. Illiquidity and stock returns: cross-section and time-series effects. *Journal of Financial Markets* 5, 31-56
- Ang, A., Hodrick, R.J., Xing, Y., Zhang, X., 2006. The cross-section of volatility and expected returns. *The Journal of Finance* 61, 259-299
- Ang, A., Hodrick, R.J., Xing, Y., Zhang, X., 2009. High idiosyncratic volatility and low returns: International and further U.S. evidence. *Journal of Financial Economics* 91, 1-23
- Ang, A., Liu, J., Schwarz, K., 2008. Using individual stocks or portfolios in tests of factor models. *SSRN Electronic Journal*
- Antoniou, C., Doukas, J.A., Subrahmanyam, A., 2015. Investor sentiment, beta, and the cost of equity capital. *Management Science* 62, 347-367
- Asness, C.S., Frazzini, A., Gormsen, N.J., Pedersen, L.H., 2018. Betting against correlation: Testing theories of the low-risk effect. *Journal of Financial Economics* (Forthcoming)
- Asness, C.S., Frazzini, A., Pedersen, L.H., 2012. Leverage aversion and risk parity. *Financial Analysts Journal* 68, 47-59
- Baker, M., Bradley, B., Wurgler, J., 2011. Benchmarks as limits to arbitrage: Understanding the low-volatility anomaly. *Financial Analysts Journal* 67, 40-54
- Baker, M., Wurgler, J., 2006. Investor sentiment and the cross-section of stock returns. *The Journal of Finance* 61, 1645-1680
- Baker, M., Wurgler, J., 2007. Investor sentiment in the stock market. *Journal of Economic Perspectives* 21, 129-151
- Bali, T.G., Brown, S., Murray, S., Tang, Y., 2017. A lottery demand-based explanation of the beta anomaly. *Journal of Financial and Quantitative Analysis* (Forthcoming)
- Bali, T.G., Cakici, N., Whitelaw, R.F., 2011. Maxing out: Stocks as lotteries and the cross-section of expected returns. *Journal of Financial Economics* 99, 427-446
- Barillas, F., Shanken, J., 2018. Comparing asset pricing models. *The Journal of Finance* 73, 715-754
- Black, F., 1972. Capital market equilibrium with restricted borrowing. *The Journal of Business* 45, 444-455
- Blitz, D., Vidojevic, M., 2017. The profitability of low-volatility. *Journal of Empirical Finance* 43, 33-42
- Bollerslev, T., 1990. Modelling the coherence in short-run nominal exchange rates: a multivariate generalized ARCH model. *The Review of Economics and Statistics*, 498-505
- Brunnermeier, M.K., Pedersen, L.H., 2009. Market liquidity and funding liquidity. *Review of Financial Studies* 22, 2201-2238
- Cederburg, S., O'Doherty, M.S., 2016. Does it pay to bet against beta? On the conditional performance of the beta anomaly. *The Journal of Finance* 71, 737-774
- Engle, R., 2002. Dynamic conditional correlation: A simple class of multivariate generalized autoregressive conditional heteroskedasticity models. *Journal of Business & Economic Statistics* 20, 339-350
- Fama, E.F., French, K.R., 1992. The cross-section of expected stock returns. *The Journal of Finance* 47, 427-465
- Fama, E.F., French, K.R., 1993. Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics* 33, 3-56
- Fama, E.F., French, K.R., 2006. The value premium and the CAPM. *The Journal of Finance* 61, 2163-2185
- Fama, E.F., French, K.R., 2012. Size, value, and momentum in international stock returns. *Journal of Financial Economics* 105, 457-472
- Fama, E.F., French, K.R., 2015. A five-factor asset pricing model. *Journal of Financial Economics* 116, 1-22

- Fama, E.F., French, K.R., 2017. International tests of a five-factor asset pricing model. *Journal of Financial Economics* 123, 441-463
- Fama, E.F., French, K.R., 2018. Choosing factors. *Journal of Financial Economics* 128, 234-252
- Fama, E.F., MacBeth, J.D., 1973. Risk, return, and equilibrium: Empirical tests. *Journal of Political Economy*, 607-636
- Frazzini, A., Pedersen, L.H., 2014. Betting against beta. *Journal of Financial Economics* 111, 1-25
- Friend, I., Blume, M., 1970. Measurement of portfolio performance under uncertainty. *American Economic Review* 60, 561-575
- Goyal, A., Welch, I., 2008. A comprehensive look at the empirical performance of equity premium prediction. *Review of Financial Studies* 21, 1455-1508
- Harvey, C.R., Siddique, A., 2000. Conditional skewness in asset pricing tests. *The Journal of Finance* 55, 1263-1295
- Haugen, R.A., Heins, A.J., 1975. Risk and the rate of return on financial assets: Some old wine in new bottles. *Journal of Financial and Quantitative Analysis* 10, 775-784
- Hong, H., Sraer, D.A., 2016. Speculative betas. *The Journal of Finance* 71, 2095-2144
- Hou, K., Loh, R.K., 2016. Have we solved the idiosyncratic volatility puzzle? *Journal of Financial Economics* 121, 167-194
- Jegadeesh, N., Titman, S., 1993. Returns to buying winners and selling losers: Implications for stock market efficiency. *The Journal of Finance* 48, 65-91
- Korajczyk, R.A., Sadka, R., 2004. Are momentum profits robust to trading costs? *The Journal of Finance* 59, 1039-1082
- Lemmon, M., Portniaguina, E., 2006. Consumer confidence and asset prices: some empirical evidence. *Review of Financial Studies* 19, 1499-1529
- Li, Y., 2018. Investment and profitability versus value and momentum: The price of residual risk. *Journal of Empirical Finance* 46, 1-10
- Lintner, J., 1965. The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets. *The Review of Economics and Statistics*, 13-37
- Liu, J., Stambaugh, R.F., Yuan, Y., 2018. Absolving beta of volatility's effects. *Journal of Financial Economics* 128, 1-15
- Næs, R., Skjeltorp, J.A., Ødegaard, B.A., 2011. Stock market liquidity and the business cycle. *The Journal of Finance* 66, 139-176
- Novy-Marx, R., Velikov, M., 2015. A taxonomy of anomalies and their trading costs. *The Review of Financial Studies* 29, 104-147
- Novy-Marx, R., Velikov, M., 2018. Betting against betting against beta. Working Paper
- Patton, A.J., Timmermann, A., 2010. Monotonicity in asset returns: New tests with applications to the term structure, the CAPM, and portfolio sorts. *Journal of Financial Economics* 98, 605-625
- Qiu, L., Welch, I., 2004. Investor sentiment measures (No. w10794). National Bureau of Economic Research
- Schneider, P., Wagner, C., Zechner, J., 2015. Low risk anomalies? CFS Working Paper
- Sharpe, W.F., 1964. Capital asset prices: A theory of market equilibrium under conditions of risk. *The Journal of Finance* 19, 425-442
- Welch, I., 2019. Simpler better market betas. Working Paper