

# Taking the Cochrane-Piazzesi Term Structure Model Out of Sample: More Data, Additional Currencies, and FX Implications

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## ABSTRACT

We examine the statistical term structure model of Cochrane and Piazzesi (2005) and its affine counterpart, developed in Cochrane and Piazzesi (2008), in several out-of-sample analyzes. The model's one-factor forecasting structure across bonds with two, three, four, and five years to maturity characterizes the term structures of additional major currencies in samples ending in 2003. In post-2003 data such one-factor structures again characterize each currency's term structure, but we reject equality of the coefficients across the two samples. We derive currency return forecasting implications from the Cochrane and Piazzesi (2008) affine model showing that the term structure forecasting variables in each currency should predict cross-currency investments, but we find no support for these predictions in either pre-2004 or post-2003 data, whereas the interest differentials do predict currency returns. Here too, though, we find strong evidence of parameter instability as the parameter estimates on the interest differentials change sign. In recursive out-of-sample forecasts of excess rates of return on bonds in each currency, the Cochrane and Piazzesi

(2008) term structure forecasting models fail to beat forecasts from the historical average excess rates of return. Graphical analysis indicates that the instability in the forecasting models' parameters begins in the global financial crisis.

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*JEL Codes:* G12, G15

## **1 Introduction**

In two seminal papers, Cochrane and Piazzesi (2005) and Cochrane and Piazzesi (2008) document a strong one-factor structure in the unconstrained predictability of one-year-ahead excess returns on U.S. dollar zero-coupon bonds of several maturities. The first principal component of the four unconstrained expected excess returns explains 99.7% of the variance of these expected returns. This finding motivates a constrained model for which Cochrane and Piazzesi (2005) note (p. 142), "The same function of forward rates forecasts holding period returns at all maturities. Longer maturities just have greater loadings on this same function." They model this constrained system with a two-step estimation in which they first estimate the forecasting factor, which is labeled the 'CP factor' in much of the subsequent literature, by regressing the average future annual excess rates of return on two, three, four, and five year bonds onto a set of forward rates or forward spreads. Then, they regress each excess return on the CP forecasting factor to get the factor loadings. The constrained model fits the unconstrained expected excess return data remarkably well. They also demonstrate that their bond market forecasting factor predicts excess returns in the U.S. stock market, which strengthens the case that it is capturing risk premiums. Cochrane and Piazzesi (2008) reverse engineer an affine term structure model (ATSM) that has the forecasting properties uncovered in the constrained regressions.

This paper examines whether analogous one-factor forecasting structures exist in the predictability of the excess returns on zero-coupon bonds denominated in other currencies, and we find that they do. For all currencies in the G10 except the New Zealand dollar, we find that the first principal component of the four unconstrained expected returns explains

over 99% of the variance in expected returns in our first sample. We initially examine samples that end in 2003, the end of the sample in Cochrane and Piazzesi (2005). For the constrained model, we find that the factor loadings are quite similar across currencies, ranging between 0.37 and 0.47 for the two-year bond, between 0.80 and 0.87 for the three-year bond, and between 1.19 and 1.23 for the four-year bond. In contrast to this similarity, we find the parameter estimates on the forward spreads that generate the CP factors to be quite dissimilar across the different currencies. These estimates could reflect differences in the ultimate sources of risk premiums due to alternative monetary and fiscal policies and other macroeconomic shocks across the countries, they could reflect short samples with a corresponding failure of rational expectations econometrics, or they could be the result of multicollinearity, which seems particularly likely in some countries where we see large coefficients of approximately equal and opposite signs.

We then examine data from 2004-2016, and we again find strong one-factor forecasting structures in all nine currencies with the first principal component of the four unconstrained expected returns explaining over 95% of the variance in the unconstrained expected returns in each case. While the factor loadings in the second sample are quite similar to those of the first sample, ranging between 0.28 and 0.38 for the two-year bond, between 0.73 and 0.82 for the three-year bond, and between 1.22 and 1.29 for the four-year bond, the estimates of the slope coefficients on the CP factors change quite a bit. Consequently, we generally reject the hypothesis of equality of the coefficients in the CP factors across the two samples. For example, the coefficient estimate on the U.S. two-year forward spread changes from 2.32, with a standard error of (1.63), to -7.57 (2.62), while the coefficient on the Japanese two-year forward spread changes from -14.99 (3.75) to 3.33 (1.73).

Because foreign exchange rates and the term structures of interest rates in the two currencies are closely linked in theory to the stochastic discount factors of the two currencies, we derive new predictions from the Cochrane and Piazzesi (2008) ATSM for the predictability of excess rates of return on uncovered foreign currency investments. We find that the CP factors from the bond markets of the two currencies and their squared values should forecast the excess rate of return on uncovered foreign currency investments between the two currencies. Because tests of uncovered interest rate parity generally find that interest rate differentials predict excess currency returns, which is not true in our theoretical derivation of

currency risk premiums from the affine model, we investigate whether the CP factors of the two currencies drive out the interest rate differentials in predicting excess currency returns. We find that they do not as the interest differentials remain the only significant predictors. For example, in forecasting the excess return to investing the U.S. dollar in the British pound, we find a coefficient on the pound-dollar interest differential of 1.86 (0.89) while the coefficients on the USD and GBP CP factors are 0.24 (0.62) and -0.24 (1.20), respectively. In this first sample, the coefficients on the interest differentials are also mostly greater than one, which is consistent with the typical findings in the literature that rejects the uncovered interest rate parity hypothesis and motivates the carry trade. In this analysis, though, we also show substantial differences in estimated coefficients across our two samples with the coefficients on the interest differentials generally reversing sign in the second sample, which is consistent with the deterioration of the carry trade during and after the global financial crisis.

Although the Cochrane and Piazzesi (2005) model characterizes the bond market data quite well in any given sample, as noted above, we generally reject that the parameters are the sample across our two samples. As an additional investigation of the stability of the model, we explore the recursive out-of-sample predictions of the Cochrane and Piazzesi (2005) model. Recursive forecasts of excess rates of return in the bond markets from the estimated Cochrane and Piazzesi (2005) model are generally unable to beat the recursive forecasts from the historical averages of excess rates of return on bonds. While these findings are perhaps unsurprising given that the out-of-sample period contains the global financial crisis, they demonstrate the necessity of modeling risk premiums while allowing for structural change. We leave this challenging task for future research. For completeness, we also show that recursive estimation and forecasting from the empirical version of our international extension of the Cochrane and Piazzesi (2008) model similarly fails to beat historical average excess currency returns in out-of-sample tests. This is completely unsurprising given the in-sample failure of the CP factors to predict currency returns and the changes in the signs of the coefficients across the two samples.

The last sections of the paper relate our findings to the existing literature and discuss some alternative modeling approaches that may improve our understanding of the term structure of interest rates and the predictability of bond and currency returns.

## 2 The Cochrane-Piazzesi Term Structure Model

In presenting the model, we mostly adopt the notation of Cochrane and Piazzesi (2005). The presentation can be thought of as referring to the term structure of a generic currency. For simplicity, we suppress currency subscripts in laying out the basic term structure model.

The natural logarithm of the price of a pure discount bond at time  $t$  that matures in  $n$  years and pays one unit of currency at that time is denoted  $p_t^{(n)}$ . The time subscript  $t$  indexes years, in which case months, which are the observation interval of the data, are indicated with  $(1/12)$  fractions of a year. The continuously compounded annualized yield on an  $n$ -year bond is therefore

$$y_t^{(n)} \equiv -\frac{1}{n} p_t^{(n)}.$$

The natural logarithm of the one-year forward rate at time  $t$  for loans between  $t + n - 1$  and  $t + n$  is

$$f_t^{(n)} \equiv p_t^{(n-1)} - p_t^{(n)}.$$

The forward spreads between these forward rates and the one-year yield are

$$f s_t^{(n)} \equiv f_t^{(n)} - y_t^{(1)}.$$

The continuously compounded rate of return from buying an  $n$ -year bond at time  $t$  and selling it one year later is

$$r_{t+1}^{(n)} \equiv p_{t+1}^{(n-1)} - p_t^{(n)},$$

in which case the excess rate of return is

$$r x_{t+1}^{(n)} \equiv r_{t+1}^{(n)} - y_t^{(1)}.$$

The average of four excess rates of return on bonds with two through five years to maturity is

$$\overline{r x}_{t+1} \equiv (1/4) \sum_{n=2}^5 r x_{t+1}^{(n)}.$$

Bold symbols without superscripts indicate vectors or matrices. For example, the vector of excess rates of return on bonds with two through five years to maturity is

$$\mathbf{r} x_{t+1} \equiv [r x_{t+1}^{(2)}, r x_{t+1}^{(3)}, r x_{t+1}^{(4)}, r x_{t+1}^{(5)}]^T.$$

When used as right-hand-side variables in a regression, such vectors include a constant. For example,

$$\mathbf{fs}_t \equiv \left[ 1, f s_t^{(2)}, f s_t^{(3)}, f s_t^{(4)}, f s_t^{(5)} \right]^T.$$

Whereas Cochrane and Piazzesi (2005) use the levels of the forward rates as forecasting variables for the excess rates of return on bonds, we follow Cochrane and Piazzesi (2008) and use the averages of the three most recent monthly spreads as the forecasting variables:<sup>1</sup>

$$\overline{\mathbf{fs}}_t \equiv (1/3) \sum_{j=0}^2 \mathbf{fs}_{t-(j/12)}.$$

The unconstrained forecasting system for the excess rates of return in a particular currency's bond market can therefore be written as

$$\mathbf{rx}_{t+1} = \boldsymbol{\beta} \overline{\mathbf{fs}}_t + \boldsymbol{\varepsilon}_{t+1}, \quad (1)$$

where  $\boldsymbol{\beta}$  represents the  $(4 \times 5)$  matrix of responses of excess returns to the forward spreads. Cochrane and Piazzesi (2005, 2008) motivate their constrained one-factor model of expected bond returns from the finding that the first principal component of the unconstrained expected returns in the system of equations (1) explains over 99% of the variance of these expected returns.

This constrained model of a vector of expected returns was first developed by Hansen and Hodrick (1983) and Gibbons and Ferson (1985) who postulated that a set of expected returns could be proportional to a common unobserved factor,  $v_t$ :

$$E_t(\mathbf{rx}_{t+1}) = \mathbf{b} v_t, \quad (2)$$

where  $\mathbf{b} \equiv [b_2, b_3, b_4, b_5]^T$ . By projecting the unobserved factor onto some observed information, in this case  $\overline{\mathbf{fs}}_t$ , one can write

$$v_t = \boldsymbol{\gamma}^T \overline{\mathbf{fs}}_t + \xi_t, \quad (3)$$

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<sup>1</sup>Cochrane and Piazzesi (2008) note that levels of forward rates have near unit root components which are unlikely to match up with rational risk premiums. Forward spreads are more likely to be stationary and hence to capture risk premiums. See also the discussion in Cochrane (2015) who advocates using moving averages of forward spreads to avoid spurious predictability due to measurement error in the yields.

where by the properties of linear prediction, the error term,  $\xi_t$ , is orthogonal to the right-hand-side variables.

Substituting equation (3) into equation (2) and assuming rational expectations produces a constrained single factor forecasting system that can be written as

$$\mathbf{r}x_{t+1} = \mathbf{b}\boldsymbol{\gamma}^T \overline{\mathbf{f}}\mathbf{s}_t + \boldsymbol{\varepsilon}_{t+1}, \quad (4)$$

where  $\boldsymbol{\varepsilon}_{t+1}$  now represents both the rational expectations forecast errors for each equation plus  $\mathbf{b}\xi_t$ . Estimation can be done with the generalized method of moments (GMM) of Hansen (1982) because  $\boldsymbol{\varepsilon}_{t+1}$  is orthogonal to  $\overline{\mathbf{f}}\mathbf{s}_t$ . Because  $\mathbf{b}$  and  $\boldsymbol{\gamma}^T$  are multiplied together, some identifying constraint must be imposed on the estimation, and we follow Cochrane and Piazzesi (2005) in imposing the constraint on  $\mathbf{b}$  that the average of the  $b_n$ 's equals one:

$$(1/4) \sum_{n=2}^5 b_n = 1.$$

Whereas the unconstrained model in equation (1) has 20 parameters, the constrained model in equation (4) has 8 free parameters, 5 in  $\boldsymbol{\gamma}$  and 3 in  $\mathbf{b}$ .

As Cochrane and Piazzesi (2005) note, estimation of the constrained model can be done in two steps. The first step is an OLS regression of the average excess rate of return on the four long-horizon bonds on the average of the forward spreads as in

$$\overline{rx}_{t+1} = \boldsymbol{\gamma}^T \overline{\mathbf{f}}\mathbf{s}_t + \overline{\boldsymbol{\varepsilon}}_{t+1}. \quad (5)$$

This imposes the constraint that the average of the  $b_n$ 's equals one.<sup>2</sup> The second step involves OLS regressions without constant terms of three individual excess rates of return on the fitted value from equation (5):

$$rx_{t+1}^{(n)} = b_n \left( \widehat{\boldsymbol{\gamma}}^T \overline{\mathbf{f}}\mathbf{s}_t \right) + \boldsymbol{\varepsilon}_{t+1}^{(n)}. \quad (6)$$

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<sup>2</sup>Rather than equal-weighting the excess returns in the first step of the constrained model, Cochrane and Piazzesi (2008) develop an alternative weighting system that relies on an eigenvalue decomposition of the covariance matrix of the expected excess returns from the unrestricted regressions and takes the weights on the excess returns to be the normalized eigenvector associated with the dominant eigenvalue. We follow the approach in the original paper because it is the primary way in which the CP factor has been estimated in the literature, but we note that the alternative approach provides additional insights and may be more useful in understanding other asset markets.

We use the two-year, three-year, and four-year maturities.

### 2.1 The Affine Model with Restrictions

Before discussing the results of estimating the constrained model, we first introduce the affine term structure model that Cochrane and Piazzesi (2008) reverse engineer to be consistent with the forecasting properties from the constrained regressions of excess returns of the long-term bonds on forward spreads.

In a generic ATSM the continuously compounded short-term interest rate is postulated to be a linear function of a  $K$ -dimensional vector of state variables,  $\mathbf{X}_t$  :

$$r_t = \delta_0 + \delta_1^\top \mathbf{X}_t.$$

The state variables are assumed to follow a first-order vector autoregression:

$$\mathbf{X}_{t+1} = \boldsymbol{\mu} + \boldsymbol{\Phi} \mathbf{X}_t + \boldsymbol{\Sigma} \mathbf{v}_{t+1}.$$

The vector of innovations,  $\mathbf{v}_{t+1}$ , is assumed to be  $N(0, \mathbf{I}_K)$ , and the covariance matrix of the state variables is  $\boldsymbol{\Sigma} \boldsymbol{\Sigma}^\top$ . The natural logarithm of the stochastic discount factor is specified to be

$$m_{t+1} = -r_t - \frac{1}{2} \boldsymbol{\lambda}_t^\top \boldsymbol{\lambda}_t - \boldsymbol{\lambda}_t^\top \mathbf{v}_{t+1}, \quad (7)$$

and the innovations to the state variables are thus potential sources of risks. Finally, the prices of these risks are also postulated to be affine functions of the state variables:

$$\boldsymbol{\lambda}_t = \boldsymbol{\lambda}_0 + \boldsymbol{\lambda}_1 \mathbf{X}_t,$$

where  $\boldsymbol{\lambda}_0$  is  $K \times 1$ , and  $\boldsymbol{\lambda}_1$  is  $K \times K$ .

The solution of such an affine term structure model uses the basic no-arbitrage asset pricing model,

$$E_t \left( M_{t+1} R_{t+1}^{(n)} \right) = 1, \quad (8)$$

where  $M_{t+1} = \exp(m_{t+1})$  and  $R_{t+1}^{(n)} = \exp\left(r_{t+1}^{(n)}\right)$ . Substituting for  $M_{t+1}$  and  $R_{t+1}^{(n)}$  in equation (8) and solving the conditional expectation provides



the solution of the ATSM in which the natural logarithms of the bond prices are found to be affine functions of the state variables:

$$p_t^{(n)} = A_n + \mathbf{B}_n^\top \mathbf{X}_t. \quad (9)$$

The recursive formulas for the  $A_n$  and  $\mathbf{B}_n$  coefficients in equation (9) are given in Appendix B.

From the solution of the the ATSM, one finds that the expected excess rates of return on bonds are also affine functions of the state variables:

$$E_t \left( r x_{t+1}^{(n)} \right) = -(1/2) \mathbf{B}_{n-1}^\top \Sigma \Sigma^\top \mathbf{B}_{n-1} + \mathbf{B}_{n-1}^\top \Sigma \lambda_0 + \mathbf{B}_{n-1}^\top \Sigma \lambda_1 \mathbf{X}_t. \quad (10)$$

The three terms on the right-hand side of equation (10) are a Jensen's inequality term related to the variance of the rate of return, a constant risk premium, and a time-varying risk premium. In the general ATSM without constraints on the parameters, time-varying expected excess rates of returns on bonds would be driven by the  $K$  state variables. This would be inconsistent with the empirical finding that only one state variable is required to forecast economically interesting variation in expected excess returns.

To reconcile the theoretical analysis with the empirical findings, Cochrane and Piazzesi (2008) postulate that the term structure of interest rates depends on four state variables, but they constrain the prices of risks such that only one of these variables drives expected excess rates of return. At least since Litterman and Scheinkman (1991) it has been known that time variation in zero-coupon bond yields can be effectively modeled with the first three principal components of the yields, which are a level effect,  $l_t$ , a slope effect,  $s_t$ , and a curvature effect,  $c_t$ . Hence, these three variables are present as state variables. The fourth state variable is the "return forecasting factor," that is, the CP factor:

$$x_t \equiv \widehat{\gamma}^\top \overline{\mathbf{f}}_t. \quad (11)$$

The state vector can therefore be written as  $\mathbf{X}_t = (x_t, l_t, s_t, c_t)^\top$ .<sup>3</sup> Because Cochrane and Piazzesi (2005) empirically find a very strong one-factor structure in the unconstrained model in equation (1), Cochrane and Piazzesi

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<sup>3</sup>Cieslak and Povala (2015) develop a similar ATSM with three state variables: the expected or 'trend' rate of inflation, a real factor orthogonal to expected inflation, and a forecasting variable that only affects the prices of the two risks.

(2008) place a set of restrictions on the prices of risks,  $\lambda_t$ , such that a one-factor structure emerges in equation (10). Since  $x_t$  is the only variable that can predict expected returns, the columns of the  $\lambda_1$  matrix other than the first must be zeros. Cochrane and Piazzesi (2008) also find that the  $b_n$  coefficients of the empirical model line up nicely with the covariances of the excess returns with the innovations to the level factor. This motivates the full set of restrictions such that

$$\lambda_t = \begin{bmatrix} 0 \\ \lambda_{0l} \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ \lambda_{1l} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_t \\ l_t \\ s_t \\ c_t \end{bmatrix}. \quad (12)$$

Thus, although innovations in the four state variables drive the zero-coupon yields and bond prices at all maturities, the only innovation that affects the bond market's stochastic discount factor and hence affects expected rates of return on bonds is the innovation in the level of the term structure, denoted  $v_{l,t+1}$ , and the time varying price of this risk is driven by the return forecasting factor. That is,

$$\lambda_t^\top v_{t+1} = \begin{bmatrix} 0 \\ (\lambda_{0l} + \lambda_{1l}x_t) v_{l,t+1} \\ 0 \\ 0 \end{bmatrix}. \quad (13)$$

Substituting from equation (12) into equation (10) gives

$$E_t \left( r x_{t+1}^{(n)} \right) = -(1/2) \mathbf{B}_{n-1}^\top \Sigma \Sigma^\top \mathbf{B}_{n-1} + \mathbf{B}_{n-1}^\top \Sigma \begin{bmatrix} 0 \\ (\lambda_{0l} + \lambda_{1l}x_t) \\ 0 \\ 0 \end{bmatrix}. \quad (14)$$

While equation (14) is quite close to the constrained econometric model in equation (4) in that each expected return loads with a different coefficient onto the common forecasting factor, the constrained model makes the additional assumption that the constant terms in the equations share the same proportionality as the slope coefficients. The Jensen's inequality terms do not scale in the same way, which makes this assumption technically incorrect. Because these terms are generally considered to be small, in

what follows we ignore this issue and follow the approach of Cochrane and Piazzesi (2008).<sup>4</sup>

### 3 Estimation Results for Nine Term Structures

In this section we estimate the Cochrane and Piazzesi (2005) empirical model for the zero-coupon government bond yields of nine of the G-10 currencies: the Australian dollar, AUD; the Canadian dollar, CAD; the Swiss franc, CHF; the euro, EUR, spliced with data from the Deutsche mark; the British pound, GBP; the Japanese yen, JPY; the Norwegian krone, NOK; the Swedish krona, SEK; and the USD. After reviewing the available term structure data for the New Zealand dollar, we viewed it as unreliable and therefore did not include it in our analysis. Sources of data are described in Appendix A.

We present the results in two sections corresponding to samples of data that would have been available when the original model was first estimated and samples of data that subsequently became available. Because the last observation on the dependent variable in the the first data set is December 2003, we refer to these data as the pre-2004 sample. We begin observations on the dependent variable in the second data set in December 2004 to avoid overlap with the first data set, and we refer to these data as the post-2003 sample. To allow for samples that coincide with the exchange rate data, the dependent variables for the first sample begin in 1974:12 for the USD, the GBP and the EUR; in 1989:03 for the CHF; in 1987:03 for the CAD; in 1986:03 for the JPY; in 1988:04 for the AUD; in 1988:03 for the SEK; and in 1999:03 for the NOK. The first sample is particularly short for the NOK, so we do not think those results are particularly informative, but we choose to include the results simply because the NOK is included in the post-2003 analysis.

#### 3.1 Results with Pre-2004 Data

Table 1 reports the estimation of the constrained model in equation (4) with the two-step OLS procedure described above.

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<sup>4</sup>The Online Appendix presents some results of a model that relaxes this restrictive assumption by allowing for separate constants at each maturity. We find that the relevant parameters associated with the time-varying forecasts of the two models are quite close and inference is quite similar.

Table 1: The Single Factor Cochrane and Piazzesi (2005) Model

CUR	$b_2$	$b_3$	$b_4$	$\gamma_1$	$\gamma_2$	$\gamma_3$	$\gamma_4$	$\gamma_5$	$\chi^2(4)$	$R^2$	%PC1
USD1	0.46 (0.02)	0.86 (0.02)	1.23 (0.01)	-0.04 (0.63)	2.32 (1.63)	3.65 (1.33)	1.73 (0.95)	-4.67 (0.58)	100.10 (0.00)	0.39	0.997
USD2	0.35 (0.06)	0.78 (0.06)	1.24 (0.01)	0.06 (0.69)	-7.57 (2.61)	2.17 (1.76)	2.57 (0.60)	-0.80 (0.61)	25.44 (0.00)	0.37	0.973
GBP1	0.44 (0.05)	0.86 (0.03)	1.21 (0.02)	1.04 (0.53)	-5.40 (4.31)	11.24 (11.72)	-6.54 (14.23)	0.39 (5.97)	9.46 (0.05)	0.09	0.996
GBP2	0.37 (0.08)	0.82 (0.06)	1.23 (0.02)	0.17 (0.76)	-8.25 (7.48)	-5.32 (19.84)	27.01 (24.71)	-15.88 (10.43)	67.63 (0.00)	0.29	0.964
EUR1	0.44 (0.03)	0.86 (0.03)	1.21 (0.01)	1.02 (0.90)	-1.77 (5.07)	-4.19 (14.33)	16.47 (19.38)	-10.50 (9.17)	9.66 (0.05)	0.12	0.992
EUR2	0.34 (0.06)	0.77 (0.05)	1.22 (0.01)	1.61 (1.49)	-21.01 (14.67)	35.23 (33.29)	-24.91 (35.70)	6.39 (14.36)	6.56 (0.16)	0.08	0.973
CHF1	0.47 (0.08)	0.86 (0.04)	1.19 (0.03)	0.78 (1.45)	-1.45 (11.88)	-9.62 (30.09)	26.03 (38.64)	-14.60 (18.01)	22.24 (0.00)	0.24	0.994
CHF2	0.28 (0.06)	0.74 (0.06)	1.25 (0.01)	0.12 (0.47)	-10.41 (2.88)	16.56 (8.60)	-14.21 (11.62)	5.73 (5.46)	164.11 (0.00)	0.40	0.983
CAD1	0.46 (0.03)	0.86 (0.03)	1.20 (0.01)	1.64 (0.84)	0.53 (2.34)	-5.16 (5.25)	19.68 (6.95)	-13.59 (4.03)	34.29 (0.00)	0.23	0.993
CAD2	0.38 (0.07)	0.81 (0.06)	1.22 (0.02)	1.59 (1.04)	3.77 (3.60)	-13.41 (6.40)	21.50 (6.49)	-10.71 (2.78)	17.68 (0.00)	0.16	0.948
JPY1	0.37 (0.02)	0.81 (0.02)	1.23 (0.00)	1.00 (1.23)	-14.98 (3.76)	17.33 (4.16)	-0.29 (2.06)	-5.00 (1.87)	25.87 (0.00)	0.25	0.997
JPY2	0.30 (0.03)	0.73 (0.04)	1.29 (0.02)	0.40 (0.22)	3.33 (1.75)	-1.04 (2.70)	4.04 (1.74)	-3.31 (1.12)	36.36 (0.00)	0.34	0.994
AUD1	0.44 (0.02)	0.87 (0.02)	1.21 (0.01)	0.82 (1.07)	-32.92 (12.15)	107.79 (42.03)	-138.85 (61.52)	61.21 (29.98)	15.94 (0.00)	0.11	0.996
AUD2	0.33 (0.11)	0.81 (0.08)	1.22 (0.02)	1.50 (1.02)	3.03 (2.59)	2.68 (2.08)	-4.15 (1.46)	0.90 (1.11)	17.56 (0.00)	0.12	0.970

(Continued)

Table 1: (Continued)

CUR	$b_2$	$b_3$	$b_4$	$\gamma_1$	$\gamma_2$	$\gamma_3$	$\gamma_4$	$\gamma_5$	$\chi^2(4)$	$R^2$	%PC1
SEK1	0.39 (0.05)	0.80 (0.03)	1.21 (0.02)	1.27 (1.43)	30.34 (13.92)	-68.63 (39.50)	73.43 (51.60)	-29.16 (23.65)	5.51 (0.24)	0.11	0.998
SEK2	0.37 (0.08)	0.80 (0.06)	1.22 (0.02)	1.87 (1.10)	15.10 (8.60)	-40.53 (17.47)	47.55 (19.23)	-19.82 (8.42)	11.40 (0.02)	0.12	0.974
NOK1	0.44 (0.03)	0.87 (0.02)	1.21 (0.01)	1.97 (1.23)	32.85 (19.32)	-127.27 (69.50)	200.04 (98.33)	-101.71 (46.53)	116.38 (0.00)	0.42	0.987
NOK2	0.37 (0.13)	0.79 (0.12)	1.22 (0.02)	0.99 (0.97)	-7.11 (10.76)	6.46 (23.26)	-2.84 (23.36)	0.63 (8.99)	2.65 (0.62)	0.07	0.870

**Description:** The Table reports coefficient estimates for the two-step estimation of the constrained single factor model for two sample periods. The sample periods for the dependent variables in the first sample labeled CUR1 all end in 2003:12. These samples begin in 1974:12 for the USD, the GBP and the EUR; in 1989:03 for the CHF; in 1987:03 for the CAD; in 1986:03 for the JPY; in 1988:04 for the AUD; in 1988:03 for the SEK; and in 1999:03 for the NOK. For the second sample period labeled CUR2 the dependent variables for all currencies begin in 2004:12 and end in 2016:12. The first step in the estimation involves OLS regressions of the average one-year excess rates of return on bonds with two through five years to maturity,  $\bar{r}x_{t+1}$ , on a constant and the average of the current value and two monthly lags of the four forward spreads,  $\bar{fs}_t$ :

$$\bar{r}x_{t+1} = \gamma^T \bar{fs}_t + \bar{\varepsilon}_{t+1}.$$

The second step involves OLS regressions of the individual excess rates of return on bonds with two through four years to maturity on the fitted value from the first step:

$$r x_{t+1}^{(n)} = b_h \left( \hat{\gamma}^T \bar{fs}_t \right) + \varepsilon_{t+1}^{(n)}$$

Standard errors in the first step are based on the usual GMM versions from OLS orthogonality conditions, and the standard errors in the second step allow for the estimation error in the first step. All standard errors are constructed with 18 Newey-West (1987) lags and are in parentheses. The  $\chi^2(4)$  statistic tests the hypothesis that  $\gamma_2$  through  $\gamma_5$  equal zero with  $p$ -values in angled brackets. The  $R^2$  is from the first step regression. The column labeled %PC1 presents the percentage of the variance of the unconstrained estimates of the four excess rates of return explained by their first principal component.

**Interpretation:** There is strong evidence for a single factor forecasting structure of expected excess returns for all currencies in each of the two non-overlapping samples. The coefficient estimates of the  $b$ 's are remarkably similar across currencies and across the two samples. The estimates of the  $\gamma$ 's are not similar across currencies, and we observe large differences in the  $\gamma$  estimates across the two samples.

Both samples are included in Table 1 with the notation CUR1 or CUR2 indicating the currency of denomination of the bonds and either the first or second sample period, respectively. We report asymptotic GMM standard errors that account for the overlapping forecasts and the fact that the second step in the estimation uses estimated coefficients from the first step.<sup>5</sup>

Although the unconstrained results are not reported because of the large number of parameters, the first thing to notice in Table 1 is the strong support for the single factor forecasting structure of expected excess returns in each of the nine term structures in the unconstrained estimations. The far right column labeled %PC1 presents the proportion of the variance of the four unconstrained estimates of the excess rates of return, denominated in the particular currency of that row, that is explained by their first principal component. For all the currencies in the first sample, the first principal component explains at least 98.8% of the variance of these expected excess returns. This evidence represents strong support for the one-factor forecasting model of expected excess bond returns in each of the currencies.

The second noteworthy aspect of Table 1 is the remarkable similarity in the coefficient estimates of  $b_2$ ,  $b_3$ , and  $b_4$  across currencies. The estimated values of  $b_2$  range from 0.37 for the JPY to 0.47 for the CHF. The estimated values of  $b_3$  range from 0.80 for the SEK to 0.87 for the AUD. The estimated values of  $b_4$  range from 1.19 for the CHF to 1.23 for the USD and the JPY. From equation (14) we see that the estimated values of the  $b_n$ 's in an ATSM differ because of the different values of  $\mathbf{B}_{n-1}^T \Sigma$  associated with the CP factor. The recursive solution for the  $\mathbf{B}_n^T$  in equation (B3) indicates that values of  $\mathbf{B}_n^T$  change as  $\Phi^*$ , the risk neutral autocorrelation matrix of the state variables, is raised to higher powers. Thus, the finding of similar values of the  $b_n$ 's across countries indicates that if we were to estimate an ATSM for each currency, the resulting  $\Phi^*$  estimates would be quite similar across currencies. At this point, we leave this as a conjecture for future research.

While there is considerable variety in the estimates of the  $\gamma'_j$ s across

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<sup>5</sup>The standard errors could be constructed as in Hansen and Hodrick (1980), by equally weighting the 11 lagged covariances that are non-zero by construction when forecasting annual excess returns with overlapping monthly data. These standard errors are not guaranteed to be positive definite, and in fact in some cases they were not. Consequently, we rely on Newey and West (1987) standard errors using 18 lags as in Cochrane and Piazzesi (2005).

the different currencies, the  $\chi^2(4)$  statistics for all currencies except the SEK provide strong rejections of the currency-by-currency null hypothesis that the time-varying, right-hand-side variables have no collective forecasting power. Particularly large values of coefficients for the AUD, SEK, and NOK are an indication of multicollinearity. Although Cochrane and Piazzesi (2005) found a clear "tent" pattern in their projection of average returns onto the levels of the five forward rates, we only see this pattern in projections onto the four forward spreads for the USD and JPY data.

There are at least three reasons why the estimates of the  $\gamma$ 's might differ across currencies. The first explanation takes a rational expectations econometrics view and recognizes that the forward spreads capture the risk exposures of a country as represented by the reduced form coefficients from an ATSM. Underlying structural differences in the nature of risks would consequently manifest themselves in different  $\gamma$ 's. Monetary and fiscal policies certainly differ across countries, and we do not attempt to relate the underlying coefficients of the ATSM to more structural coefficients in equations such as the Taylor (1993) rule.

Alternatively, a second reason would take the perspective of Bekaert *et al.* (2001) who argue that the rational expectations econometrics perspective may be too strong. Developed countries, such as those studied here, may actually be following the same time series rule, but the realizations of the shocks hitting the economies may have differed across countries. It may take a very long sample for a particular economy to experience all of the possible realizations from the policy rule with their *ex ante* frequencies that investors anticipated during the sample. It is certainly true that *ex post* experiences with inflation have differed across the countries, although at a casual level, all countries now seem to be converging to relatively low rates of expected inflation.

As an example of this last perspective, it is notable in Table 1 that the  $R^2$ 's from the first-step regression of the average return on the forward spreads are the highest for the USD and JPY. Bekaert *et al.* (2001) argue that the decline in U.S. inflation under Federal Reserve Chairmen Volcker and Greenspan represents a one-sided realization that made the *ex post* returns on investments in long-term bonds better than was anticipated.<sup>6</sup> Inflation

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<sup>6</sup>See Bauer and Rudebusch (2017) for an analysis of the U.S. term structure that allows for declining stochastic trends in both the long-run expected rate of inflation and the equilibrium real interest rate.

in Japan during much of the sample was also surprisingly low. Thus, the Japanese situation could be similar to the U.S. in that the stagnation in the Japanese economy and its ultimate experiences with deflation resulted in surprisingly good ex post returns on long-term Japanese bonds even though bond yields were quite low to start.

A third reason is offered by Cochrane and Piazzesi (2008) who note that the construction of the CP factor is sensitive to how the term structure data are derived. While the USD data are constructed using actual prices and the bootstrap method of Fama and Bliss (1987), zero-coupon term structure data on other currencies result from sequential, cross-sectional estimation using the flexible functional form approaches of Nelson and Siegel (1987) or Svensson (1995). In comparing forecasts of USD data from Gürkaynak *et al.* (2007) that are constructed from the flexible form of Svensson (1995) to forecasts from the Fama and Bliss (1987) data, Cochrane and Piazzesi (2008) find evidence of multicollinearity in the former and more predictive power in the latter. Consistent with these findings, we noted above that several of the currencies show evidence of multicollinearity in the forecasting variables.

### 3.2 Results with Post-2003 Data

We now discuss the results for the second sample from 2004 to 2016 that are also presented in Table 1 in the rows labeled CUR2. While the one-factor structure of expected excess returns, estimated from unconstrained regressions, is not quite as strong in this sample, we still see that the first principal component of the unconstrained expected returns explains between 86.6% of the variance for the NOK and 99.4% for the JPY. The similarity in the coefficient estimates of  $b_2$ ,  $b_3$ , and  $b_4$  is maintained, and differences from the estimates in the first sample are generally small. The estimated values of  $b_2$  now range from 0.28 for the CHF to 0.38 for the CAD; the estimated values of  $b_3$  range from 0.73 for the JPY to 0.82 for the GBP and the CAD; and the estimated values of  $b_4$  range from 1.22 for the EUR, CAD, AUD, SEK, and NOK to 1.29 for the JPY. The estimates of the  $\gamma$ 's appear much different in the second sample compared to the first. Differences are particularly large for the GBP, EUR, CHF, JPY, AUD, and NOK.

As a first step in analyzing the out-of-sample performance of the Cochrane and Piazzesi (2005) model, Table 2 presents tests of the equality of the  $\mathbf{b}$ 's



and  $\gamma$ 's across the two samples on a currency-by-currency basis.

For the  $\mathbf{b}$ 's, even though the coefficient estimates are quite similar across the two samples, their small standard errors lead to rejections of equality of the three coefficients for the EUR at the 1% marginal level of significance, for the CHF at the 3% level, and for the JPY at smaller than the 1% level. The tests of the  $\gamma$ 's reject equality of the parameter estimates across the two periods for the USD, the JPY, and the NOK at less than the 1% level, for the GBP at the 9% level, and for the AUD at the 10% level. These findings provide the first evidence of instability in the forecasting relations.

### 3.3 Correlation Matrix and Variance Decomposition of Country CP Factors

Because one-factor forecasting structures characterize each of the term structures quite well, a natural question to ask is how correlated are the various CP factors. Table 3 provides correlation matrices for the respective currency-specific CP factors for the two sample periods.

Panel A of Table 3 presents the results for the first sample, and we find that 26 of the 36 correlations are positive, but only the GBP-CHF correlation of .63 is larger than .5. Of the nine negative ones, the JPY-NOK correlation is the most negative at -.30. The last column in Table 3 labeled %PC( $i$ ) reports the percent of the variance of the nine country-specific CP factors that is explained by the respective principal components. The first three principal components explain 82% of the total variance. While this evidence is suggestive that global risk factors may be at work in explaining the ability of the CP factors to forecast excess bond returns, it is certainly not definitive.<sup>7</sup>

When we examine the post-2003 samples in Panel B of Table 3, we find that six of the 36 correlations are negative, and the largest positive correlation is now the GBP-NOK correlation of .54, which is the only correlation greater than .5. Twelve of the correlations change sign, and the largest switch is the GBP-EUR correlation which increased from -.19 to .39. The share of the variance explained by the first three principal components falls to 69%. These changes in correlations are another indication of instability in the model.

We will examine out-of-sample forecasting of bond returns below, but first, we examine some international implications of the model.

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<sup>7</sup>Jotikasthira *et al.* (2015) investigate the determinants of the correlations across several major currency term structures.

Table 2: Tests of Equality of Coefficients for the Two Samples

CUR	$\chi^2(3)$ for $\mathbf{b}$ 's	$\chi^2(5)$ for $\boldsymbol{\gamma}$ 's
USD	5.15 (0.16)	81.77 (0.00)
GBP	4.77 (0.19)	9.52 (0.09)
EUR	11.64 (0.01)	4.39 (0.49)
CHF	8.69 (0.03)	5.47 (0.36)
CAD	5.46 (0.14)	7.16 (0.21)
JPY	26.52 (0.00)	25.87 (0.00)
AUD	2.83 (0.42)	9.36 (0.10)
SEK	1.12 (0.77)	1.27 (0.94)
NOK	5.84 (0.12)	35.65 (0.00)

**Description:** The Table reports two statistics that test the equality of the coefficient estimates in the Cochrane and Piazzesi, 2005 models for the two samples estimated in Table 1. The first test examines the  $\mathbf{b}$  coefficients and is distributed as a  $\chi^2(3)$ . The second test examines the  $\boldsymbol{\gamma}$  coefficients and is distributed as a  $\chi^2(5)$ . The  $p$ -values are in angled brackets. The first sample periods for the dependent variables all end in 2003:12. These samples begin in 1974:12 for the USD, the GBP, and the EUR; in 1989:03 for the CHF; in 1987:03 for the CAD; in 1986:03 for the JPY; in 1988:04 for the AUD; in 1988:03 for the SEK; and in 1999:03 for the NOK. The second sample period for the dependent variables is 2004:12 to 2016:12 for all currencies.

**Interpretation:** These findings provide the first evidence of instability in the forecasting relations. While we cannot reject the hypotheses that the  $\mathbf{b}$ 's and  $\boldsymbol{\gamma}$ 's are equal across the two samples for the CAD and the SEK, for the other currencies we find strong rejections of equality of the  $\mathbf{b}$ 's for the EUR, the CHF, and the JPY; and we find strong rejections of equality of the  $\boldsymbol{\gamma}$ 's for the USD, the JPY, and the NOK, and slightly weaker evidence of inequality of the  $\boldsymbol{\gamma}$ 's for the GBP and the AUD.

Table 3: Correlation Matrices and Variance Decompositions of Country CP Factors

Panel A: Pre-2004 Data										
	GBP	EUR	CHF	CAD	JPY	AUD	SEK	NOK	i	%PC(i)
USD	0.06	0.23	0.26	0.37	0.22	0.35	0.13	-0.02	1	0.40
GBP	1.00	-0.19	0.63	0.49	0.45	0.10	-0.24	0.27	2	0.29
EUR		1.00	0.12	-0.11	0.18	-0.08	0.44	0.44	3	0.13
CHF			1.00	0.19	0.34	-0.16	-0.20	0.34	4	0.09
CAD				1.00	0.24	0.12	0.10	-0.07	5	0.04
JPY					1.00	0.02	0.03	-0.30	6	0.03
AUD						1.00	-0.09	0.18	7	0.01
SEK							1.00	0.37	8	0.01
NOK								1.00	9	0.00

Panel B: Post-2003 Data										
	GBP	EUR	CHF	CAD	JPY	AUD	SEK	NOK	i	%PC(i)
USD	0.42	-0.07	0.12	0.34	0.35	0.12	0.12	0.18	1	0.37
GBP	1.00	0.39	0.34	0.40	-0.11	0.08	0.19	0.49	2	0.18
EUR		1.00	0.33	0.19	-0.32	0.08	0.10	0.35	3	0.14
CHF			1.00	0.17	0.08	-0.01	0.14	0.44	4	0.09
CAD				1.00	0.37	0.15	0.12	0.03	5	0.08
JPY					1.00	0.06	-0.00	-0.29	6	0.06
AUD						1.00	0.14	-0.32	7	0.04
SEK							1.00	0.13	8	0.02
NOK								1.00	9	0.01

**Description:** The Table presents the correlation matrices of the CP factors, the fitted return forecasting variables from the term structure regressions for the different currencies and the two sample periods in Table 1. In Panel A, the sample periods for the dependent variables all end in 2003:12. The samples begin in 1974:12 for the USD, the GBP, and the EUR; in 1989:03 for the CHF; in 1987:03 for the CAD; in 1986:03 for the JPY; in 1988:04 for the AUD; in 1988:03 for the SEK; and in 1999:03 for the NOK. Because the samples for the different currencies are of different lengths, the correlations are estimated over the shorter of the two samples. In Panel B, the sample periods for the dependent variables are all 2004:12 to 2016:12. The last column labeled %PC reports the percent of variance explained by the  $i$ -th principal component.

**Interpretation:** Although the estimated correlations between the different currency CP factors are generally positive in both samples, they are relatively small in magnitude, and in the post-2003 data, twelve out of 36 correlations change sign from the first sample. In the first sample, the first three principal components explain 82% of the correlation matrix, which suggests that global risk factors may explain the ability of the CP factors to forecast excess bond returns, but such evidence is certainly not definitive. In the second sample, the share of variance explained by the first three principal components drops to 69%, suggesting instability in the model.

#### 4 International Implications

This section derives some implications of the Cochrane and Piazzesi (2005) and Cochrane and Piazzesi (2008) model for foreign exchange markets. Doing so requires the introduction of subscripts for the currencies, and we subscript the USD variables with a one and variables denominated in an arbitrary foreign currency with a  $j$ . We define exchange rates as  $S_{ij,t}$ , which represents the currency  $j$  price of base currency  $i$  at time  $t$ . The continuously compounded rate of appreciation of base currency  $i$  relative to currency  $j$  between times  $t$  and  $t + 1$  is denoted  $\Delta s_{ij,t+1}$ .

We first argue that tight restrictions between the term structure models of the two currency markets and the relative rate of currency appreciation are not supported empirically.<sup>8</sup> Then, we consider some less constrained empirical predictions.

To understand this argument, consider the basic no arbitrage asset pricing equation for a particular currency  $i$  that prices all returns denominated in that currency as in equation (8); but now, let  $Q_{i,t+1}$  represent this general SDF that prices these generic returns,  $R_{i,t+1}$ , which include returns on all currency  $i$  denominated assets and not just the bond returns of equation (8). Thus, we have

$$E_t(Q_{i,t+1}R_{i,t+1}) = 1. \quad (15)$$

The difference between the SDF in equation (15),  $Q_{i,t+1}$ , that prices all currency  $i$  returns and the SDF in equation (8),  $M_{i,t+1}$ , that only prices the currency  $i$  bond returns is that  $Q_{i,t+1}$  can contain risks that are orthogonal to the risks that are priced in the term structure of interest rates. Analytically, we can decompose  $Q_{i,t+1}$  as

$$Q_{i,t+1} = M_{i,t+1}Z_{i,t+1}. \quad (16)$$

Consistency of the two no arbitrage conditions requires that  $E_t(Z_{i,t+1}) = 1$ , because the risk free rate is correctly priced by  $M_{i,t+1}$ ;  $E_t(M_{i,t+1}Z_{i,t+1}) = E_t(M_{i,t+1})E_t(Z_{i,t+1})$ , because  $Z_{i,t+1}$  and  $M_{i,t+1}$  are orthogonal; and for the return on an  $n$ -period bond,  $R_{i,t+1}^{(n)}$ ,  $E_t(Z_{i,t+1}R_{i,t+1}^{(n)}) = E_t(Z_{i,t+1})E_t(R_{i,t+1}^{(n)})$

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<sup>8</sup>See Backus *et al.* (2001) for a discussion of the links between fully specified SDF's and the rate of currency appreciation when financial markets are complete, and see Brandt and Santa-Clara (2002) and Brandt *et al.* (2006) for discussions of the effects of incomplete markets.

because  $M_{i,t+1}$  contains all risks priced in the bond market making  $Z_{i,t+1}$  orthogonal to  $R_{i,t+1}^{(n)}$ .

#### 4.1 Implications for the Innovation in Currency Appreciation

If financial markets are complete, it is well known that there is a tight relation between the rate of appreciation of currency  $i$  relative to currency  $j$  and the difference between the natural logarithms of the stochastic discount factor of currency  $i$ ,  $q_{i,t+1}$ , and the stochastic discount factor of currency  $j$ ,  $q_{j,t+1}$ :

$$\Delta s_{ij,t+1} = q_{i,t+1} - q_{j,t+1}. \quad (17)$$

From equation (17) we see that the innovation in the rate of appreciation of currency  $i$  relative to currency  $j$  should be completely explained by the difference in the innovations in the risks present in the natural logarithms of the two currencies stochastic discount factors.

To draw out the international implications of the Cochrane and Piazzesi (2008) ATSM, we substitute for the  $q$ 's in equation (17) to find

$$\Delta s_{ij,t+1} = m_{i,t+1} + z_{i,t+1} - m_{j,t+1} - z_{j,t+1}, \quad (18)$$

where  $z_{i,t} \equiv \ln(Z_{i,t})$ . Notice that if the Cochrane and Piazzesi (2008) ATSM correctly characterized the term structure in each currency, if asset markets were complete, and if the term structure SDF's contained all the sources of risks, then the  $z$ 's could be eliminated from equation (18). After substituting for the innovations in the  $m$ 's from equation (13), the innovation in the rate of appreciation of currency  $i$  relative to currency  $j$  would be

$$\Delta s_{ij,t+1} - E_t(\Delta s_{ij,t+1}) = (\lambda_{j,0l} + \lambda_{j,0l} x_{j,t}) v_{j,l,t+1} - (\lambda_{i,0l} + \lambda_{i,0l} x_{i,t}) v_{i,l,t+1}. \quad (19)$$

Thus, the innovation in  $\Delta s_{ij,t+1}$  would be fully explained by the innovations in  $m_{j,t+1}$  and  $m_{i,t+1}$ . In the Cochrane and Piazzesi (2008) ATSM, the innovation in the SDF of a currency is the innovation in the level factor of the term structure interacted with a constant and with the predetermined CP factor.

We investigate this issue for rates of appreciation of the USD versus the other eight currencies in Table 4. Because the exact fit of equation (19)

would be unlikely to hold, we run regressions with the expectation that if the model were true, we would have quite significant explanatory power. We proxy the innovation in the rate of appreciation of the USD with respect to currency  $j$  with the excess rate of return on a USD investment in the currency  $j$  money market,  $-\Delta s_{1j,t+1} + r_{j,t} - r_{1,t}$ . We proxy the innovations in the level factors with the changes in the levels, as represented by the first principal components of the term structures, because these first principal components are highly serially correlated. For simplicity, we also just report results for the full sample periods associated with each currency.

In the regressions in Table 4 the  $R^2$ 's range from 2% for the CAD and the CHF to 23% for the JPY. This represents strong evidence that the constrained Cochrane and Piazzesi (2008) term structure models do not span the spaces of risks that characterize the rates of currency depreciation, which we interpret as evidence for the presence of additional risks in the SDF's that price all assets.

These results are consistent with the analysis of Sarno *et al.* (2012) who estimate four-factor, latent variable ATSM's for the bond markets of two currencies and find that while the bonds are priced very well, the variation of the rate of currency appreciation from the implied ATSM stochastic discount factors does not match well with the actual rate of currency appreciation. Of course, the results could also indicate that financial markets are incomplete as in the analysis of Brandt and Santa-Clara (2002), but the exchange rate volatility puzzle first discussed in Brandt *et al.* (2006) suggests that the stochastic discount factors should be relatively highly correlated.

#### 4.2 Implications for expected cross-currency investments

To investigate expected rates of return on cross-currency investments that are implied by the Cochrane and Piazzesi (2008) ATSM model but with  $Z_{i,t+1}$  present, let  $Z_{i,t+1}$  be log-normally distributed. Then, we can assume that the stochastic process for  $z_{i,t+1}$  is

$$z_{i,t+1} = -\frac{1}{2} \boldsymbol{\lambda}_{z_i,t}^\top \boldsymbol{\lambda}_{z_i,t} - \boldsymbol{\lambda}_{z_i,t}^\top \mathbf{v}_{z_i,t+1}, \quad (20)$$

where  $\mathbf{v}_{z_i,t+1}$  is a vector of risks that are distributed  $N(0, I)$  and that are orthogonal to the vector of risks,  $\mathbf{v}_{i,t+1}$ , that drive the term structure of interest rates in that currency.

Table 4: Explaining Currency Market Excess Returns with Changes in Level Factors

CUR $j$	$\beta_0$	$\beta_1$	$\beta_2$	$\beta_3$	$\beta_4$	$R^2$
GBP	0.67 (1.49)	0.56 (0.64)	-0.13 (0.22)	-0.64 (0.58)	0.48 (0.22)	0.04
EUR	-0.37 (1.65)	-0.70 (0.59)	-0.02 (0.25)	0.63 (1.11)	-0.47 (0.57)	0.04
CHF	0.04 (1.89)	-0.10 (1.06)	-0.34 (0.41)	0.03 (0.85)	-0.06 (0.65)	0.02
CAD	1.58 (1.36)	0.27 (0.87)	-0.02 (0.30)	0.17 (0.77)	0.15 (0.22)	0.02
JPY	-0.97 (2.05)	1.51 (1.03)	-0.78 (0.23)	-3.02 (1.26)	0.90 (0.48)	0.22
AUD	3.91 (2.05)	0.09 (0.92)	-0.28 (0.39)	0.84 (1.49)	0.49 (0.65)	0.13
SEK	0.65 (1.78)	0.31 (1.32)	-0.68 (0.61)	2.35 (0.74)	-0.47 (0.49)	0.20
NOK	1.01 (2.11)	-0.04 (1.55)	-1.17 (0.71)	1.26 (3.31)	-0.07 (4.05)	0.12

**Description:** The Table presents estimation results for the regression,

$$-\Delta s_{1j,t+1} + r_{j,t} - r_{1,t} = \beta_0 + \beta_1 \Delta l_{1,t+1} + \beta_2 \Delta l_{1,t+1} \cdot x_{1,t} + \beta_3 \Delta l_{j,t+1} + \beta_4 \Delta l_{j,t+1} \cdot x_{j,t} + \varepsilon_{1j,t+1},$$

where the dependent variable is the excess rate of return in USD on an annual investment in the money market of currency  $j$ , which is our proxy for the innovation in the rate of dollar appreciation. The regressors are the contemporaneous changes in the first principal components of the yields for the USD,  $\Delta l_{1,t+1}$ , and for currency  $j$ ,  $\Delta l_{j,t+1}$ , and the interaction of these variables with their respective currency-specific CP factors, which are the term structure excess return forecasting variables,  $x_{1,t}$  and  $x_{j,t}$ . Standard errors are in parentheses. The sample periods for the dependent variables all end in 2016:12. The samples begin in 1974:12 for the GBP and the EUR; in 1989:03 for the CHF; in 1987:03 for the CAD; in 1986:03 for the JPY; in 1988:04 for the AUD; in 1988:03 for the SEK; and in 1999:03 for the NOK. The samples all end in 2016:12.

**Interpretation:** The small  $R^2$ 's in regressions for models proxying the predictions of the Cochrane and Piazzesi, 2008 ATSM suggest that these variables do not span the spaces of risks that characterize the rates of currency depreciation, suggesting the possible presence of additional risks that price all assets.

Substituting for the SDF's in equation (18) from equations (7) and (20) and rearranging terms gives the excess rate of return in currency  $i$  on a one-year investment in the money market of currency  $j$ :

$$-\Delta s_{ij,t+1} + r_{j,t} - r_{i,t} = \frac{1}{2} \left( \lambda_{i,t}^\top \lambda_{i,t} - \lambda_{j,t}^\top \lambda_{j,t} \right) + \frac{1}{2} \left( \lambda_{z_{i,t}}^\top \lambda_{z_{i,t}} - \lambda_{z_{j,t}}^\top \lambda_{z_{j,t}} \right) + \lambda_{i,t}^\top \mathbf{v}_{i,t+1} - \lambda_{j,t}^\top \mathbf{v}_{j,t+1} + \lambda_{z_{i,t}}^\top \mathbf{v}_{z_{i,t+1}} - \lambda_{z_{j,t}}^\top \mathbf{v}_{z_{j,t+1}}. \quad (21)$$

Taking the conditional expectation of equation (21) gives

$$E_t \left( -\Delta s_{ij,t+1} + r_{j,t} - r_{i,t} \right) = \frac{1}{2} \left( \lambda_{i,t}^\top \lambda_{i,t} - \lambda_{j,t}^\top \lambda_{j,t} \right) + \frac{1}{2} \left( \lambda_{z_{i,t}}^\top \lambda_{z_{i,t}} - \lambda_{z_{j,t}}^\top \lambda_{z_{j,t}} \right). \quad (22)$$

The right-hand side of equation (22) is the expected excess rate of return to borrowing one unit of currency  $i$ , investing that amount in the currency  $j$  money market, and bearing the foreign exchange risk.<sup>9</sup> By imposing the constraints of the one-factor forecasting model for the two bond markets in equation (12), we find

$$\lambda_{j,t}^\top \lambda_{j,t} = \left( \lambda_{j,0l} + \lambda_{j,1l} x_{j,t} \right)^2 = \lambda_{j,0l}^2 + 2\lambda_{j,0l} \lambda_{j,1l} x_{j,t} + \lambda_{j,1l}^2 x_{j,t}^2. \quad (23)$$

Substituting from equation (23), for both currencies  $i$  and  $j$ , into equation (22) implies that the CP forecasting factors,  $x_{i,t}$  and  $x_{j,t}$ , from the bond markets of the two currencies and their squared values should forecast the excess rate of return to investing a unit of currency  $i$  in the currency  $j$  money market while bearing the foreign exchange risk.

In contrast to the predictability implied by this ATSM approach, the literature on uncovered interest rate parity predicts that the excess rate of return to investing a unit of currency  $i$  in the currency  $j$  money market should be unpredictable, and empirical evidence that rejects this hypothesis often focuses on the interest differential,  $r_{j,t} - r_{i,t}$ , as a predictor, as in the analysis of Fama (1984).

To investigate the predictability of excess currency returns we blend these two specifications as in the following forecasting regression in which we use the USD as the base currency:

$$-\Delta s_{1j,t+1} + r_{j,t} - r_{1,t} = \phi_0 + \phi_1 x_{1,t} + \phi_2 x_{j,t} + \phi_3 (r_{j,t} - r_{1,t}) + \epsilon_{1j,t+1}^s. \quad (24)$$

<sup>9</sup>As in equation (10), one can also express this time-varying expected excess rate of return in terms of a Jensen's inequality term and a logarithmic risk premium term.



In equation (24) we leave out the squared values of the CP forecasting factors as these are numerically small, and we also do not impose any constraints on the regression coefficients of the CP factors that arise from the term structure theory because we do not observe  $\lambda_{z_i,t}^\top \lambda_{z_i,t} - \lambda_{z_j,t}^\top \lambda_{z_j,t}$ . Although equation (23) demonstrates that tight restrictions related to the prices of risks and the forecasts of excess currency returns implied by the CP factors arise when everything is observable, these return forecasting variables may also enter the determination of the prices of risks,  $\lambda_{z_i,t}$  and  $\lambda_{z_j,t}$ , or they may simply be correlated with the variables that drive these prices of risks, which are not observed, in which case OLS regression of the excess rate of return on  $x_{i,t}$  and  $x_{j,t}$  does not isolate the pure effect of these variables that arises strictly from the fact that they are the determinants of the prices of the term structure risks,  $\lambda_{i,t}$  and  $\lambda_{j,t}$ . Any restrictions arising from an ATSM specification of  $\lambda_{i,t}$  and  $\lambda_{j,t}$  are lost in the general regression specification in equation (24) because the determinants of  $\lambda_{z_i,t}$  and  $\lambda_{z_j,t}$  are not included in the regression. Table 5 presents the estimated coefficients for equation (24) with their asymptotic standard errors in parenthesis for the two sample periods.<sup>10</sup>

Here, we only present the regressions associated with forecasts of the USD excess rates of return from investments in the eight different currencies relegating the results of the remaining 28 non-USD, non-redundant, cross-currency excess rate of return forecasting regressions to the Online Appendix.<sup>11</sup> The statistical significance of the estimates of the coefficients is quantified with three different tests. The  $\chi^2(2)$  statistic tests the null hypothesis that  $\phi_1$  and  $\phi_2$  equal zero, which tests whether the USD CP factor and the currency  $j$  CP factor have forecasting power for the USD excess rate of return on an investment in the currency  $j$  money market. The first  $\chi^2(1)$  statistic tests the null hypothesis that  $\phi_3$  equals zero, which tests whether the interest differential has forecasting power; and the second  $\chi^2(1)$  statistic tests the null hypothesis that  $\phi_3$  equals one.

This latter hypothesis is motivated by the typical finding in tests of

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<sup>10</sup>Appendix C derives the standard errors of the parameters in equation (24). These standard errors allow for the fact that the forecasting variables are estimated in first stage regressions.

<sup>11</sup>The findings in these investigations are quite similar to the USD results included in the paper and are consequently not discussed here. Also, note again that for completeness we present the results for the NOK, but because the first sample for the NOK is particularly short, we do not interpret them.

Table 5: Forecasts of USD Excess Rates of Return in Currency Markets

CUR $j$	$\phi_0$	$\phi_1$	$\phi_2$	$\phi_3$	$\chi^2(2)$ $\phi_1, \phi_2=0$	$\chi^2(1)$ $\phi_3=0$	$\chi^2(1)$ $\phi_3=1$	$R^2$
GBP1	-2.92 (3.08)	0.24 (0.62)	-0.24 (1.20)	1.86 (0.89)	0.26 (0.88)	4.31 (0.04)	0.92 (0.34)	0.09
GBP2	8.60 (5.79)	-1.89 (1.22)	-2.84 (1.91)	-3.68 (1.95)	2.93 (0.23)	3.57 (0.06)	5.78 (0.02)	0.35
EUR1	-1.45 (3.56)	1.09 (0.73)	1.28 (1.35)	1.69 (0.88)	2.75 (0.25)	3.70 (0.05)	0.62 (0.43)	0.16
EUR2	8.95 (8.03)	-1.93 (2.26)	-4.21 (3.59)	-0.87 (1.85)	1.98 (0.37)	0.22 (0.64)	1.02 (0.31)	0.18
CHF1	3.07 (2.92)	0.09 (1.98)	-0.49 (1.90)	2.47 (0.89)	0.07 (0.97)	7.71 (0.01)	2.73 (0.10)	0.20
CHF2	1.71 (3.20)	0.50 (1.24)	-1.22 (0.97)	0.14 (1.15)	1.65 (0.44)	0.01 (0.90)	0.57 (0.45)	-0.00
CAD1	0.52 (2.52)	0.13 (0.97)	-0.90 (0.60)	1.46 (0.74)	2.34 (0.31)	3.87 (0.05)	0.38 (0.54)	0.24
CAD2	2.22 (3.96)	-3.43 (1.59)	3.17 (2.44)	-3.88 (3.12)	5.63 (0.06)	1.54 (0.21)	2.44 (0.12)	0.35
JPY1	10.75 (5.17)	3.13 (1.17)	-2.84 (1.31)	3.94 (0.94)	9.11 (0.01)	17.39 (0.00)	9.68 (0.00)	0.58
JPY2	-8.59 (4.89)	0.65 (2.27)	12.01 (6.29)	0.41 (1.25)	8.40 (0.01)	0.11 (0.74)	0.22 (0.64)	0.17
AUD1	-3.26 (4.13)	0.24 (2.12)	-0.57 (1.18)	2.50 (0.83)	0.27 (0.87)	9.10 (0.00)	3.27 (0.07)	0.23
AUD2	9.62 (10.69)	-2.18 (2.51)	-1.66 (3.16)	-0.68 (2.96)	0.89 (0.64)	0.05 (0.82)	0.32 (0.57)	0.05
SEK1	-1.03 (5.64)	-0.76 (2.61)	0.60 (1.72)	0.90 (1.29)	0.16 (0.92)	0.49 (0.49)	0.01 (0.94)	0.03
SEK2	7.00 (7.00)	-2.34 (2.80)	-2.61 (2.09)	-1.52 (1.69)	2.19 (0.33)	0.81 (0.37)	2.23 (0.14)	0.16
NOK1	-9.92 (5.13)	2.54 (1.16)	0.24 (1.48)	4.91 (1.46)	5.78 (0.06)	11.24 (0.00)	7.12 (0.01)	0.70
NOK2	1.47 (6.21)	-2.28 (2.83)	2.55 (6.88)	-1.90 (2.29)	0.65 (0.72)	0.69 (0.41)	1.61 (0.20)	0.11

(Continued)

Table 5: (Continued)

**Description:** The Table presents estimation results for the regression

$$-\Delta s_{1j,t+1} + r_{j,t} - r_{1,t} = \phi_0 + \phi_1 x_{1,t} + \phi_2 x_{j,t} + \phi_3 (r_{j,t} - r_{1,t}) + \epsilon_{1j,t+1}^s,$$

where the dependent variable is the excess USD rate of return on an annual investment in the money market of currency  $j$ . The regressors are the CP factors, the fitted return forecasting variables from the term structure regressions for the USD and for currency  $j$ , and the difference in the one-year yields between currency  $j$  and the USD. Standard errors are in parentheses, and  $p$ -values are in angled brackets. The  $\chi^2(2)_{\phi_1, \phi_2=0}$  statistic tests the null hypothesis that  $\phi_1$  and  $\phi_2$  equal zero. The  $\chi^2(1)_{\phi_3=0}$  and  $\chi^2(1)_{\phi_3=1}$  statistics test the null hypothesis that  $\phi_3$  equals either zero or one, respectively. The results for the first sample period are labeled CUR1, and the dependent variables all end in 2003:12. The first samples begin in 1974:12 for the GBP and the EUR; in 1989:03 for the CHF; in 1987:03 for the CAD; in 1986:03 for the JPY; in 1988:04 for the AUD; in 1988:03 for the SEK; and in 1999:03 for the NOK. The results for the second sample period are labeled CUR2, and the dependent variables all begin in 2004:12 and end in 2016:12.

**Interpretation:** For both samples we are generally unable to reject the null hypothesis that USD and foreign currency CP factors are not significant predictors of the excess USD rate of return on foreign money market investments. Consistent with earlier literature on uncovered interest rate parity, in the first sample we generally strongly reject the null hypothesis that excess returns in foreign money markets are unpredictable using interest differentials. In the post-2003 sample, we are generally unable to reject the null hypothesis that excess returns in foreign money markets are unpredictable using interest differentials. These latter results are consistent with the post-2007 deterioration of returns on foreign currency carry trades.

uncovered interest rate parity that the slope coefficient in a regression of the rate of appreciation of the USD relative to currency  $j$ ,  $\Delta s_{1j,t+1}$ , on the interest differential,  $r_{j,t} - r_{1,t}$ , often produces estimated coefficients that are significantly negative. That is, the typical Fama (1984) regression is

$$\Delta s_{1j,t+1} = \alpha + \beta (r_{j,t} - r_{1,t}) + \epsilon_{1j,t+1}^s, \quad (25)$$

and the null hypothesis of uncovered interest rate parity is  $\beta = 1$ . Estimates of  $\beta$  are typically negative. Our specification multiplies this regression by  $-1$ , adds the interest differential to both sides, and includes two other variables. Thus, the relation between the two slope coefficients is  $-\beta + 1 = \phi_3$ . The historical finding that estimated  $\beta$ 's in regression (25) are negative translates into  $\phi_3 > 1$  in our analysis.

For the first sample, most of the coefficients on the CP factors are smaller than their standard errors, and only for the tests associated with the JPY do we find sufficiently large test statistics to reject, at the .05 marginal level of significance or smaller, the null hypothesis that the USD CP factor and the foreign currency CP factor are not significant determinants of the excess rates of return on investments in the foreign money markets.

In contrast, for the first sample, we can reject that the  $\phi_3$  coefficients are equal to zero at least at the .05 marginal level of significance for all currencies except the SEK. The  $\phi_3$  estimates are systematically larger than one, but we are only able to reject that they equal one for the JPY at less than the .01 level and for the AUD at the .07 level. The adjusted  $R^2$ 's for several of the currencies are also substantial. For the currencies with significant estimates of  $\phi_3$ , the  $R^2$ 's range from .09 for the GBP to .58 for the JPY.

These results are completely consistent with the literature on the FX carry trade, which is a strategy that borrows low interest rate currencies and lends high interest rate currencies. The dependent variable in the regressions is the USD return on the carry trade when  $r_{j,t} > r_{1,t}$ , and the highly positive values of the slope coefficients indicate that expected USD carry trade profits are conditionally high when  $r_{j,t} - r_{1,t}$  is conditionally high.<sup>12</sup>

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<sup>12</sup>See Daniel *et al.* (2017) for a recent review of the literature on the risks of the carry trade at the monthly holding period horizon. Lustig *et al.* (2017) find that investing in the carry trade with longer term bonds while maintaining the one-month holding period is unattractive as the term premiums offset the currency premiums.

How does the model do in the post-2003 sample? The answer is not particularly well. The results for the USD excess returns are also presented in Table 5. Only for the CAD and the JPY are the  $p$ -values of the  $\chi^2(2)$  statistic testing the significance of the CP factors smaller than .06. The point estimates of  $\phi_3$  become negative for six of the currencies, and we are generally unable to reject that  $\phi_3$  equals zero. One exception is the GBP which now has a significantly negative  $\phi_3$ . The fact that many of the coefficients on the CP factors and the interest differentials change signs across the two samples clearly supports the conclusion that the second sample containing the financial crisis is quite different from the pre-2004 sample. These results are also consistent with the post-2007 deterioration in the returns to the carry trade for major currencies.

## 5 Out-of-Sample Results

The previous sections examined the predictability of excess returns in bond and foreign exchange markets with classical asymptotic distribution theory. Inoue and Kilian (2005) argue that such an approach is actually more powerful than out-of-sample experiments, yet such experiments are routinely done and are considered to be a good indicator of instability in the underlying forecasting model. This section consequently examines whether the Cochrane and Piazzesi (2005) model can forecast the excess rates of return in bond and foreign currency markets out of sample. As above, we use the sample period that would have been available when the original paper was written as the in-sample period and treat the post-2003 sample beginning in January 2004 and ending in December 2016 as the out-of-sample period.<sup>13</sup>

We follow Welch and Goyal (2007) and Campbell and Thompson (2007) in assessing the models' out-of-sample forecasts by examining two statistics. The first is the  $R^2$  that compares the mean squared error of the conditional forecasts of excess returns from the term structure model to the mean squared error from assuming that the conditional forecasts of the excess returns are the conditional sample means using data up to that point in

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<sup>13</sup>Because the expected one-year excess return on the two-year bond can be written as  $E_t(rx_{t+1}^{(2)}) = -E_t(y_{t+1}^{(1)} - y_t^{(1)}) + (f_t^{(2)} - y_t^{(1)})$ , if the forward spread fails to predict the excess bond return, it must predict the change in the short-term rate. We thank John Cochrane for reminding us of this important caveat which is discussed in Cochrane and Piazzesi (2006).

time. Analytically, if  $\hat{r}_t$  represents the  $t$ -th out-of-sample forecast from the Cochrane and Piazzesi (2005) model using parameters estimated with all the historical data available at that time, and if  $\bar{r}_t$  represents the analogous forecast from the historical sample mean, using the same sample period, then with  $T_{os}$  total out-of-sample observations, the mean squared error from the CP forecasts is

$$MSE_{CP} = \frac{1}{T_{os}} \sum_{t=1}^{T_{os}} (r_t - \hat{r}_t)^2, \quad (26)$$

and the mean squared error from the historical mean forecasts is

$$MSE_{HM} = \frac{1}{T_{os}} \sum_{t=1}^{T_{os}} (r_t - \bar{r}_t)^2. \quad (27)$$

The  $R^2$  is then defined as

$$R^2 = 1 - \frac{MSE_{CP}}{MSE_{HM}} \quad (28)$$

The second closely related statistic is the Clark and McCracken (2005)  $MSE - F$  which tests for the equality of the two forecasts:

$$MSE - F = T_{os} \frac{MSE_{HM} - MSE_{CP}}{MSE_{CP}}. \quad (29)$$

Panel A of Table 6 presents the out-of-sample forecasts of excess bond returns for the Cochrane and Piazzesi (2005) model estimated separately for each currency. The first forecast is 2004:01, and the last is 2016:12.

The results are quite mixed. The model's forecasts are worse than the forecasts based on the historical mean at all maturities for the USD, the EUR, the JPY, the AUD, and the NOK. Only for the GBP do the model forecasts beat historical mean forecast for all maturities. For the CHF, the CAD, and the SEK, the results are mixed across maturities.

To better understand the inability of the forecasts from the estimated Cochrane and Piazzesi (2005) models to beat the forecasts from the historical means, the solid line in Figure 1 presents the evolution of the difference in the cumulative sum of squared errors (SSE) from the forecasts based

Table 6: Out-of-Sample Forecasts for Excess Bond Returns from Cochrane-Piazzesi Models

CUR	$R^2$				MSE-F			
	$rx_{t+1}^{(2)}$	$rx_{t+1}^{(3)}$	$rx_{t+1}^{(4)}$	$rx_{t+1}^{(5)}$	$rx_{t+1}^{(2)}$	$rx_{t+1}^{(3)}$	$rx_{t+1}^{(4)}$	$rx_{t+1}^{(5)}$
Panel A: Basic Models vs. Historical Means								
USD	-1.51	-1.95	-1.90	-1.68	-87.26	-95.86	-94.97	-90.85
GBP	0.06	0.10	0.11	0.09	9.29	16.50	18.14	15.01
EUR	-0.10	-0.17	-0.25	-0.31	-13.20	-20.65	-28.77	-34.07
CHF	-0.16	0.05	0.05	0.01	-19.75	7.64	7.99	0.73
CAD	0.01	0.00	-0.06	-0.15	1.36	0.62	-7.59	-19.13
JPY	-0.23	-0.13	-0.17	-0.21	-26.65	-16.27	-20.58	-24.94
AUD	-2.27	-2.25	-2.20	-2.08	-100.63	-100.37	-99.73	-97.85
SEK	-0.13	-0.06	0.02	0.07	-16.61	-8.11	2.98	11.26
NOK	-0.97	-1.09	-1.07	-0.97	-71.47	-75.62	-74.86	-71.41
Panel B: Models With Free Constants vs. Historical Means								
USD	-0.66	-1.16	-1.23	-1.15	-57.91	-77.90	-79.97	-77.67
GBP	-0.05	-0.08	-0.11	-0.14	-6.73	-10.30	-14.83	-18.12
EUR	-0.11	-0.15	-0.26	-0.36	-13.80	-19.23	-29.79	-38.44
CHF	-0.31	0.04	0.11	0.08	-34.43	6.61	17.40	13.27
CAD	-0.22	-0.15	-0.16	-0.23	-25.94	-18.55	-20.13	-27.27
JPY	-0.21	-0.10	-0.12	-0.11	-25.21	-13.39	-16.08	-14.86
AUD	-1.91	-2.15	-2.44	-2.56	-95.18	-99.03	-102.88	-104.30
SEK	-0.11	-0.12	-0.10	-0.08	-14.05	-15.73	-13.21	-11.14
NOK	-0.61	-0.83	-0.98	-1.06	-55.20	-65.79	-71.84	-74.56

**Description:** The Table reports two statistics that compare the out-of-sample forecasts from recursive estimations of two versions of the Cochrane and Piazzesi, 2005 model for the excess rates of returns on bonds denominated in different currencies compared to the forecasts based only on the historical mean excess rates of return. Panel A contains results for the basic model, and Panel B contains results for models estimated with free constant terms. The first statistic is the  $R^2$ , which is calculated as one minus the ratio of the mean squared error of the CP forecasts to the mean squared error of the historical mean. The second statistic tests the equality of the forecasts and is the Clark and McCracken, 2005  $MSE - F$  statistic. The sample periods for the dependent variables during the initial in-sample estimation all end in 2003:12, which is the end of the Cochrane and Piazzesi, 2005 sample. The samples begin in 1974:12 for the USD, the GBP, and the EUR; in 1989:03 for the CHF; in 1987:03 for the CAD; in 1986:03 for the JPY; in 1988:04 for the AUD; in 1988:03 for the SEK; and in 1999:03 for the NOK. The out-of-sample periods are all 2004:01 to 2016:12.

**Interpretation:** For all currencies and for both types of the Cochrane and Piazzesi, 2005 models, the out-of-sample forecasts of excess bond returns from the models are inferior to the forecasts from the historical means of excess returns.

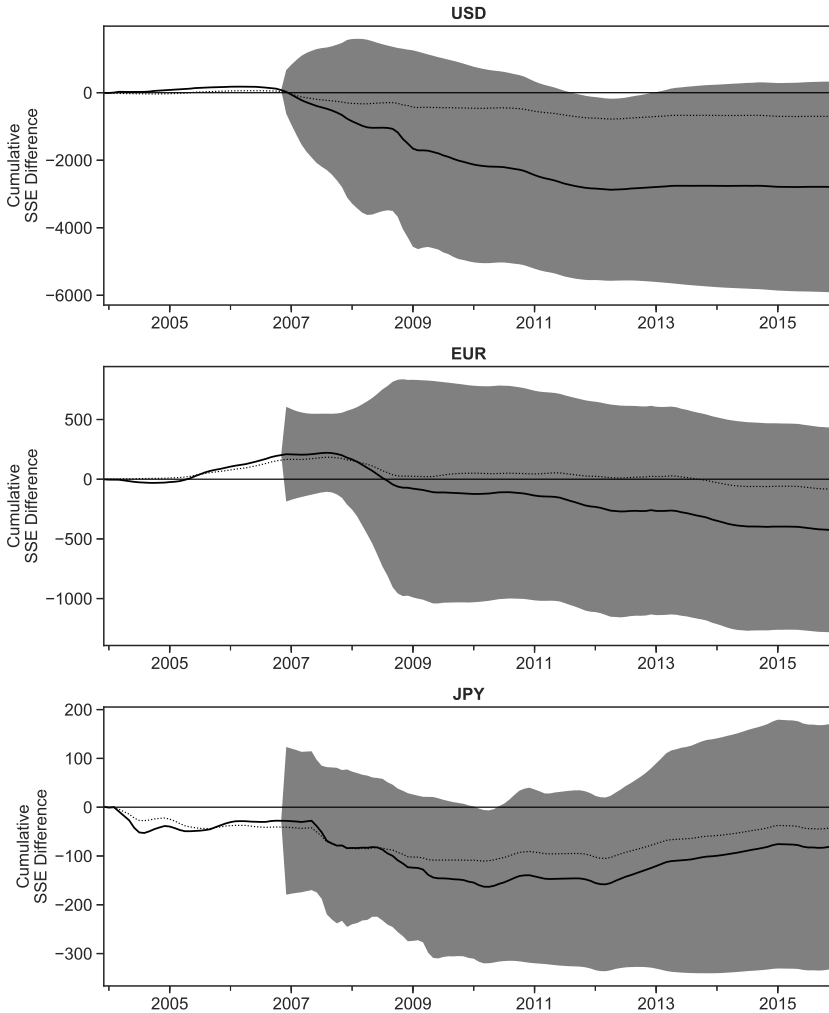


Figure 1: Out-of-Sample Results

**Description:** The figures plot the differences in the out-of-sample cumulative sums of squared errors from forecasts based on the historical means and the Cochrane-Piazzesi models for excess returns on the five-year USD, EUR, and JPY bonds. An increase in the cumulative SSE (the solid line) implies that CP model performed better in a given period. The shaded regions show two standard error bands for the SSE. The dotted line shows in-sample results from estimation over the full sample.

**Interpretation:** The Figures demonstrate that the relative deterioration in the model forecasts for the USD and the EUR begins in 2007 and 2008 with the onset of the global financial crisis. While the point estimates favor the historical means, the large standard errors remind us that distinguishing between out-of-sample forecasts is inherently difficult.



on the historical mean relative to the CP models' forecasts for the excess returns on the five-year USD, EUR, and JPY bonds.<sup>14</sup>

An increase in the cumulative SSE implies that the CP model forecasts performed better in a given period. The shaded regions show two standard error bands for the SSE and begin after the first 24 months of the out-of-sample period. The dotted line shows in-sample forecasting results from estimation over the full sample.

One sees that for the USD and the EUR, the forecasts of the model were equal to or marginally better than the forecasts from the historical means until 2007 for the USD and 2008 for the EUR, and then both models experienced a deterioration in forecasting ability during the global financial crisis. The model forecasts for the JPY were immediately inferior to those of the historical mean but were recovering in the later part of the sample. Although the point estimates clearly do not favor the models' forecasts, given the volatility of excess returns, we are generally unable to reject the hypothesis that the forecasts are the same. Finally, the dotted lines indicate that in-sample forecasts of the models from a constant set of parameters were generally inferior to the historical mean during this period.

### 5.1 An Alternative Model with Free Constants

In discussing the relation of the Cochrane and Piazzesi (2008) ATSM to the empirical model in Cochrane and Piazzesi (2005), we noted that the former does not constrain the constant terms to have the same factor of proportionality across maturities as is imposed by the latter. To see whether relaxing this constraint which formally nests the historical mean model as a constrained version of the larger model, we recursively estimated the model with free constant terms for each maturity.<sup>15</sup>

The results of these out-of-sample forecasts are presented in Panel B of Table 6. All of the  $R^2$ 's except for maturities 3, 4, and 5 for the CHF are negative. We conclude that the forward spreads, when used in this way, provide no useful out-of-sample forecasting power for the excess bond returns.

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<sup>14</sup>Figures for all maturities and bonds denominated in other currencies are available in the Online Appendix.

<sup>15</sup>The equations of the free constant model are described in the Online Appendix which also reports the parameter estimates. These estimates do not differ substantively from the estimates reported in the paper for the basic model.

## 5.2 Constraining Parameters Across Currencies

In out-of-sample forecasting situations, it is often advised to limit the number of free parameters that are estimated. We experimented with this Occam's razor intuition and recursively estimated the free constant model, described in the previous subsection, subject to the restriction that the  $\gamma$  parameters are the same for all countries. The out-of-sample forecasting results for this constrained model are better than the historical mean for the CAD, the GBP, and the JPY, but worse for the other currencies and for all maturities. These results are consequently presented in the Online Appendix.

## 5.3 Evolution of the USD Parameters

The failure of the model in the out-of-sample forecasting experiments and the rejection of equality of coefficients across sub-periods suggests substantial parameter instability. While a full analysis of this issue is not something we have space to accomplish in this article, Figure 2 presents the recursive estimates of the parameters of the Cochrane and Piazzesi (2005) model for the USD term structure as they evolve in the out-of-sample estimation period.<sup>16</sup> The estimates of the  $b_n$  parameters remain incredibly stable as the four lines are virtually horizontal.

It is also clear that beginning in 2008 with the advent of the financial crisis, the estimated  $\gamma(2)$  changes over the course of two years from positive to negative, the estimated  $\gamma(3)$  begins a slow decline, and the estimated  $\gamma(5)$  experiences a steady increase. The estimated  $\gamma(4)$  is reasonably constant after a blip in 2009. Because these are recursive estimates that use all of the sample to that point in time, they are more stable than would be recursive rolling estimates that use the same sample size at each point in time. In that sense, the slow evolution masks more dramatic changes.

## 5.4 Out-of-Sample Forecasts of Currency Returns

Given the inability of the CP factors to forecast excess rates of return in the currency markets reported in Table 5 and the changes in the signs of the estimated coefficients on the interest differentials in the in-sample regressions, the reader should expect that this specification will not be

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<sup>16</sup>Comparable Figures for the other currencies for both the basic model and the free constant model are available in the Online Appendix.

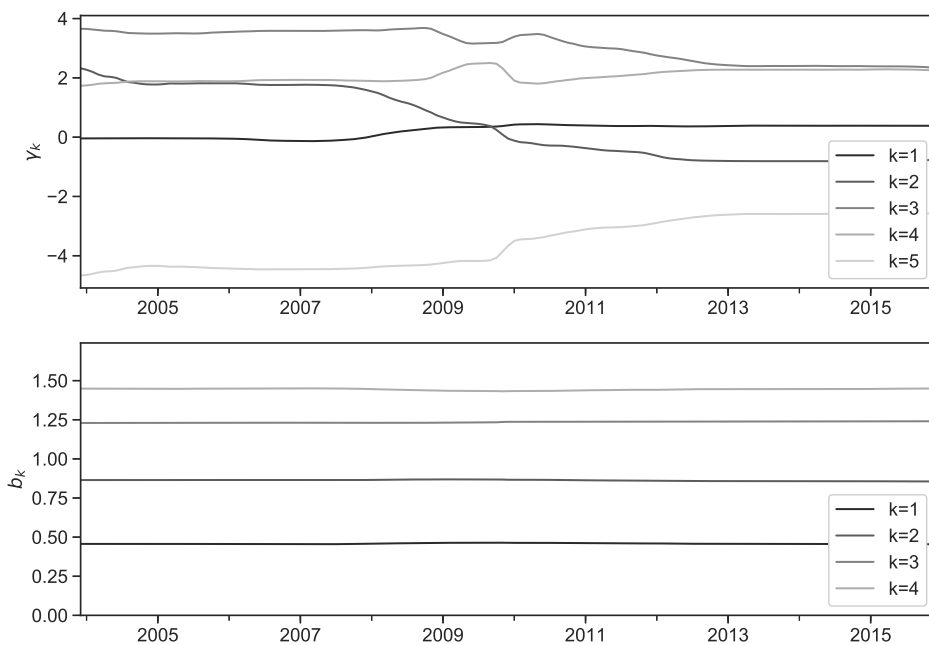


Figure 2: Evolution of the USD Parameters

**Description:** The figures present the evolution of the estimated  $\gamma(k)$  parameters in the top and the  $b(k)$  parameters in the bottom as the sample is extended from 2003:12 to 2016:12.

**Interpretation:** The stability of the  $b(k)$  parameter estimates is remarkable while it is clear that the instability of the  $\gamma(k)$  parameters is mostly related to the period between 2008 and 2013 after which the coefficients appear to have stabilized.

Table 7: Out-of-Sample Forecasts for Excess Foreign Exchange Returns: Cochrane-Piazzesi Factors and Interest Differentials vs. Historical Means

CUR	$R^2$	MSE-F
GBP	-0.08	-10.61
EUR	-0.25	-28.72
CHF	-0.18	-21.92
CAD	-0.16	-19.96
JPY	-0.43	-43.34
AUD	-0.14	-17.26
SEK	-0.02	-3.31
NOK	-0.33	-35.61

**Description:** The Table reports two statistics that compare the out-of-sample forecasts from recursive estimation of equation (24) for the excess return in USD on one-year investments in the money markets of different currencies to the forecasts based on the historical mean excess rates of return on those currencies. The first statistic is the  $R^2$ , which is calculated as one minus the ratio of the mean squared error of the CP forecasts to the mean squared error of the historical mean. The second statistic tests the equality of the forecasts and is the Clark and McCracken, 2005  $MSE - F$  statistic. The sample periods for the dependent variables during the initial in-sample estimation all end in 2003:12, which is the end of the Cochrane and Piazzesi, 2005 sample. The samples begin in 1974:12 for the USD, the GBP, and the EUR; in 1989:03 for the CHF; in 1987:03 for the CAD; in 1986:03 for the JPY; in 1988:04 for the AUD; in 1988:03 for the SEK; and in 1999:03 for the NOK. The out-of-sample periods are all 2004:01 to 2016:12.

**Interpretation:** Out-of-sample forecasts of currency returns using the Cochrane-Piazzesi forecasting factors and interest differentials are generally inferior to forecasts from the historical means of the currency returns.

useful in out-of-sample of the excess return. For completeness, we present these results in Table 7.

The results are indeed as anticipated as the out-of-sample forecasts from the model are unable to beat the historical mean excess returns of all currencies versus the USD.

## 6 Related Literature

The Cochrane and Piazzesi (2005) and Cochrane and Piazzesi (2008) papers generated a vast literature. In this section we briefly review what we consider to be the most important contributions in the literature that

are related to our paper.<sup>17</sup>

Dahlquist and Hasseltoft (2013) and Dahlquist and Hasseltoft (2016) and Sekkel (2011) were the first to extend the Cochrane and Piazzesi (2005) model to the bond markets of additional currencies. Dahlquist and Hasseltoft (2013) examine the bond markets of the USD, the CHF, the EUR, and the GBP, as well as examining the USD returns on the foreign bonds. They use a sample period from January 1975 to December 2009, and the CP factor is constructed from projections onto the five forward rates as in the original paper. They estimate local currency CP factors, and they also construct a global CP factor as a GDP-weighted average of the local CP factors. They find that the global CP factor provides some additional explanatory power relative to the local CP factors. In Dahlquist and Hasseltoft (2016) they extend their analysis adding the bond markets of the AUD, the CAD, the DKK, the JPY, the NOK, and the SEK; and they employ a sample period from December 1999 to December 2013. They find support for the model in all currencies, but they do not investigate the stability of the coefficients. We similarly find that the model works well in our two non-overlapping samples, but we are able often able to reject that the parameters are the same in the two samples.

Wright (2011) examines the term structures of interest rates for the G-10 countries by estimating ATSMs as in Joslin *et al.* (2014). He studies the implied risk premiums or term premiums, defined as the difference between the long-term yields and expectations of future spot interest rates, finding that these term premiums have generally declined in most countries over the sample period from January 1990 to May 2009. Bauer *et al.* (2014) dispute these conclusions noting that after correcting for small sample bias in the coefficient estimates, the term premiums show a pronounced countercyclical pattern as was found by Cochrane and Piazzesi (2005). We have not attempted to bias-correct our coefficient estimates, and doing so could affect our conclusions.

Sekkel (2011) uses the Wright (2011) data to estimate the Cochrane and Piazzesi (2005) model, but he projects the excess returns only onto the one, three, and five year forward rates. He finds that the performance of the model deteriorates during the global financial crisis. We use averages

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<sup>17</sup>Because the Cochrane and Piazzesi (2005) and Cochrane and Piazzesi (2008) papers have 1,400 and 328 Google Scholar citations, respectively, as of March 2019, our literature review must be highly selective.

of four forward spreads, and we did not experiment with leaving out the two-year spread. We doubt this affects our results, but the financial crisis clearly strongly influences our findings.

Consistent with the finding of Cochrane and Piazzesi (2005) that the CP factor is not spanned by the first three principal components of bond yields, Duffee (2011) documents that almost half of the variation in U.S. dollar (USD) bond risk premiums cannot be detected using the cross-section of yields. He finds that fluctuations in this hidden component have strong forecasting power for both future short-term interest rates and excess bond returns. The hidden component is negatively correlated with aggregate economic activity, but macroeconomic variables explain only a small fraction of variation in the hidden factor.

Koijen *et al.* (2017) model the stochastic discount factor as depending on the Cochrane and Piazzesi (2005) forecasting factor as well as the return on the stock market and the level of the term structure of interest rates. They demonstrate that such a model does well in simultaneously pricing returns on value and growth stocks in addition to USD zero-coupon bonds. We suspect that the results in Koijen *et al.* (2017) are affected by the instability in the CP factors that we document, but we have not investigated how their model would perform in the different samples we investigate.

Kessler and Scherer (2009), Thornton and Valente (2012), Zhu (2015), and Sarno *et al.* (2016) perform out-of-sample forecasting analyses with the Cochrane and Piazzesi (2005) model. Kessler and Scherer (2009) assess the performance of trading strategies based on a one-month forecast horizon using data from seven currencies (the AUD, CAD, CHF, EUR, GBP, JPY, and the USD) for the sample period February 1997 to July 2007. They use either a 36 or 60 month rolling window to estimate the parameters of the forecasting equation implying that they have either 88 or 64 true out-of-sample forecasts. They find slightly positive but only marginally significant trading profits. Our out-of-sample results are for an annual holding period and show no evidence of useful predictability.

Thornton and Valente (2012) investigate the out-of-sample predictability of USD bond excess returns and assess the economic value of the forecasting ability of empirical models based on Fama and Bliss (1987) and Cochrane and Piazzesi (2005). Their results show that the information content of forward rates does not generate systematic economic value to investors in a dynamic asset allocation exercise. Furthermore, they find that

the models do not outperform the no-predictability benchmark, and their relative performance deteriorates over time. We find similar results for the bond markets of other currencies and for the foreign currency markets.

Zhu (2015) explores the forecasting ability of a global CP factor constructed as the forecast of the average returns on the two through five year maturity bonds averaged over four currencies (the EUR, JPY, GBP, and USD) when regressed on the four individual currency CP factors. The full sample period is January 1980 to December 2011, and the out-of-sample period begins in January 1992. In contrast to our findings, Zhu (2015) finds statistically significant out-of-sample forecasts that beat the historical mean return for all four countries. This is true even when only the local currency CP factors are included in the analysis and when the out-of-sample period is restricted to the global financial crisis, 2008-2011. Because these results are so inconsistent with ours, we tried to replicate some of the findings in Zhu (2015). Although we were able to match his summary statistics with our data, we were not able to replicate his out-of-sample findings.

Sarno *et al.* (2016) investigate ATSMs for the USD bond market and find that their implied time-varying risk premiums do not provide important increases in utility to investors over and above inferences about expected future spot interest rates implied by the expectations hypothesis of the term structure with constant risk premiums. While we find strong in-sample evidence of time-varying risk premiums in our two samples, the changing nature of the parameters and the lack of out-of-sample predictability is consistent with their findings.

Turning to the international implications of the modeling, Sarno *et al.* (2012) find that separately estimated ATSMs for two currencies, both of which provide very small pricing errors for zero-coupon bonds denominated in those currencies, are not highly correlated with the relative rate of appreciation of those currencies in the foreign exchange market. These results clearly support our approach to testing the international implications of the Cochrane and Piazzesi (2008) model that allows for risks in the general stochastic discount factor that are not priced in the bond markets.

Our results are also related to the vast literature examining the uncovered interest rate parity (UIRP) hypothesis. Although Chinn and Meredith (2004) provide support for UIRP at the annual horizon, our results are more consistent with the conclusions of Bekaert *et al.* (2007), who argue that UIRP is violated at longer horizons just as is typically the case at the shorter monthly horizon, although here too we find evidence of parameter

instability.

## 7 Conclusions

In this paper we document substantive instabilities in the empirical analysis of risk premiums in bond and foreign exchange markets. There are many directions that research on time varying risk premiums in bond and foreign exchange markets could go. Here we review some recent approaches.

One puzzle we uncover is the observation that there is a strong one-factor structure to the forecasts of expected returns in the bond markets in a particular sample of data, but a different one-factor structure in another sample. Modeling the sources of these structural changes should be high on the research agenda of fixed income research. It is also puzzling that the CP factors in two currencies have such strong predictability in their respective bond markets but not in the foreign exchange market between the two currencies. Investigating why these markets are not more closely linked should also be on the research agenda.

In extending the Cochrane and Piazzesi (2005) model to additional currencies and considering its international implications, we have not addressed the term structure literature arguing that macroeconomic variables, such as inflation and employment, have additional forecasting power over and above that available in bond yields. In this regard we cite two recent critiques of this literature. First, Ghysels *et al.* (2018) find that several studies touting the significantly improved forecasting performance of macroeconomic variables above that provided by yields overstate their importance because the studies use revised data. Ghysels *et al.* (2018) find that use of real time U.S. data substantially reduces the implied predictive power. Second, Bauer and Hamilton (2017) argue that after taking account of small sample distortions in the test statistics induced by the use of macro variables with trends, the evidence for additional predictability from macro variables is much weaker.

It is interesting though that Jotikasthira *et al.* (2015) document highly correlated yield curve fluctuations across different currencies. They argue that common macroeconomic shocks influence bond yields both through a monetary policy channel and through a risk compensation channel. Using data from the U.S., the UK, and Germany, they find that world inflation and the level of the U.S. yield curve explain over two-thirds of the covariation



of yields at all maturities and that these effects operate largely through the risk compensation channel for long-term bonds.

In a related finding, Pericoli and Taboga (2012) propose a two-country no-arbitrage term-structure model to analyze the joint dynamics of bond yields, macroeconomic variables, and the exchange rate. The model demonstrates how exogenous shocks to the exchange rate affect the yield curves, how bond yields co-move in different countries and how the exchange rate is influenced by interest rates, macroeconomic variables and time-varying bond risk premiums. Upon estimating the model with U.S. and German data, they find that time-varying bond risk premiums account for a significant portion of the variability of the exchange rate. As we mentioned above, the correlations we find between the CP factors denominated in different currencies are certainly suggestive that global risk premiums are driven by common international investors, but we leave this issue for future research because addressing these issues in our multiple currency context is beyond what can be accomplished in a given article.

As in the original analysis of Cochrane and Piazzesi (2005), we have focused exclusively on the annual forecasting horizon. Most bond market ATSMs are estimated at the monthly horizon and typically find that monthly risk premiums are driven by more than one state variable. In contrast, we find the strong one-factor structure originally documented by Cochrane and Piazzesi (2005) at the annual horizon. Examining the dynamics of the state variables in monthly models and seeing whether they imply a single state variable at the annual horizon would be an interesting project. Recent papers that examine multiple horizons include Bacchetta and Van Wincoop (2010), Engel (2016), Lustig *et al.* (2017), and Chernov and Creal (2018) who find interesting patterns in expected returns at different horizons.

We also have only employed data on bonds with a maximum of five years to maturity as in the original paper of Cochrane and Piazzesi (2005). We treat these data sets as providing yields on actual bonds, as in most of the term structure literature. In contrast, Pancost (2018) uses the raw price data on all outstanding U.S. Treasury bonds rather than fitted zero-coupon yield curves constructed either by the bootstrap method of Fama and Bliss (1987) or the functional form approaches of Nelson and Siegel (1987) or Svensson (1995). Pancost (2018) finds that the two methods of fitting yield curves do well for maturities less than five years, but during the financial crisis there appears to be two separate yield curves in actual data for bonds with maturities between five years and 12 years corresponding to whether

the bonds have been outstanding for more or less than 15 years. He also finds that prices of risk estimated from vector autoregressions of bond market factors do not forecast the returns on actual Treasury bonds.

While ATSMs are typically developed under the assumption of rational expectations, it may be the case that behavioral finance with its time varying sentiments could be responsible for our findings of model instability. One recent empirical analysis with a behavioral slant is Brooks and Moskowitz (2017), who examine quarterly returns on bonds denominated in AUD, CAD, EUR, GBP, JPY, SEK, and USD. Using panel data methods with time fixed effects, they argue that measures of value, carry, and momentum dominate the CP factor in forecasting excess returns. While we are forecasting actual excess returns on a currency by currency basis, the panel data approach of Brooks and Moskowitz (2017) implies that they are forecasting deviations from cross-sectional average returns.

Another alternative approach to these issues would rely on the analysis of Krishnamurthy and Vissing-Jorgensen (2011) who argue that the supply of U.S. Treasury securities affects the level and slope of the yield curve. Do such changes in quantities also affect the risk premiums in the other bond markets and in the foreign exchange markets? Valchev (2017) answers this question affirmatively. It is natural to think that major changes in monetary and fiscal policies, including the quantitative easing done by major central banks during the international financial crisis, could induce the changes in the parameters of the CP factors and the resulting changes in forecasting power that we observe. Actually demonstrating this empirically is a challenging task. Evidence of substantive structural change in the international financial markets can also be found in the deviations from covered interest rate parity documented by Du *et al.* (2018) and Rime *et al.* (2017).<sup>18</sup>

Rather than focusing on time varying risk premiums, Valchev (2017) and Jiang *et al.* (2018) are two recent papers that empirically explore differential time-varying liquidity premiums, or non-pecuniary returns, on government bonds as explanations of rates of currency depreciation.

Engel (2016) notes that countries with high real interest rates tend to have currencies that are stronger than can be accounted for by the path of expected real interest differentials under UIRP. He observes that these

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<sup>18</sup>Andersen *et al.* (2018) provide a theoretical explanation for deviations from covered interest rate parity in a world with highly levered, risky financial market makers.

two findings have contradictory implications for the relationship of the foreign-exchange risk premium and interest-rate differentials and shows that existing models appear unable to account for both puzzles. He then introduces a model, in which short-term assets can have liquidity premiums as in Nagel (2016), that potentially reconciles the two sets of findings. Our findings of significant differences in parameters across samples that exclude and include the global financial crisis and the illiquidity observed in many asset markets during the financial crisis as well as the role that government bonds as safe assets play in the financial system are certainly suggestive that more research along these lines is warranted. Ultimately, we need to ask if these liquidity based models are able to explain the instabilities that we document.

Our econometric analysis also is conducted under the standard assumption that investors have rational expectations and that the data are stationary and ergodic. It has long been recognized that changes in monetary policy regimes can cause problems with econometric analysis of the term structure. Fuhrer (1996) argues that investors are aware of changes in regimes but do not anticipate future changes, which he views as a compromise between full rationality and learning. Bekaert *et al.* (2001) argue that so-called peso problems, caused by differences between the frequency of realizations of the data and the conditional distributions investors had at the time that they set bond prices, could be responsible for the anomalies observed in the term structure literature.

The necessity for investors to learn about changes in monetary policy, the rate of inflation, or the real interest rate are also important areas of recent research that relaxes the rational expectations assumption. Piazzesi *et al.* (2015) note that professional forecasts of interest rates differ from those based on regressions. They build on the insights of Froot (1989) who argued that evidence against the expectations hypothesis of the term structure was plausibly due to the failure of the rational expectations assumption imposed in the tests rather than to failures of the expectations hypothesis itself.

Giacoletti *et al.* (2016) argue that marginal investors in the bond market act as Bayesian learners to form prospective real-time views about bond market risks. While the sources of risks are the first three principal components of the yield curve, knowledge of the extent of disagreement among professionals is informative about how today's yield curve will impact its future shape and thus the prices of risks.

The studies cited here provide some interesting directions in which research can go. Most of these papers do not investigate time variation in the parameters of their empirical models. Our paper provides a set of challenging empirical results demonstrating more attention should be devoted to this type of analysis. The paper also provides interesting empirical evidence showing an absence of links that should theoretically be present between the term structures of interest rates in two currencies and the currency market between them.

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## Appendix

### A Data

Data on the term structures of interest rates for the different currencies were obtained from several sources. The USD data are from the CRSP Fama-Bliss database. This is the same source as Cochrane and Piazzesi (2005). For yields from the non-USD term structures, we obtained data from Jonathan Wright's web site. These data were used in Wright (2011). We updated the data from the web sites of the respective central banks. The monthly term structure data all end in December 2016. The data begin in June 1952 for the USD, in January 1970 for the GBP, in January 1973 for the EUR spliced with the Deutsche mark prior to 1999; in March 1988 for the CHF; in January 1986 for the CAD; in January 1985 for the JPY; in February 1987 for the AUD; in January 1987 for the SEK; and in January 1998 for the NOK. The exchange rate data are from the St. Louis Federal Reserve Bank FRED database. The sample period is January 1973 to December 2016 for all exchange rates. The Online Appendix provides additional detail about the exact sources of the data including URLs where the data may be updated. All data and programs are also available in this paper's online *Critical Finance Review* depository.

### B The Affine Model Solutions

The solutions to the coefficients of the natural logarithms of the bond prices in the affine model given in equation (9) are the following difference equations:

$$A_n = A_{n-1} - \delta_0 + \mathbf{B}_{n-1}^\top (\boldsymbol{\mu} - \boldsymbol{\Sigma} \boldsymbol{\lambda}_0) + (1/2) \mathbf{B}_{n-1}^\top \boldsymbol{\Sigma} \boldsymbol{\Sigma}^\top \mathbf{B}_{n-1} \quad (\text{B1})$$

$$\mathbf{B}_n^\top = -\delta_1^\top + \mathbf{B}_{n-1}^\top (\boldsymbol{\Phi} - \boldsymbol{\Sigma} \boldsymbol{\lambda}_1) \quad (\text{B2})$$

with initial conditions  $A_0 = 0$  and  $\mathbf{B}_0 = \mathbf{0}$ . By defining  $\boldsymbol{\Phi}^* = (\boldsymbol{\Phi} - \boldsymbol{\Sigma} \boldsymbol{\lambda}_1)$ , we can write equation (B2) as

$$\mathbf{B}_n^\top = -\delta_1^\top (\mathbf{I} - \boldsymbol{\Phi}^*)^{-1} (\mathbf{I} - \boldsymbol{\Phi}^{*n}) \quad (\text{B3})$$

### C The Standard Errors

This appendix derives the standard errors for the two-step estimation of the term structure models that generate the CP forecasting factors and the corresponding forecasting equation for the excess return on investing USD in the currency  $j$  money market. Let  $\bar{\varepsilon}_{1,t+1}$  and  $\bar{\varepsilon}_{j,t+1}$  be the error terms in equation (5) for the term structure regressions associated with the USD and currency  $j$ , respectively. The error term,  $\varepsilon_{j,t+1}^s$ , from the currency market is defined in equation (24). Let  $\mathbf{h}_{1j,t} \equiv [1, x_{1,t}, x_{j,t}, r_{j,t} - r_{1,t}]^\top$  represent the vector of regressors in equation (24), where  $x_{j,t} = \widehat{\gamma}_j^\top \overline{\mathbf{f}}_{j,t}$  is the return forecasting variable from the estimation of equation (5) for currency  $j$ , and let  $\boldsymbol{\phi}$  represent the vector of parameters in equation (24). Then, the orthogonality conditions associated with the forecasts of the average excess returns in the two bond markets and the excess rate of return in the currency market are

$$\begin{aligned} E[\overline{\mathbf{f}}_{1,t} \cdot \bar{\varepsilon}_{1,t+1}] &= \mathbf{0} \\ E[\overline{\mathbf{f}}_{j,t} \cdot \bar{\varepsilon}_{j,t+1}] &= \mathbf{0} \\ E[\mathbf{h}_{1j,t} \cdot \varepsilon_{j,t+1}^s] &= \mathbf{0}. \end{aligned} \tag{C1}$$

The parameter vector is  $\boldsymbol{\theta} = [\boldsymbol{\gamma}_1^\top, \boldsymbol{\gamma}_j^\top, \boldsymbol{\phi}^\top]^\top$ . Let  $\mathbf{g}_T(\boldsymbol{\theta})$  denote the sample mean of the orthogonality conditions in the system of equations given in (C1). Because the system is just identified, these sample orthogonality conditions can be set to zero, and the asymptotic variance of the parameter estimates can be estimated as

$$\mathbf{V}(\boldsymbol{\theta}) = \frac{1}{T} \mathbf{D}_T^{-1} \mathbf{S}_T \mathbf{D}_T^{-1\top} \tag{C2}$$

where

$$\mathbf{D}_T = \frac{\partial \mathbf{g}_T(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}^\top} \tag{C3}$$

is the sample estimate of the Jacobian of the orthogonality conditions,  $\mathbf{D}$ , which is defined below, and

$$\mathbf{S}_T \equiv \mathbf{C}_0 + \sum_{k=1}^K \frac{K-k}{K} (\mathbf{C}_k + \mathbf{C}_k^\top), \tag{C4}$$

is the sample estimate of the variance of the orthogonality conditions. The autocovariances are estimated with

$$\mathbf{C}_k \equiv \frac{1}{T} \sum_{t=k+1}^T \mathbf{g}_t \mathbf{g}_{t-k}^\top \quad (\text{C5})$$

where  $\mathbf{g}_t$  is the vector of observations on the orthogonality conditions are time  $t$ , and we use  $K = 18$ .

The derivatives in equation (C3) are sample estimates of

$$\mathbf{D} = \begin{bmatrix} -E(\overline{\mathbf{f}}_{1,t} \overline{\mathbf{f}}_{1,t}^\top) & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -E(\overline{\mathbf{f}}_{j,t} \overline{\mathbf{f}}_{j,t}^\top) & \mathbf{0} \\ \mathbf{D}_1 & \mathbf{D}_2 & \mathbf{D}_3 \end{bmatrix}$$

where  $\mathbf{D}_1 \equiv \nabla_{\gamma_1^\top} E(\varepsilon_{j,t+1}^s \mathbf{h}_{1j,t})$ ,  $\mathbf{D}_2 \equiv \nabla_{\gamma_j^\top} E(\varepsilon_{j,t+1}^s \mathbf{h}_{1j,t})$ , and  $\mathbf{D}_3 \equiv \nabla_{\phi^\top} E(\varepsilon_{j,t+1}^s \mathbf{h}_{1j,t})$ , respectively. We estimate  $\mathbf{D}_T$  numerically using Python's numdifftools package.

From the structure of the  $\mathbf{D}$  matrix and the partitioned inverse formula, one sees that the variances of the estimates of  $\gamma_1$  and  $\gamma_j$  are not affected by the estimation of  $\phi$  whereas the variances of the latter parameters are affected by the estimation of the former.