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Liquidity Risk and Asset Pricing

Hongtao Li[1]
Robert Novy-Marx[2]
Mihail Velikov[3*]

[1]*Guggenheim Partners, USA; hongtao.li@outlook.com*
[2]*University of Rochester, USA; robert.novy-marx@simon.rochester.edu*
[3]*Federal Reserve Bank of Richmond, USA; mihail.velikov@rich.frb.org*

ABSTRACT

Pastor and Stambaugh's (PS 2003) aggregate liquidity innovations can be closely replicated, as can their traded factor based on historically estimated liquidity betas, which performs even stronger out of sample. This factor's performance is highly sensitive to construction details, however, and exhibits significantly weaker performance when rebalanced at its natural monthly frequency, or when constructed using either more or less extreme sorts. Their predicted liquidity risk factor is more difficult to replicate, and difficult to interpret because characteristics chosen to predict liquidity risk introduce mechanical relations to other known anomalies. Contrary to the claims of PS, liquidity risk appears essentially unrelated to momentum.

Keywords: Asset pricing, Liquidity, Factor models, Momentum

JEL Codes: G11, G12

*The views expressed in this paper are those of the authors and do not necessarily reflect the position of the Federal Reserve Bank of Richmond or the Federal Reserve System. Novy-Marx provides consulting services to Dimensional Fund Advisors, an investment firm headquartered in Austin, Texas with strong ties to the academic community. The thoughts and opinions expressed in this article are those of the authors alone, and no other person or institution has any control over its content.

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1 Introduction

Intuitively it seems like liquidity should impact asset prices. Given two otherwise identical securities, market participants will pay more for, and thus expect lower returns from, the more liquid asset. It is harder, however, to precisely define liquidity. Liquidity is a multi-dimensional concept. It encompasses both spreads and the price impact of trading, depends on the desired speed of execution, and all of these factors vary over time. One stock may be less expensive to trade on the margin, but more difficult to trade in large quantities. Another may be even more expensive to trade in large quantities when demanding quick execution, but less expensive when traded patiently. Yet another may be less expensive to trade today, but become more difficult to trade when market liquidity evaporates. Given these subtleties, many adopt an attitude for liquidity akin to Supreme Court Justice Potter Stewart's "you know it when you see it" doctrine for recognizing pornography (*Jacobellis v. Ohio*, 1964).

Identifying empirically important asset pricing implications of liquidity requires a more precise definition, however, and Pastor and Stambaugh (PS 2003) represents one of the most important steps in this direction. PS propose an empirical measure of liquidity risk and evaluate the role it plays in generating cross-sectional variation in expected stock returns. Their measure "captures a dimension of liquidity associated with the strength of volume-related return reversals," essentially quantifying the extent to which the price impact of order flow is transitory. The measure is based on the idea that when liquidity is low, then any given trade will move prices more, but only temporarily. When aggregated to the market level, their measure exhibits "a number of sharp declines, many of which coincide with market downturns and apparent flights to quality," suggesting that it may be a priced state variable. PS are particularly interested in how a stock's exposure to aggregate liquidity, measured by its price sensitivity to fluctuations in the level of aggregate liquidity, affects its expected rate of return.

They present evidence that "expected stock returns are related cross-sectionally to the sensitivities of stock returns to innovations in aggregate liquidity," showing that "stocks that are more sensitive to aggregate liquidity have substantially higher expected returns." Perhaps most intriguingly, they suggest a significant relation between liquidity risk and momentum, presenting evidence that a liquidity risk factor "accounts for half of the profits" earned by momentum over their sample.

This paper attempts to evaluate these claims. It does so by revisiting their original evidence, replicating many of their tests, and by extending their results 16 years beyond their original sample. We are able to replicate their aggregate liquidity series with a high degree of precision. We can also reproduce their traded liquidity risk factor based on stocks' estimated betas to aggregate liquidity innovations. The factor performs even stronger out of sample, which is particularly remarkable given McLean and Pontiff's (2016) reported average 26% out-of-sample drop in average returns across 97 strategies from the academic literature.

The PS liquidity factor's performance is sensitive, however, to details of factor construction. In particular, both the choice to rebalance annually at the end of December, and the choice to construct the factor using extreme portfolios from a decile sort, significantly contribute to factor performance. Rebalancing annually in another month, or at the natural monthly frequency at which the stocks' liquidity betas can be easily estimated, significantly diminishes factor performance. Similarly, constructing the factor using either less or more extreme portfolio sorts yields performance reductions. A liquidity risk factor constructed using the most commonly employed methodology, that introduced by Fama and French (1993) for their three-factor model, earns a positive but insignificant 10 bps per month (t-statistic of 1.28). Overall the evidence is only suggestive that expected stock returns may be weakly cross-sectionally related to the sensitivities of stock returns to innovations in aggregate liquidity.

The PS results using predicted, as opposed to historically observed, liquidity risk, are both more difficult to replicate and more difficult to interpret. These employ a measure of liquidity risk that uses six stock-level characteristics in addition to the historically observed liquidity beta to estimate liquidity risk. We are able to qualitatively replicate most of the results employing predicted liquidity risk, but are unable to achieve the same quantitative precision. Because PS do not maintain a publicly available time-series for these results, as they do for the results based on historically estimated liquidity risk, it is difficult to ascertain if there are any undocumented methodological choices driving any differences.

Pastor and Stambaugh also use momentum variables, both share price (which is mechanically correlated with past performance) and prior 6 month stock returns, when predicting liquidity risk. This makes it difficult to interpret their claim that liquidity risk accounts for half of momentum. Adding a factor based on liquidity risk predicted using momentum variables certainly does reduce momentum's alpha, at least over the PS sample, but this reflects the power of the momentum variables, not liquidity risk, to help explain momentum's performance. When constructed without using momentum variables to predict liquidity, the predicted liquidity risk factor is basically orthogonal to momentum over the PS sample. This suggests that all of the power of the PS predicted liquidity risk factor to price momentum derives from its use of momentum variables to predict liquidity risk. This interpretation is also supported by the out of sample evidence. In the post-PS sample, the momentum variables are estimated to play a much less important role predicting liquidity risk, and over this sample the predicted liquidity risk factor does not help price momentum.

2 Replicating the Time-Series of Aggregate Liquidity

Following Pastor and Stambaugh (2003), we calculate each stock's liquidity, measured as the estimated extent to which price changes associated with order flow

	γ	Posted Spread	Effective Spread	Amihud	ME
Posted spread	−0.02				
Effective spread	−0.03	0.39			
Amihud	−0.05	0.17	0.64		
ME	0.05	−0.31	−0.76	−0.92	
Volatility	−0.02	0.77	0.57	0.25	−0.42

Table 1: Liquidity Measures Correlations.

Description: This table reports pairwise Spearman rank correlations between different liquidity measures. γ is the PS liquidity measure, estimated from Eq. (1), which proxies for the price impact of trade. Effective spread is the estimated transaction cost measure from Hasbrouck (2009). Posted spread is the month-end bid-ask spread, divided by the midpoint from CRSP. Amihud is the Amihud (2002) illiquidity measure. ME is market equity. Volatility is estimated for each stock each month using daily returns. The sample covers January 1968 to December 2015.

Interpretation: PS γ is uncorrelated with other liquidity measures.

on a given day tend to reverse the next. That is, a stock's liquidity is the extent to which the dollar volume traded in a given day in which the stock outperforms (or underperforms) the broad market appears to predict underperformance (or outperformance) the following day.

Specifically, liquidity for stock i in month t is calculated, using CRSP daily data, as the OLS estimate of $\gamma_{i,t}$ in the regression

$$r^e_{i,d+1,t} = \theta_{i,t} + \phi_{i,t} r_{i,d,t} + \gamma_{i,t} sign(r^e_{i,d,t}) \cdot v_{i,d,t} + \epsilon_{i,d+1,t}, \tag{1}$$

where $r_{i,d,t}$ and $r_{m,d,t}$ are the returns to stock i and the CRSP value-weighted market on day d in month t, respectively, $r^e_{i,d,t} \equiv r_{i,d,t} - r_{m,d,t}$ is the extent to which the stock outperforms the market, and $v_{i,d,t}$ is the volume traded in the stock that day, measured in millions of dollars.[1] The liquidity measure can be interpreted as the estimated "cost" from trading \$1 million of the stock, measured as the magnitude of adverse return the trader expects to experience as the transient price impact of her trade reverses. Conceptually the magnitude of this measure is driven by the extent to which trade moves price from "intrinsic value," so it may be viewed as a proxy for the slope of the price impact function.[2]

Table 1 reports time-series average cross-sectional Spearman correlations of γ, the PS individual stock-level liquidity measure, with liquidity measures that can be

[1]While not explicit in Pastor and Stambaugh (2003), d may be zero, in which case the right-hand variables correspond to the observations from the last trading day of the preceding month.

[2]The PS gamma is also superficially similar to, though quite distinct from, Kyle's (1985) stock "resiliency," defined as "the speed with which prices recover from a random, uninformative shock" (p. 1316). For Kyle's measure, larger reversals in deviations from fundamental prices indicate more resiliency, and thus higher liquidity; for the PS measure, larger reversals indicate that trade generates larger deviations from fundamental prices, and thus lower liquidity.

constructed using CRSP data. These include posted spreads (bid-ask spread scaled by the midpoint), the effective spread measure of Hasbrouck (2009), and Amihud (2002) illiquidity measure. They also include market capitalization and volatility, stock characteristics that correlate significantly with spreads. The table shows that the PS-γ is essentially uncorrelated with all the other liquidity measures, which are generally highly correlated with each other. This is perhaps unsurprising, as all these other measures primarily capture spreads, while Goyenko *et al.* (2009) note that "Pastor and Stambaugh's Gamma… [is] not appropriate to use as [a] proxy for effective or realized spreads" and should be "more naturally thought of as [a] price impact measure." Goyenko *et al.* (2009) also report low (or negative) correlations between the PS-γ and spread measures constructed using Trade and Quote (TAQ) data, but significant, though modest, positive correlations between the PS-γ and price impact measures constructed using TAQ data. The possibility of course remains that the PS-γ captures a dimension of liquidity distinct from both spreads and price impact.

2.1 Marketwide Liquidity

Aggregate liquidity for month t is calculated as simply the equal-weighted average of the estimated liquidities for all "eligible" stocks,

$$\hat{\gamma}_t = \frac{1}{N}\sum_{t=1}^{N}\hat{\gamma}_{i,t}. \tag{2}$$

The eligible universe includes NYSE and AMEX domestic common stocks (share codes 10 and 11), with share prices not less than \$5 or greater than \$1,000 at the end of the previous month.[3] PS also require that a stock has more than 15 observations to estimate individual liquidity in any given month. This restriction must be relaxed in the late sample, due to market closures following 9/11. Finally, while not noted by PS, they delete zero-volume observations when estimating Eq. (1), and doing so here is crucial to generating a high correspondence between our results and those reported in their paper.[4]

Aggregate liquidity can be interpreted as the expected return reversal cost from a \$1 million trade spread equally across the eligible stocks. A \$1 million trade is more substantial early in the sample, because both the number of eligible stocks and their average market capitalizations increase over the sample. PS consequently also compute a scaled liquidity series, $(m_t/m_1)\hat{\gamma}_t$, where m_t is the market

[3]PS exclude NASDAQ stocks (exchange code 3 or 33 at end of previous year) in constructing the aggregate liquidity measures, because "NASDAQ returns and volume data are available from CRSP for only part of this period (beginning in 1982)."

[4]Determining this fact required implementing numerous variations on the methodology described in PS. This involved labor far beyond what could reasonably be expected for casual replication, and was only possible because of the public aggregate liquidity series maintained by PS, which allowed us to infer which variations were important for generating a close correspondence.

capitalization of eligible stocks at the end of month $t-1$, and month 1 corresponds to the beginning of the sample, August 1962. Scaled aggregate liquidity thus can be interpreted as the expected return reversal cost associated with trading a fixed proportion of the market capitalization of the eligible universe, spread equally across this universe's stocks, where the fixed proportion is the inverse of the universe's initial market capitalization measured in millions of dollars.

Innovations to aggregate liquidity are calculated as the unexpected changes in scaled aggregate liquidity. That is, they are the residuals from the regression predicting changes in scaled average stock liquidity using both last month's changes and last month's level, i.e., L_t from the regression

$$\Delta\hat{\gamma}_t = a + b\Delta\hat{\gamma}_{t-1} + c\left(\frac{m_{t-1}}{m_t}\right)\hat{\gamma}_{t-1} + 100L_t, \tag{3}$$

where $\Delta\hat{\gamma}_t = \left(\frac{m_t}{m_1}\right)\frac{1}{N_t}\sum_{i=1}^{N_t}(\hat{\gamma}_{i,t} - \hat{\gamma}_{i,t-1})$, and i indexes the stocks eligible in both months t and $t-1$. The 100 on L_t is an arbitrary scaling. PS construct this for each month from August 1962 to December 1999, and the series through December 2015 is available both on Pastor's website and from WRDS.[5]

Figure 1 shows the publicly available innovations to aggregate liquidity maintained by Pastor, together with our replication of this series (thick gray and thin black lines, respectively; left-hand scale). The difference between the two is included in the same figure below (dotted line; right-hand scale). The figure shows a close correspondence between the two series. In the PS sample (August 1962 to December 1999) the two series are 98.9% correlated and statistically indistinguishable. In the post-PS sample (January 2000 to December 2015) the correlation is still high, 93.2%, but lower than in the PS sample. We do not know the full reason for this correlation reduction, but suspect undocumented methodological changes. For example, to construct the aggregate liquidity level series in September 2001 we had to relax the required minimum 15 daily observations due to the market closures following 9/11, but the posted series does not specify how it deals with these closures.

We must reemphasize the importance of deleting zero-volume observations when replicating the results of PS, a methodological step undocumented in that paper. While the correlation between the series shown in Figure 1 (our liquidity innovation series and the one publicly available from WRDS) is 98.9% over the PS sample, if we retain zero-volume observations when calculating individual stock liquidities this correlation falls to 39.2%.

This dramatic correlation reduction results almost entirely from increasing the number of small and illiquid stocks used in the aggregate liquidity estimation.[6] The

[5]The liquidity innovation series, as well as adjusted aggregate liquidity and the performance of tradable liquidity factors, can be found at http://faculty.chicagobooth.edu/lubos.pastor/research/.

[6]Reestimating aggregate liquidity each month using only those firms with at least 15 non-zero volume observations, but retaining zero-volume observations, has little impact on the results. In

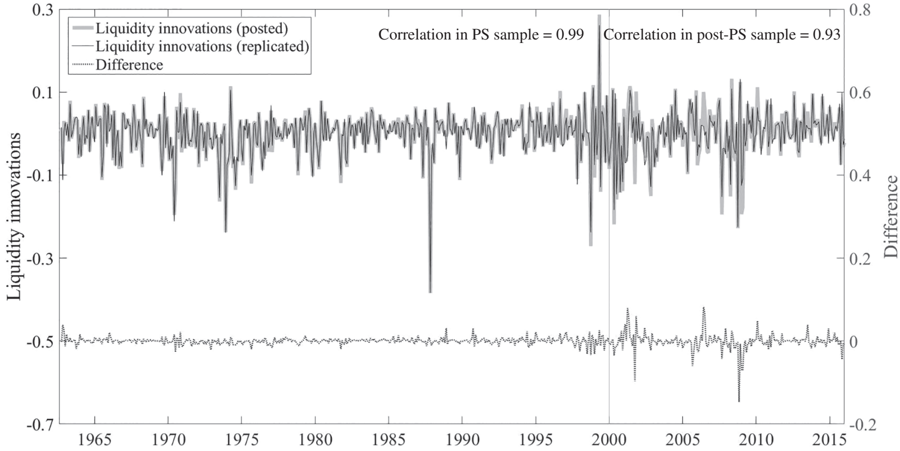

Figure 1: Aggregate Liquidity Innovations.

Description: The figure shows the Pastor and Stambaugh (2003) innovations to aggregate liquidity available from WRDS (thick gray line, left-hand scale), together with our replication of these innovations (thin black line). The difference is plotted underneath (dotted line, right-hand scale).

Interpretation: PS aggregate liquidity innovations series can be closely replicated.

zero-volume observations represent a significant minority of the data, 4.6 million out of a total of 52 million firm-day observations, or almost 9% of the total. Dropping the zero-volume observations reduces the number of daily observations below the required 15 in roughly a tenth of firm-months, and is more likely to exclude the smallest and least liquid stocks from the sample. As a result, retaining zero-volume observations expands the number of firms used in the aggregate liquidity estimation from between 966 and 2,208 to between 1,093 and 2,265.[7] In half of the additional firm-month observations stock-level liquidity is estimated using 10 or fewer daily observations, while in a fifth of cases it is estimated using five or fewer daily observations. Excluding these observations may make sense, as stock-level liquidity estimates based on so few observations may be unreliable.[8]

this case, the correlation between the estimated liquidity innovation series and the series available from WRDS is 98.4%, nearly as high as the 98.9% correlation observed when deleting zero-volume observations.

[7]There are small differences in the set of eligible stocks employed by us and PS even when we delete zero-volume observations. We do not know the precise differences, but PS report between 951 and 2,188 eligible stocks over their sample, slightly fewer than we use. They also report that the eligible universe is 34 times as large (in nominal dollars) at the end of their sample as at the beginning, whereas we only see a 32.4-fold increase in the market capitalization of the universe over the same period.

[8]Moreover, Eq. (1), which is used to estimate stock-level liquidity, regresses returns on lagged returns and lagged volume. Daily returns associated with zero-volume (either concurrent or 1-day

The excluded firms are on average less than one thirtieth the size of those employed in the liquidity estimation, and on average make up only 0.44% of total NYSE market capitalization.

3 Tradable Liquidity Risk Factors

Pastor and Stambaugh are interested in "whether marketwide liquidity is a state variable important for asset pricing" (PS abstract), i.e., whether exposure to aggregate liquidity shocks is priced. A direct method for investigating this hypothesis is to analyze the performance of tradable liquidity risk factors. These factors are essentially liquidity risk mimicking portfolios, which buy stocks that are sensitive to aggregate liquidity shocks, while hedging market exposure by shorting stocks less sensitive to this risk. These long/short strategies are designed to be market neutral, but highly exposed to changes in aggregate market liquidity. If investors are willing to pay more for stocks that remain relatively easy to trade when marketwide liquidity evaporates, but demand a premium in the form of higher expected returns to hold stocks that become hard to trade at those times, then we should expect the tradable liquidity factor to deliver high average returns.

Pastor and Stambaugh consider two methodologies for estimating stocks' exposures to aggregate liquidity shocks. One methodology simply uses stocks' estimated betas to liquidity innovations using 5 year return histories, and is termed "historical liquidity risk." The other predicts individual stock liquidity risk using these historically estimated betas together with six additional stock-level characteristics, and is termed "predicted liquidity risk." The paper emphasizes the performance of the factor based on predicted liquidity risk, but we begin here with the factor based on historical liquidity risk for three main reasons. First, while PS spend more time investigating the performance of the factor based on predicted liquidity risk, the authors only maintain the factor based on historical liquidity risk. This seems to suggest that they currently think it the more important factor. Its availability also provides a measuring stick against which we can compare our factor, which is an invaluable tool when undertaking replication. Second, the historical beta methodology is simpler, which also facilitates replication. Third, there are issues with the variables used to predict liquidity risk, which make it difficult to interpret some of the predicted liquidity risk factor results, particularly those pertaining to the relation between liquidity risk and price momentum. These issues are discussed in detail in Subsection 4.3.

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lagged) are calculated using bid-ask midpoints, as opposed to only traded prices, and may thus themselves be unreliable.

3.1 Factor Based on Estimated Historical Liquidity Risk

Pastor and Stambaugh estimate an asset's liquidity risk as its loading on aggregate liquidity innovations. That is, for each stock i liquidity risk is taken as the estimate of β_i^L from the time-series regression

$$r_{i,t}^e = \beta_i^0 + \beta_i^L L_t + \beta_i^M \mathrm{MKT}_t + \beta_i^S \mathrm{SMB}_t + \beta_i^H \mathrm{HML}_t + \epsilon_{i,t}, \tag{4}$$

where $r_{i,t}^e$ is the stock's return in month t in excess of T-bill returns, and MKT_t, SMB_t, and HML_t are the contemporaneous returns to the Fama and French (1993) factors. When constructing the liquidity risk factor they estimate each stock's liquidity risk beta annually, at the end of the year, using 5 years of monthly data, requiring a full 5 years of data to make the estimate. They include NASDAQ in addition to NYSE and AMEX common stocks, but continue to exclude stocks with prices less than \$5 or in excess of \$1,000.

Pastor and Stambaugh carefully try to avoid any look-ahead bias when constructing their traded liquidity factors. The liquidity risk innovations available on WRDS, however, come out of the time-series regression given in Eq. (3) estimated using data through the end of 2015, and thus use information only available at the end of 2015 when calculating innovations at the start of the sample in the 1960s. Estimated liquidity risk betas consequently cannot be calculated by regressing stock returns onto the posted aggregate liquidity innovations. Instead, when estimating liquidity risk betas in any month t using Eq. (4), the liquidity risk innovations used in the regression must themselves be reestimated with Eq. (3) using only data available up until time t.

The traded liquidity factor is then constructed as a simple long/short strategy based on the estimated liquidity risks of the individual stocks. At the end of each year stocks are sorted into ten portfolios (each with an equal number of stocks) on the basis of their liquidity risk betas estimated over the preceding 5 years. The traded liquidity factor buys the capitalization weighted decile of stocks with the highest estimated liquidity risk, while shorting the decile with the lowest estimated liquidity risk. The start date, 1967, is determined by the 5 year data requirement when estimating betas.

We are interested in replicating the results of PS, so we follow them in ignoring delisiting returns. This is equivalent to assuming that we do not trade stocks which will delist before the next rebalance date, which requires perfect foresight with respect to future delistings, and thus introduces a look-ahead bias in the factor's performance. This issue is largely inconsequential here, but has a significant impact on the results related to momentum, which we discuss further in Section 4.3.

Figure 2 shows the performance of traded liquidity factors, both the PS version publicly available on WRDS (dotted line) and as replicated here (solid line). Specifically, it shows the growth of a \$1 investment in each of the factors, ignoring

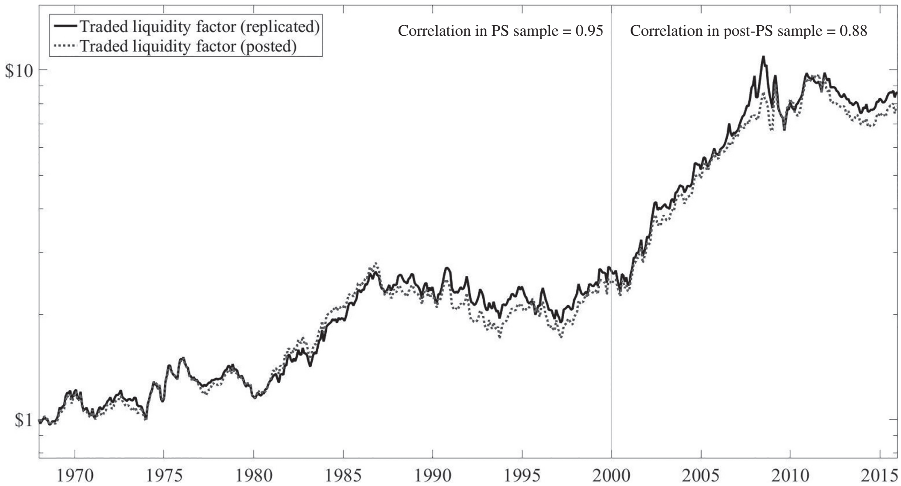

Figure 2: Traded Liquidity Factor Performance.

Description: The figure shows the growth over time of $1 invested in the Pastor and Stambaugh (2003) traded liquidity risk factor available on WRDS (dotted line) and the factor replicated in this paper (solid line). Factors are long/short extreme deciles of stocks (using name breaks) with the highest and lowest liquidity risk betas, estimated using 5 years of monthly data. Portfolios are capitalization weighted, and rebalanced annually at the end of each December.

Interpretation: PS traded liquidity factor can be replicated, and works even better out of sample.

transaction costs, from January 1968 to the end of 2015.[9] The figure again shows a close correspondence between our results and those of PS. The returns to the two series are 95% correlated in the PS sample (over which our aggregate liquidity innovation series are 99% correlated), and 88% correlated in the post-PS sample (over which our aggregate liquidity innovation series are 93% correlated). Perhaps most notably, the figure shows strong average performance in the post-PS sample. This is somewhat surprising, given McLean and Pontiff's (2016) evidence of an average 32% post-publication decline in anomaly performance. The factor's impressive late sample performance is concentrated in the early half of this period, between the NASDAQ peak and the start of the 2008 financial crisis.

Table 2 shows the two factors' average monthly excess returns, and results of time-series regressions of these factors on the Fama and French (1993, 2015) three- and five-factor models and the Carhart (1997) four-factor model, both in and out of the PS sample (January 1968 to December 1999, and January 2000

[9]These are long/short factors, so technically it shows the growth of a dollar put into a strategy that puts cash into risk-free collateral, while taking long and short positions of equal magnitudes in the high and low liquidity risk portfolios, after a funding charge equal to the risk-free rate.

	Posted Liquidity Risk Factor				Replicated Liquidity Risk Factor			
	(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**	**(7)**	**(8)**
Panel A: Pastor–Stambaugh Sample, January 1968 to December 1999								
α	0.28	0.26	0.37	0.20	0.30	0.28	0.37	0.20
	[1.73]	[1.60]	[2.14]	[1.16]	[1.90]	[1.73]	[2.22]	[1.19]
MKT		−0.03	−0.03	−0.02		−0.01	−0.01	0.00
		[−0.63]	[−0.65]	[−0.45]		[−0.23]	[−0.24]	[0.03]
SMB		−0.14	−0.17	−0.11		−0.13	−0.16	−0.09
		[−2.50]	[−2.86]	[−1.77]		[−2.37]	[−2.70]	[−1.54]
HML		0.14	0.10	0.20		0.11	0.09	0.17
		[2.10]	[1.59]	[2.14]		[1.80]	[1.33]	[1.83]
UMD			−0.10				−0.09	
			[−2.13]				[−1.94]	
RMW				0.26				0.29
				[2.35]				[2.69]
CMA				−0.01				0.02
				[−0.09]				[0.16]
Panel B: Post Pastor–Stambaugh Sample, January 2000 to December 2015								
α	0.69	0.69	0.68	0.53	0.70	0.72	0.66	0.53
	[2.38]	[2.38]	[2.31]	[1.76]	[2.35]	[2.38]	[2.21]	[1.71]
MKT		−0.01	0.01	0.07		0.01	0.09	0.10
		[−0.19]	[0.15]	[0.84]		[0.17]	[1.25]	[1.26]
SMB		0.15	0.14	0.32		0.06	0.02	0.26
		[1.69]	[1.51]	[2.93]		[0.69]	[0.18]	[2.35]
HML		−0.13	−0.12	−0.21		−0.12	−0.09	−0.20
		[−1.43]	[−1.33]	[−1.55]		[−1.24]	[−0.94]	[−1.44]
UMD			0.04				0.15	
			[0.76]				[2.62]	
RMW				0.37				0.43
				[2.47]				[2.85]
CMA				−0.13				−0.18
				[−0.74]				[−0.99]

Table 2: Alphas and Factor Loadings of Posted and Replicated Liquidity Risk Factors.

Description: The table reports results from time-series regressions of the returns of the liquidity risk factors onto various factor models. The posted Pastor–Stambaugh liquidity risk factor is downloaded from Lubos Pastor's website (http://faculty.chicagobooth.edu/lubos.pastor/research/). The replicated factor is long/short extreme deciles of value-weighted portfolios sorted on liquidity betas estimated at the end of each year, using 5 years of monthly data, from a time-series regression of stock returns onto the three Fama and French factors and aggregate liquidity innovations, which are themselves reestimated each year using only data available at that time.

Interpretation: Alphas and factor loadings of the liquidity risk factor can be closely replicated. Returns to the liquidity risk factor can be accounted for by the Fama and French five-factor model.

to December 2015, respectively).[10] The table shows similar alphas and factor loadings for the posted and replicated factor in every case. Somewhat surprisingly, the only significant alphas observed in the PS sample are the excess returns

[10]Strict replications of the major tables from PS are provided in the Appendix.

relative to the four-factor model that includes momentum, and even this is only marginally significant. Consistent with Figure 2, the liquidity factors exhibit stronger performance in the post-PS sample, but even over this sample are not significant relative to the five-factor model.

3.2 Issues with Factor Construction

Ideally empirical results should be insensitive to the specific choices made regarding the details of the empirical design. That is, results should be robust to alternative specifications. The performance of the PS traded liquidity factor is sensitive to at least two specific design choices.

First, PS rebalance the traded factor annually, at the end of December, despite the fact that individual stock-level liquidity risk can easily be estimated monthly. The choice to rebalance annually seems somewhat strange, given that PS are concerned with generating significant dispersion in postranking liquidity betas. If liquidity betas are somewhat persistent, but change over time, then ranking on estimates made with the most recent data maximizes the dispersion in postranking betas. PS instead choose to use annual rebalancing, and the specific choice they make has a significant impact on the strength of their results, especially in their sample.

Second, they build their factor from decile sorted portfolios constructed using "name" (or "all stock") breaks. That is, they put an equal number of stocks into each portfolio, regardless of the size of the stocks. This choice tends to yield more extreme portfolios than other potential choices, such as choosing breaks which yield an equal number of NYSE stocks in each portfolio ("NYSE breaks," popular in academia), or breaks which yield equal total market capitalization in each portfolio ("cap breaks," popular in industry). The choice to use decile portfolios for factor construction is also relatively extreme.

3.2.1 Rebalancing Issues

The importance of annual December rebalancing for realized factor performance can be seen in Figure 3. The figure shows the performance of liquidity factors that are constructed identically, except in their rebalancing. The top, dashed line shows the cumulative returns to the replicated PS factor, which rebalances annually in December; the middle, solid line shows the factor rebalanced at the natural, monthly frequency; the bottom, dotted line shows the factor that rebalances annually, but at the end of January instead of at the end of December. The performances of the three factors are remarkably different, with the factor constructed using the PS rebalancing convention dramatically outperforming the others, especially in the PS sample. In no sample does the factor rebalanced in January have a significant alpha relative to any of the factor models. The factor

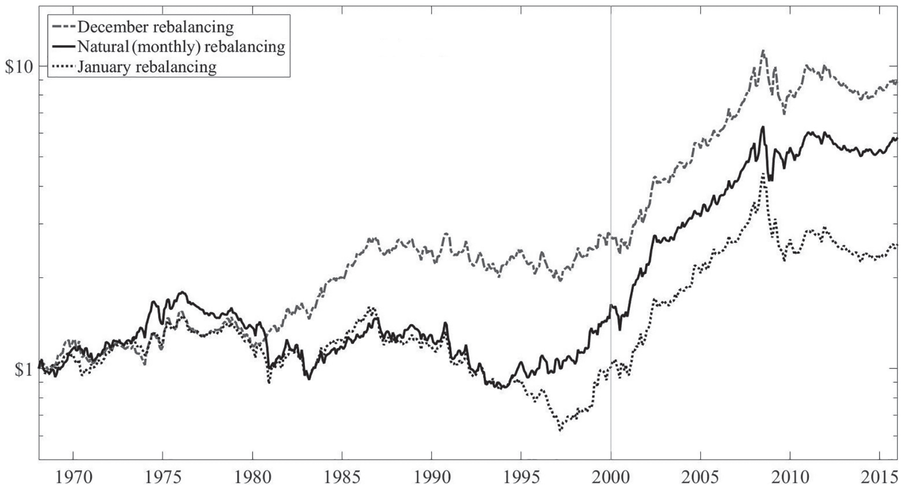

Figure 3: Cumulative Performance of Liquidity Factors with Alternative Rebalancing.

Description: The figure shows the growth over time of $1 invested in liquidity factors rebalanced annually in December, as in PS (top, dashed line), at the natural monthly frequency at which the liquidity betas are easily estimated (middle, solid line), and annually in January (bottom, dotted line). Each liquidity factor is the value-weighted returns to the long/short strategy that buys the decile of stocks with the highest estimated liquidity betas and shorts the decile with lowest estimated liquidity betas.

Interpretation: Liquidity factor performance is sensitive to rebalancing frequency.

rebalanced monthly has significant full sample performance, but performs poorly in the PS sample.

Table 3 confirms these results. For each of the three factors (rebalanced annually at the end of December, annually at the end of January, and monthly), as well as the PS factor available from WRDS, the table reports average monthly returns, and alphas relative to the three, four, and five factor models. The table reports much stronger performance for the factors rebalanced using the PS convention than it does for those constructed using equally (or more) reasonable alternative choices.[11]

3.2.2 Portfolio Construction Issues

The rebalancing dates are not the only detail of factor construction that significantly impacts the liquidity risk factor's observed performance. The choice to construct

[11] The performance of liquidity strategies hedged of their ex ante estimated exposures to the various factors at each point in time yield similar results. These are reported in Table A1.

	r^e	α_{FF3}	α_{FF4}	α_{FF5}
Panel A: Posted Liquidity Factor				
PS sample	0.28	0.26	0.37	0.20
	[1.73]	[1.60]	[2.14]	[1.16]
Post-PS sample	0.69	0.69	0.68	0.53
	[2.38]	[2.38]	[2.31]	[1.76]
Full sample	0.42	0.44	0.46	0.44
	[2.87]	[2.97]	[3.02]	[2.85]
Panel B: Replicated Liquidity Factor (Annual December Rebalancing)				
PS sample	0.30	0.28	0.37	0.20
	[1.90]	[1.73]	[2.22]	[1.19]
Post-PS sample	0.70	0.72	0.66	0.53
	[2.35]	[2.38]	[2.21]	[1.71]
Full sample	0.44	0.45	0.42	0.43
	[2.99]	[3.04]	[2.78]	[2.82]
Panel C: Liquidity Factor Constructed Using Annual January Rebalancing				
PS sample	0.06	0.05	0.23	0.10
	[0.34]	[0.30]	[1.33]	[0.55]
Post-PS sample	0.57	0.55	0.49	0.39
	[1.97]	[1.88]	[1.68]	[1.29]
Full sample	0.23	0.23	0.23	0.27
	[1.54]	[1.52]	[1.52]	[1.77]
Panel D: Liquidity Factor Constructed Using Natural (Monthly) Rebalancing				
PS sample	0.18	0.20	0.42	0.19
	[1.02]	[1.10]	[2.30]	[1.03]
Post-PS sample	0.75	0.76	0.73	0.49
	[2.55]	[2.56]	[2.45]	[1.61]
Full sample	0.37	0.38	0.45	0.36
	[2.40]	[2.42]	[2.84]	[2.24]

Table 3: Alphas of Traded Liquidity Factors with Alternative Rebalancing.

Description: Each liquidity factor is the value-weighted returns to the long/short strategy that buys the decile of stocks with the highest estimated liquidity betas and shorts the decile with lowest estimated liquidity betas. The three-factor model consists of market (MKT), size (SMB), and value (HML) factors; the four-factor model adds momentum (UMD); the five-factor model includes profitability (RMW) and investment (CMA). The Pastor–Stambaugh (PS) sample covers January 1968 to December 1999, the post-PS sample covers January 2000 to December 2015, and the full sample covers January 1968 to December 2015.

Interpretation: Liquidity factor performance is sensitive to rebalancing frequency.

the factor using the extreme portfolios from a decile sort employing name breaks also contributes to the strength of the PS results.

If covariance with aggregate liquidity innovations is truly priced, then one would expect that tests of the liquidity risk premia would be relatively insensitive to the details of factor construction. More extreme sorts on estimated liquidity risk loadings would yield more variation in liquidity risk exposure, and thus a greater return spread, but this greater spread would come with commensurately higher volatility. The net result would be similar Sharpe ratios, and similar information ratios relative to asset pricing models, and consequently similar inferences regarding the statistical significance of the results. That is, the strength of the inferences would be robust to changes in the details of factor construction.

This does not appear to be the case for the liquidity risk factor. Table 4 reports t-statistics on factor average returns (Panel A), and alphas relative to three different asset pricing models (Panels B to D), for factors constructed using 15 variations in the sorting methodology. These variations include sorts that are both less and more aggressive than the decile sort used by PS, ranging from two to 20 portfolios, and employ three different types of breaks (name breaks, NYSE breaks, and capitalization breaks). To facilitate comparison, other details of factor construction adhere to PS, in particular the choices to rebalance annually at the end of December, and to exclude stocks with prices outside the \$5 to \$1,000 range.

In all four panels of Table 4 the single most significant result comes from decile sorting using name breaks. That is, of the 15 methodologies considered, the choices employed by PS maximize the strength of the observed results. The decile sorted factor has a Sharpe ratio roughly 50% higher than the factors constructed using less extreme sorts (into two, three, or five portfolios), and commensurately higher alphas. More extreme sort leads to an even more dramatic diminution of performance, because the 5% of names with the lowest estimated liquidity risk exposures perform roughly as well as the 5% with the highest estimated exposures (average monthly excess returns of 72 vs. 73 bps). The alternative break points (NYSE or capitalization) somewhat improve the poor performance of the strategies based on 20 portfolios, but hurt the performance of the strategies based on the less extreme sorts, especially those that use the decile and quintile sorting methodology.

Table A2 replicates Table 4 using more timely liquidity risk estimates, rebalancing the portfolios at the natural monthly frequency. The table generally reports insignificant factor performance, but performance which is again always maximized under the PS convention of decile sorting using name breaks.

3.2.3 A Final Note on Rebalancing and Sorting

While no methodology completely insulates a researcher from the need to make choices, each of which potentially biases their results, methodologies that constrain the number of choices a researcher must make are less susceptible to these biases. The “best” methodology is thus arguably the one that most constrains the number of choices a researcher must make. When constructing factors, the methodology least susceptible to these overfitting biases consequently is to simply copy the most commonly employed methodology, and to do so at the natural frequency

$n^{\text{portfolio}}$	Name Break	NYSE Break	Market Cap Break
Panel A: t-Statistics on Average Excess Returns			
2	2.08	2.07	1.85
3	2.02	1.88	2.06
5	1.94	1.51	1.54
10	2.99	2.63	1.87
20	0.08	1.33	1.25
Panel B: t-Statistics on Alphas Relative to Fama and French (1993) Three-Factor Model			
2	1.77	1.76	1.54
3	1.73	1.55	1.82
5	1.94	1.48	1.53
10	3.04	2.68	1.81
20	0.08	1.40	1.27
Panel C: t-Statistics on Alphas Relative to Carhart (1997) Four-Factor Model			
2	1.95	1.99	1.82
3	2.14	1.99	2.26
5	2.19	1.74	1.77
10	2.78	2.64	1.78
20	0.34	1.57	1.22
Panel D: t-Statistics on Alphas Relative to Fama and French (2015) Five-Factor Model			
2	1.93	1.88	1.68
3	2.19	2.13	2.09
5	2.33	2.03	1.85
10	2.82	2.55	1.63
20	−0.09	1.29	0.68

Table 4: Performance of Liquidity Risk Factors Constructed Using Alternative Sorts.

Description: The table reports the t-statistics on the abnormal returns to liquidity factors constructed using 15 sorting methodology variations. Each factor is long/short the extremes value-weighted portfolios from a sort on stocks' estimated liquidity betas. Sorts are into two, three, five, 10 or 20 portfolios, using name, NYSE, or market cap breakpoints. Following PS, portfolios are rebalanced annually, at the end of December. Panel A reports t-statistics on average excess returns; Panel B reports t-statistics on alphas relative to Fama and French (1993) three-factor model; Panel C reports t-statistics on alphas relative to Carhart (1997) four-factor model; and Panel D reports t-statistics on alphas relative to Fama and French (2015) five-factor model. The sample covers January 1968 to December 2015.

Interpretation: Liquidity factor performance is sensitive to portfolio construction method.

of the signal used in factor construction. The dominant methodology for factor construction is the one introduced by Fama and French (1993), and the liquidity risk betas are naturally estimated at a monthly frequency.

That is, the essentially decision-free choice for liquidity risk factor construction, given historical liquidity beta as a measure of liquidity risk, is to replicate UMD, the momentum factor maintained by Ken French, replacing past stock performance with historical liquidity beta.[12] This factor uses monthly rebalancing and a less extreme sort than that employed by PS. Given the results of the previous two subsections, it would be surprising if the factor performed as strongly as the PS liquidity factor. The results are not surprising. Over the full sample, January 1968 to December 2015, the liquidity risk factor constructed using the Fama and French (1993) methodology earns less than ten bps per month, and these average returns are insignificant (t-statistic of 1.28).

4 Predicted Liquidity Risk

Despite only maintaining the return series for the traded liquidity risk factor based on historical liquidity betas, PS focus on the performance of the factor based on predicted liquidity risk. Analyzing this factor is important, at least in part because it is crucial to one of the most intriguing claims made in PS, that liquidity risk explains half of momentum profits.

4.1 Predicting Liquidity Betas

To calculate the predicted liquidity risk betas, PS use a two-step procedure. At the end of each year, they first estimate innovations to aggregate liquidity and individual stocks' liquidity risk betas using Eqs. (3) and (4), as in the previous section. Then, each stock's beta is modeled as a linear function of observables,

$$\beta^L_{i,t-1} = \psi_{1,i} + \psi'_{2,i}\mathbf{Z}_{i,t-1}, \tag{5}$$

where $\mathbf{Z}_{i,t-1}$ is a vector of seven stock characteristics used to predict liquidity risk. These characteristics include:

1. Historical liquidity beta, estimated using all data available from months $t-60$ to $t-1$, provided at least 36 months of data are available.

2. Average liquidity, calculated as the average value of $\hat{\gamma}_{i,t}$ from months $t-6$ to $t-1$.

3. Average volume, calculated as the natural log of the stock's average dollar volume from months $t-6$ to $t-1$.

4. Past performance, calculated as the cumulative return on the stock from months $t-6$ to $t-1$.

[12] Specifically, this factor is an equal-weighted average of large and small cap liquidity risk factors. These factors each buy (sell) the top (bottom) 30% of stocks by estimated liquidity risk, using NYSE breaks, within their corresponding universes, where large and small are defined by above and below NYSE median market capitalization, respectively. The size and liquidity risk sorts are independent, portfolio returns are value weighted, and portfolios are rebalanced monthly. It is available at http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

5. Return volatility, calculated as the standard deviation of the stock's monthly return from months $t-6$ to $t-1$.

6. Price, calculated as the natural log of price per share from month $t-1$.

7. Shares outstanding, the natural log of the number of shares outstanding from month $t-1$.

Substituting the right-hand side of Eq. (5) into Eq. (4), the liquidity risk beta estimating equation, they get

$$r_{i,t} = \beta_i^0 + \beta_i^M \mathrm{MKT}_t + \beta_i^S \mathrm{SMB}_t + \beta_i^H \mathrm{HML}_t + (\psi_{1,i} + \psi_{2,i}' \mathbf{Z}_{i,t-1}) L_t + \epsilon_{i,t}. \quad (6)$$

The two-step procedure for predicting liquidity risk betas, used at the end of each year, is as follows. In the first stage, for each stock PS estimate its returns that cannot be attributed to exposure to the three Fama and French factors, but may be driven by liquidity risk exposure. That is, using all data available at the time they calculate

$$e_{i,t} = r_{i,t} - \hat{\beta}_i^M \mathrm{MKT}_t - \hat{\beta}_i^S \mathrm{SMB}_t - \hat{\beta}_i^H \mathrm{HML}_t, \quad (7)$$

where the beta estimates come from Eq. (4), the regression used to estimate individual stocks' liquidity risks.

In the second stage PS estimate a regression to predict these residual returns, which are potentially liquidity driven, using characteristics that might predict liquidity risk. Specifically, they restrict the coefficients $\psi_{1,i}$ and $\psi_{2,i}$ to be the same across stocks, and estimate them using a pooled time-series, cross-sectional regression of the residual returns on the lagged predictive characteristics interacted with aggregate liquidity innovations,

$$e_{i,t} = \psi_0 + \psi_1 L_t + \psi_2' \mathbf{Z}_{i,t-1} L_t + \nu_{i,t}. \quad (8)$$

The sample on which they estimate predicted liquidity risk includes all NYSE, AMEX, and NASDAQ common stocks with end of prior year prices between \$5 and \$1,000.

We are less successful replicating the PS results on predicted liquidity risk. Our results are qualitatively similar to, but show quantitative discrepancies from, those reported in PS. We do not know the exact reasons why. It may be that without a publicly available series against which to compare out results, like we have for aggregate liquidity innovations and the traded liquidity factor returns, we were unable to ascertain important, undocumented methodological steps.

While our coefficient estimates on the characteristics do not exactly match those reported in PS, they do generally agree on the relative importance of the predictive variables, and the direction in which these variables predict liquidity risk exposure. While a direct comparison of our estimated coefficients from Eq. (8) with those reported in PS is left for the online Appendix (Table A1), we find, in agreement

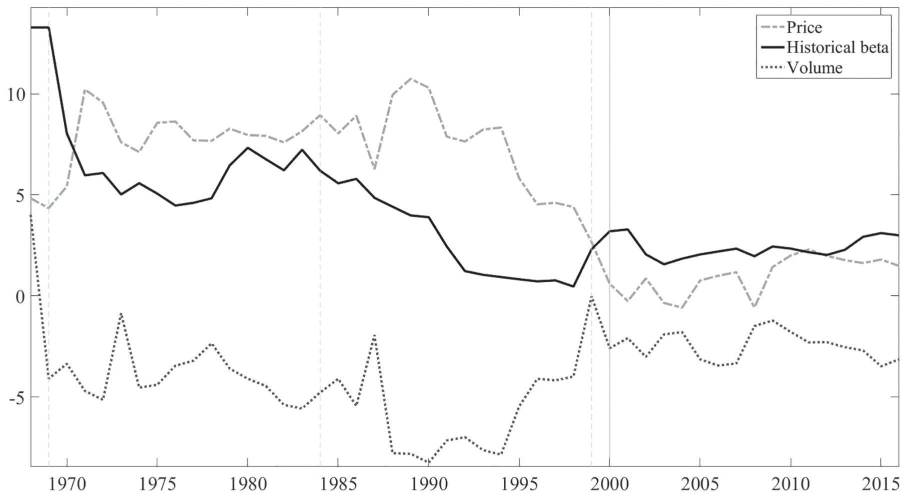

Figure 4: Importance of Most Significant Variables Predicting Liquidity Risk.

Description: The figure shows the impact, at each point in time, of a one standard deviation change at the mean for the three most significant variable predicting liquidity risk: (1) historically estimated liquidity risk (i.e., stocks' betas to innovations in aggregate liquidity); (2) log-average dollar volume traded over the preceding 6 months; and (3) log share price. Estimates come from calculating stocks' abnormal returns relative to the three Fama and French (1993) factors in a four-factor model that includes aggregate liquidity innovations, and regressing these onto the interaction of seven liquidity predicting stock characteristics and aggregate liquidity innovations (Eqs. (7) and (8)). The figure shows that high estimated historical liquidity betas, high stock prices, and low volumes, are all associated with higher predicted liquidity risk, but that the estimated sensitivity of liquidity risk to all of these variables falls over time. The solid vertical line is the end of the PS sample; the dashed vertical lines are the dates for which PS report coefficient estimates.

Interpretation: Sensitivity of liquidity risk to price, historical beta, and volume decreases in the post-PS sample.

with PS, that the three variables most important for predicting liquidity risk are historical liquidity risk (i.e., stocks' estimated betas to innovations in aggregate liquidity), (log) average dollar volume traded over the preceding 6 months, and (log) stock price. Figure 4 shows the importance of each of these variables. In particular, it shows, at each point in time, the estimated impact on the predicted liquidity risk beta of a one standard deviation change at the mean of each of these three important variables. The solid vertical line is the end of the PS sample. The dashed vertical lines correspond to the dates at which PS report coefficient estimates (their Table 2, replicated in Table A1 of the online Appendix).

The figure shows that, at least over the PS sample, historical liquidity risk and stock price are both positively associated with predicted liquidity risk, while average dollar volume traded is negatively associated with predicted liquidity risk.

That is, the stocks most sensitive to liquidity risk tend to be those that have been most sensitive in the past, those with high share prices, and those that have traded little in the preceding 6 months. The sensitivity of liquidity risk to all of these variables is much lower in the post-PS sample.

4.2 Traded Factor Based on Predicted Liquidity Risk

While our results predicting liquidity risk do not exactly match those reported in PS, they do agree qualitatively, and we still think it worthwhile to investigate the performance of a tradable liquidity risk factor based on predicted liquidity risk. In fact, it is necessary to do so to investigate one of the most important claims in PS, that liquidity risk exposure explains half of momentum, because this claim specifically relates to the predicted liquidity risk factor.

This factor is again long/short the extreme deciles from a sort on liquidity risk, measured now using its predicted, as opposed to historically observed, values. Following PS, we again rebalance these portfolios only annually, at the end of December, excluding stocks for which any of the seven predictors are missing, and those with end of preceding year share prices below \$5 or in excess of \$1,000. While PS only discuss a value-weighted factor based on historical liquidity risk, they construct both value-weighted and equal-weighted factors based on predicted liquidity risk, and focus on the performance of the equal-weighted factor. We also follow them in this regard.

Figure 5 shows the cumulative performance, ignoring transaction costs, of these traded factors based on predicted liquidity risk. The dashed gray line shows the growth over time of \$1 invested in the equal-weighted predicted liquidity risk factor, while the dotted black line shows the same for the value-weighted factor. For comparison the figure also shows results for the value-weighted factor based on historical liquidity risk (solid black line), which was previously included in Figures 2 and 3. The figure shows that the equal-weighted factor based on predicted liquidity risk generates similar average returns to the value-weighted factor based on historical liquidity risk over the PS sample, but has since underperformed. Both of these factors earned significantly higher average returns than the value-weighted factor based on predicted liquidity risk over the full sample.

4.3 Pricing Momentum

Perhaps the strongest claim made by PS is that "a liquidity risk factor accounts for half of the profits to a momentum strategy" (PS abstract). This claim is based on their Table 11, which reports a momentum strategy alpha of 16.3% per year relative to the Fama and French (1993) three-factor model, but only 8.4% per year relative to the four-factor model that augments the three-factor model with the equal-weighted predicted liquidity risk factor. That is, the inclusion of the liquidity

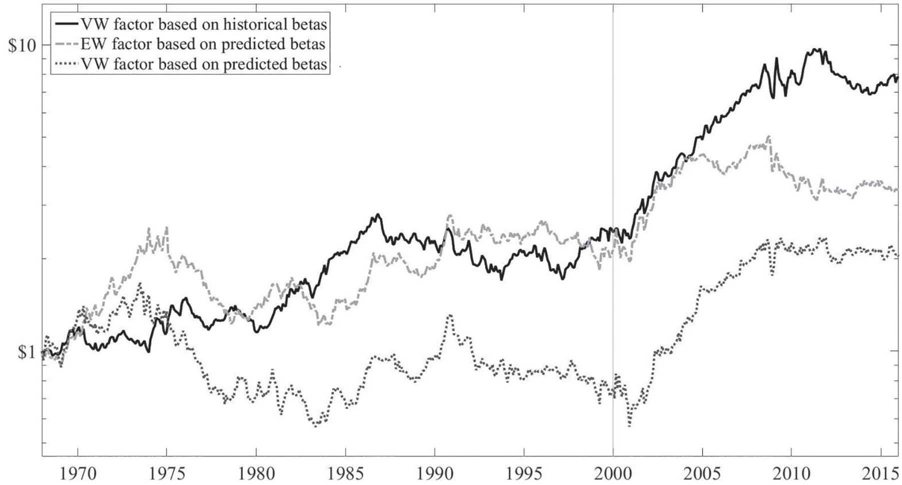

Figure 5: Cumulative Performance of Predicted Liquidity Risk Factors.

Description: The figure shows the growth over time of $1 invested in both the equal-weighted and value-weighted predicted liquidity risk factors (dashed gray and dotted lines, respectively). The growth of $1 invested in the value-weighted factor based on historical liquidity risk is also included for comparison (solid black line). Each factor buys the decile of stocks with the highest estimated liquidity risk, and shorts the decile with lowest estimated liquidity risk. All factors are rebalanced annually, at the end of December.

Interpretation: Liquidity factor based on historical beta outperforms the factors based on predicted beta in the post-PS sample.

risk factor reduces the alpha almost 8 percentage points per year, a nearly 50% reduction.

The first problem with this result is that PS have, by ignoring delisting returns, underestimated momentum strategy performance. The momentum strategy they consider is equal-weighted, and based on relatively extreme decile portfolios constructed using name breaks. The losers, having performed terribly over the preceding year, are on average quite small. The equal-weighting significantly overweights the smallest, most distressed of these losers. This presents a problem, because by ignoring delisting returns PS effectively assume that the loser portfolio is built with perfect foresight, constructed to avoid holding any stock that will delist in the coming month, stocks which on average perform terribly. By avoiding holding the worst performing stocks in any month, this loser portfolio greatly outperforms the similarly constructed, look-ahead bias free alternative. This is generally not an issue for value-weighted strategies, even those constructed within the small cap universe, because delisting stocks tend to have tiny market capitalizations and thus little impact on portfolio returns. For the decile sorted, equal-weighted momentum strategy however, which is relatively undiversified and dramatically overweights the smallest stocks, this bias adds more than 3% per year to the loser portfolio's realized average returns over the PS sample. The PS long/short momentum strategy, which shorts these losers, thus underestimates the look-ahead bias free performance of their preferred momentum strategy by the same amount. As a result, PS overstate the fraction of momentum strategy performance explained by their predicted liquidity risk factor.

Perhaps more problematic, all of the covariance between the PS liquidity risk factor and price momentum strategies, which is necessary for the factor to help price momentum, is driven by their use of momentum variables to predict liquidity risk. PS use prior 6 months' return as a liquidity risk predictor, which is obviously related to price momentum. Share price plays an even larger role, because of its high estimated importance for predicting liquidity risk. Past performance largely reflects changes in stock prices, so winners mechanically have higher average prices than losers. That is, share price is itself a momentum variable, with high prices mechanically selecting, on average, last year's winners. As a consequence, the procedure for predicting liquidity risk, which assigns higher estimated liquidity betas to higher priced stocks, mechanically introduces momentum into the predicted liquidity risk factor. The two momentum variables, stock price and prior 6 months' return, together play an important role in driving the PS results relating liquidity risk and momentum.

The important role the momentum variables play generating the positive observed covariance between the predicted liquidity risk factor and price momentum strategies can be seen in Table 5. The table shows results from time-series regressions of predicted liquidity risk factor returns onto various factor models. The first four specifications show the equal-weighted liquidity risk factor's average monthly returns, and its alphas and factor loadings relative to the Fama and French (1993) three-factor model, the Carhart (1997) four-factor model, and the Fama and French (2015) five-factor model. Panel A shows results for the PS sample, while Panel B shows them for the post-PS sample. The last four specifications (5 to 8) replicate the results for a factor that is constructed identically, except that it assigns stocks to deciles on the basis of liquidity risk estimates predicted without using the momentum variables, share price or prior 6 months return, as predictive variables. Results for the value-weighted factors, which perform less impressively, are provided in Table A3.

The most interesting aspect of the table is the difference in factor loadings on the predicted liquidity risk factors constructed using and excluding momentum variables when predicting liquidity risk. Not surprisingly, because share price is an important positive predictor of liquidity risk in the PS sample, the high liquidity risk stocks in the factor constructed including price as a predictor tend to be larger than those in the factor that excludes price as a predictor. This is reflected in Panel A of the table, observable as much lower (more negative) SMB loadings on the factor constructed using share price as a liquidity risk predictor

	Factor Constructed Using Momentum Variables When Predicting Liquidity				Factor Constructed Excluding Momentum Variables When Predicting Liquidity			
	(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**	**(7)**	**(8)**
Panel A: Pastor–Stambaugh Sample, January 1966 to December 1999								
alpha	0.45	0.71	0.44	0.59	0.12	0.22	0.17	0.17
	[2.57]	[4.90]	[3.03]	[4.04]	[0.90]	[1.76]	[1.28]	[1.32]
MKT		−0.28	−0.28	−0.27		−0.14	−0.14	−0.13
		[−7.89]	[−8.16]	[−7.45]		[−4.62]	[−4.59]	[−4.24]
SMB		−0.44	−0.40	−0.38		−0.17	−0.16	−0.15
		[−8.81]	[−8.27]	[−7.50]		[−3.91]	[−3.68]	[−3.23]
HML		−0.11	−0.02	0.02		0.02	0.04	0.05
		[−1.85]	[−0.41]	[0.24]		[0.39]	[0.70]	[0.67]
UMD			0.27				0.05	
			[6.64]				[1.42]	
RMW				0.46				0.17
				[4.82]				[1.97]
CMA				−0.03				0.02
				[−0.25]				[0.22]
Panel B: Post Pastor–Stambaugh Sample, January 2000 to December 2015								
alpha	0.35	0.42	0.44	0.73	0.43	0.55	0.55	0.58
	[1.05]	[1.49]	[1.55]	[2.56]	[1.58]	[2.27]	[2.26]	[2.40]
MKT		−0.36	−0.38	−0.50		−0.43	−0.44	−0.44
		[−5.55]	[−5.38]	[−6.74]		[−7.91]	[−7.21]	[−7.12]
SMB		0.50	0.52	0.46		0.04	0.05	0.21
		[5.71]	[5.75]	[4.50]		[0.59]	[0.60]	[2.39]
HML		−0.33	−0.34	0.05		0.02	0.02	0.21
		[−3.65]	[−3.72]	[0.41]		[0.29]	[0.27]	[1.93]
UMD			−0.04				0.00	
			[−0.82]				[−0.11]	
RMW				−0.31				0.21
				[−2.20]				[1.79]
CMA				−0.65				−0.61
				[−3.85]				[−4.34]

Table 5: Alphas and Factor Loadings of Predicted Liquidity Risk Factors.

Description: The table reports results from time-series regressions of the returns of the predicted liquidity risk factors onto various factors models. Predicted liquidity risk factors are long/short extreme deciles of equal-weighted portfolios sorted on predicted liquidity betas (value-weighed results provided in Table A3), and are rebalanced annually at the end of December. In the first four specifications liquidity risk is predicted using all seven variables employed by PS; in the last four the liquidity risk prediction does not use share price or prior 6 months' stock return.

Interpretation: The observed relation between momentum and the PS liquidity factor based on predicted beta is driven by the inclusion of momentum variables in the predicted liquidity betas.

(specifications 2 to 4) than on the factor constructed excluding share price as a predictor (specifications 5 to 8).

More importantly for how it pertains to momentum, the factor constructed using momentum variables to predict liquidity risk has a significantly higher UMD loading. Again, this should be expected, given that price, which is positively correlated with past stock performance, is the strongest predictor of liquidity risk over most of the PS sample. The factor constructed using the momentum variables to predict liquidity risk has a highly significant positive loading on UMD (0.27 with a t-statistic of 6.64), while the factor constructed excluding the momentum variables when predicting liquidity risk has a UMD loading less than a fifth as high, an insignificant loading of 0.05. The difference in loadings of 0.22 is highly significant, with a t-statistic of 7.02. This suggests that the observed relation between momentum strategies and the PS liquidity factor is primarily driven by the momentum variables used to predict liquidity risk, which mechanically bring momentum into the liquidity risk factor.

The relation between predicted liquidity risk and momentum noted by PS is also largely absent from the post-PS sample, over which the two strategies are weakly negatively correlated. Consistent with the lower estimated importance of share price as a liquidity risk predictor in the late sample, the difference in UMD loadings between the predicted liquidity risk factors constructed including and excluding price as a liquidity risk predictor is much smaller in the post-PS period.

The importance of including the momentum variables as liquidity risk predictors for the PS result that liquidity risk helps explain price momentum can be seen directly in Table 6. Panel A replicates PS Table 11, their primary evidence supporting their claim that liquidity risk explains half of momentum performance, using a momentum factor that does not have the performance attenuating look-ahead bias that results from ignoring delisting returns (a true replication of the table in PS, which ignores delisting returns, is provided in Table A7 of the online Appendix). It also extends these results to the post-PS sample. Panel B replicates these results employing predicted liquidity risk factors that are constructed without using the momentum variables' share price and prior 6 months' performance as predictors of liquidity risk ($\mathrm{LIQ}_{\mathrm{vw}}^{\text{w/o mom. vars.}}$ and $\mathrm{LIQ}_{\mathrm{ew}}^{\text{w/o mom. vars.}}$).

A somewhat attenuated version of their main result can be seen in our replication, in the first column of Panel A. Momentum's three factor alpha over the PS sample is 19.3% per year, but only 13.7% per year relative to a four-factor model that includes the equal-weighted predicted liquidity risk factor, a 29% reduction in alpha. This alpha reduction occurs because the momentum strategy loads heavily, in the PS sample, on the predicted liquidity risk factor (loading of 0.65 with a t-statistic of 7.35; PS report a loading of 0.75 with a t-statistic of 7.77). The 29% reduction we observe is short of the nearly 50% reduction reported by PS, partly because the true momentum spread, calculated accounting for delisting returns, is more than 3 percentage points per year larger and thus more difficult to explain,

	α_MOM			β_LIQ		
Explanatory Factors	**1966 to 1999**	**2000 to 2015**	**1966 to 2015**	**1966 to 1999**	**2000 to 2015**	**1966 to 2015**
Panel A: Liquidity Risk Factors Based on Predictions Made Using Momentum Variables						
FF3	19.26	7.91	16.61			
	[5.87]	[0.87]	[4.42]			
FF3 + LIQ_{vw}	17.34	7.79	15.79	0.35	0.05	0.16
	[5.38]	[0.85]	[4.20]	[4.77]	[0.25]	[2.32]
FF3 + LIQ_{ew}	13.73	9.50	14.75	0.65	−0.31	0.23
	[4.33]	[1.04]	[3.88]	[7.35]	[−1.61]	[2.64]
Panel B: Liquidity Risk Factors Based on Predictions Made Excluding Momentum Variables						
FF3 + $LIQ_{vw}^{w/o\ mom.\ var.}$	19.26	7.84	16.61	0.00	0.02	0.00
	[5.85]	[0.86]	[4.40]	[0.04]	[0.07]	[0.03]
FF3 + $LIQ_{ew}^{w/o\ mom.\ var.}$	19.02	10.70	16.68	0.09	−0.42	−0.02
	[5.77]	[1.17]	[4.40]	[0.85]	[−1.87]	[−0.14]

Table 6: Momentum Abnormal Returns.

Description: The table reports annualized alphas from time-series regressions of momentum strategy returns onto the three Fama and French factors (FF3) and liquidity risk factors based on predicted liquidity betas. The momentum strategy is long/short the extreme deciles, using name breaks, of a sort on stock past performance over the first 11 months of the preceding year (i.e., prior year's performance ignoring the most recent month, which is commonly associated with short term reversals). Momentum portfolio returns are equal-weighted, and rebalanced monthly. Data include delisting returns, which are not included in Pastor and Stambaugh (2003) but have a material impact on the performance of the momentum factor's loser portfolios (details, and results excluding delisting returns, are provided in Table A4). The explanatory factors include the Fama and French market, size, and value factors (MKT, SMB, and HML, respectively, which together make up the FF3), and value-weighted and equal-weighted predicted liquidity risk factors, constructed both including and excluding prior 6 months' stock return and share price as predictors of liquidity risk (LIQ_{vw}, LIQ_{ew}, $LIQ_{vw}^{w/o\ mom.\ var.}$, and $LIQ_{ew}^{w/o\ mom.\ var.}$).

Interpretation: The liquidity risk factor constructed without momentum variables is unrelated to momentum.

and partly because our predicted liquidity factor explains 2.4 percentage points per year less of the alpha (5.5% as opposed to 7.9%).

The first column of Panel B shows that this result is completely driven by the inclusion of momentum variables as liquidity risk predictors. Adding a predicted liquidity risk factor constructed without using momentum variables to the three-factor model yields essentially no reduction in momentum's alpha, because momentum does not significantly load on the liquidity risk factor. That is, the liquidity risk factor constructed without using the momentum variables to predict liquidity appears completely unrelated to momentum.

The PS result that liquidity risk helps explain price momentum, at least when liquidity risk is predicted using share price and past performance, is also confined to the PS sample. In the post-PS sample, over which the momentum variables are not strong predictors of liquidity risk, momentum loads negatively on their predicted liquidity risk factor. As a result, none of the liquidity risk factors helps explain momentum in the late sample. In fact, in the post-PS sample including a liquidity risk factor generally increases momentum's alpha.

5 Conclusion

Identifying an empirical measure of liquidity that has clear asset pricing implications is one of the most important goals of the liquidity literature. The payoffs to finding such a measure are high. A compelling liquidity factor, especially one that exhibited obvious relations to other anomalies, would likely see wide adoption into empirical asset pricing models, and thus be highly influential in practice as well as theory.

Pastor and Stambaugh (2003) is unquestionably an important contribution to this literature, and its primary empirical results can be replicated with a high degree of precision. It has been particularly influential by focusing on the time-varying nature of liquidity, and how exposure to changes in aggregate liquidity may be as important as the level of liquidity for asset pricing. We find only modest support, however, for the claim that liquidity risk is related cross-sectionally to expected returns, and none for the claim that liquidity risk helps explain momentum.

One possible explanation for the relatively modest role we observe liquidity risk playing in asset pricing may simply be measurement error. PS observe that important liquidity episodes appear to be associated with "flights to quality." Identifying stocks exposed to flight to quality risk, i.e., stocks that will underperform when investors take refuge in higher quality assets, represents a classic "peso problem." These episodes are by nature infrequent, meaning that liquidity risk is generally estimated over periods in which the risks go unrealized, which poses an inherently difficult challenge. It is still more difficult when measuring risk using betas, which are always imprecisely estimated at the stock-level because individual stock returns are themselves primarily idiosyncratic. The resulting errors-in-variables problem biases tests against finding a significant role for liquidity risk in asset pricing, even if liquidity risk is truly priced.

Alternatively, it may be that there is simply no large illiquidity premium to find, a possibility consistent with a large theoretical literature arguing that liquidity should only modestly impact asset prices.[13] These theoretical predictions against a

[13] Constantinides (1986) and Vayanos (1998) both argue that proportional transaction costs should significantly impact trading frequency but have only a minimal effect on prices. Heaton and Lucas (1996) find that transaction costs do not generate significant premia in an economy in which agents trade to share labor-income risk. Lo *et al.* (2001) generate moderate price discounts due to illiquidity, but the resulting return premium is still quite low. Huang (2003) finds that illiquidity premia should be inconsequential, barring other significant constraints. Amihud and Mendelson (1986) find large

significant illiquidity premium are driven by endogenous trading and long horizon investors. Illiquid assets should trade infrequently, and be held primarily by patient investors. A transaction cost of even 5% represents only a 16 basis point drag on annual returns to an investor with a 30-year horizon. If these patient investors only need compensation, in the form of higher returns, sufficient to make them indifferent between holding liquid and illiquid assets, then the return premium provided by illiquid assets need not be large.[14]

Of course, even if liquidity does not drive significant differences in expected returns, empirical measures of liquidity could still help identify any differences. A liquidity measure should help predict returns whenever firm characteristics driving risk exposure also affect liquidity, even if variations in liquidity do not cause any differences in expected returns. That is, any observed illiquidity premium could simply reflect a mechanical relation, similar to the one relating size and expected returns discussed by Berk (1995), an idea explored by Novy-Marx (2004) and Johnson (2006).[15]

The simple fact remains, however, that there is still no truly compelling evidence for a strong cross-sectional relation, causal or otherwise, between liquidity and expected returns.

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liquidity effects, but in a model that explicitly forbids long horizon investors from undertaking liquidity "arbitrage," and allowing for competition across investor type significantly reduces the magnitude of the model's predicted illiquidity premium. Similarly, Acharya and Pedersen (2005) derive a significant illiquidity premium, but in a model in which investors liquidate their entire portfolios every period.

[14]Simple theoretical models also typically predict, of course, that investors should only require a small premium to hold the market.

[15]Vayanos (2004) and Chien and Lustig (2010) provide examples of equilibrium models in which liquidity is not priced per se, but in which an illiquidity premium nevertheless arises from covariance between returns and state prices. This covariance is driven by asset sales forced by mutual fund withdrawals (Vayanos, 2004) or binding solvency constraints (Chien and Lustig, 2010).

Appendix: Additional Tables

	r^e	$r^{e,*}_{\mathrm{FF3}}$	$r^{e,*}_{\mathrm{FF4}}$	$r^{e,*}_{\mathrm{FF5}}$
Panel A: Replicated Liquidity Factor (Annual December Rebalancing)				
PS sample	0.30	0.26	0.28	0.27
	[1.90]	[1.61]	[1.76]	[1.71]
Post-PS sample	0.70	0.57	0.58	0.33
	[2.35]	[2.21]	[2.24]	[1.22]
Full sample	0.44	0.36	0.37	0.29
	[2.99]	[2.60]	[2.74]	[2.10]
Panel B: Liquidity Factor Constructed Using Natural (Monthly) Rebalancing				
PS sample	0.18	0.39	0.44	0.39
	[1.02]	[2.28]	[2.68]	[2.42]
Post-PS sample	0.75	0.39	0.38	0.24
	[2.55]	[1.61]	[1.57]	[0.94]
Full sample	0.37	0.39	0.42	0.34
	[2.40]	[2.79]	[3.09]	[2.49]

Table A1: Liquidity Factor Performance with ex ante Hedging of Other Exposures (Table 3 with Dynamic Hedging).

Description: The table reports the average monthly returns to liquidity factors hedged at each point in time of their estimated exposures to the factors employed in the three, four, and five factor models. Liquidity factors are the value-weighted returns to the long/short strategy that buys the decile of stocks with the highest estimated liquidity betas and shorts the decile with lowest estimated liquidity betas, and is rebalanced either annually, at the end of December, as in Pastor and Stambaugh (2003, Panel A), or at the natural monthly frequency (Panel B). Hedged returns ($r^{e,*}$) are the returns to a factor hedged of its ex ante estimated exposures to the factors employed in the corresponding asset pricing model. Betas are estimated at each rebalance point using the past year of daily returns to the portfolio that will be held until the next rebalance point. This procedure, also used in Novy-Marx (2016), accounts for the possibility that the test assets' factor loadings may vary over time, and is similar in spirit to the Fama and MacBeth (1973) regressions of Brennan *et al.* (1998) and Chordia and Subrahmanyam (2001), which employ individual stock returns hedged of their exposures to the three Fama and French factors estimated using 5 years of past monthly returns. The three-factor model consists of market (MKT), size (SMB), and value (HML) factors; the four-factor model adds momentum (UMD); the five-factor model includes profitability (RMW) and investment (CMA). The Pastor–Stambaugh (PS) sample covers January 1968 to December 1999, the post-PS sample covers January 2000 to December 2015, and the full sample covers January 1968 to December 2015.

Interpretation: Liquidity factor performance under dynamic hedging is also sensitive to rebalancing frequency.

$n^{\text{portfolio}}$	**Name Break**	**NYSE Break**	**Market Cap Break**
Panel A: t-Statistics on Average Excess Returns			
2	1.44	1.22	1.31
3	1.38	1.39	1.30
5	0.94	0.93	1.03
10	2.40	1.46	1.23
20	0.28	1.51	1.27
Panel B: t-Statistics on Alphas Relative to Fama and French (1993) Three-Factor Model			
2	1.07	0.86	1.09
3	1.15	1.10	1.15
5	0.93	0.88	0.94
10	2.42	1.38	1.24
20	0.39	1.74	1.28
Panel C: t-Statistics on Alphas Relative to Carhart (1997) Four-Factor Model			
2	1.62	1.43	1.77
3	1.90	1.89	1.81
5	1.46	1.41	1.56
10	2.84	1.91	1.76
20	0.73	2.15	1.60
Panel D: t-Statistics on Alphas Relative to Fama and French (2015) Five-Factor Model			
2	1.19	0.95	1.37
3	1.55	1.52	1.42
5	1.08	1.14	1.10
10	2.24	1.29	0.85
20	0.46	1.71	0.52

Table A2: Table 4 with Monthly Rebalancing.

Description: The table reports t-statistics on the abnormal returns to liquidity factors constructed using 15 sorting methodology variations. Each factor is long/short the extremes value-weighted portfolios from a sort on stocks' estimated liquidity betas. Sorts are into two, three, five, 10, or 20 portfolios, using name, NYSE, or market cap breakpoints. Portfolios are rebalanced *monthly*. Panel A reports t-statistics on average excess returns; Panel B reports t-statistics on alphas relative to Fama and French (1993) three-factor model; Panel C reports t-statistics on alphas relative to Carhart (1997) four-factor model; and Panel D reports t-statistics on alphas relative to Fama and French (2015) five-factor model. The sample covers January 1968 to December 2015.

Interpretation: Monthly rebalanced factor performance is also sensitive to portfolio construction.

	Factor Constructed Including Return and Price to Predict Liquidity				Factor Constructed Excluding Return and Price to Predict Liquidity			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Panel A: Pastor–Stambaugh Sample, January 1968 to December 1999								
Alpha	0.15	0.46	0.22	0.36	0.06	0.20	0.18	0.21
	[0.76]	[2.54]	[1.19]	[1.92]	[0.31]	[1.10]	[0.93]	[1.13]
MKT		−0.33	−0.33	−0.32		−0.22	−0.21	−0.22
		[−7.41]	[−7.49]	[−6.97]		[−4.76]	[−4.74]	[−4.63]
SMB		−0.26	−0.22	−0.21		−0.11	−0.10	−0.11
		[−4.13]	[−3.56]	[−3.22]		[−1.66]	[−1.58]	[−1.66]
HML		−0.27	−0.20	−0.17		−0.02	−0.01	−0.02
		[−3.81]	[−2.78]	[−1.67]		[−0.30]	[−0.19]	[−0.20]
UMD			0.23				0.02	
			[4.51]				[0.44]	
RMW				0.39				−0.03
				[3.17]				[−0.27]
CMA				−0.02				−0.02
				[−0.12]				[−0.12]
Panel B: Post Pastor–Stambaugh Sample, January 2000 to December 2015								
Alpha	0.32	0.22	0.22	0.56	0.50	0.39	0.39	0.45
	[0.75]	[0.72]	[0.73]	[1.82]	[1.60]	[1.53]	[1.51]	[1.66]
MKT		−0.04	−0.04	−0.19		−0.08	−0.07	−0.10
		[−0.52]	[−0.55]	[−2.42]		[−1.32]	[−1.14]	[−1.42]
SMB		0.94	0.94	0.87		0.68	0.68	0.75
		[10.04]	[9.87]	[7.94]		[8.46]	[8.25]	[7.82]
HML		−0.51	−0.51	−0.12		−0.22	−0.22	−0.08
		[−5.33]	[−5.31]	[−0.89]		[−2.63]	[−2.59]	[−0.70]
UMD			−0.01				0.01	
			[−0.20]				[0.14]	
RMW				−0.38				0.06
				[−2.52]				[0.48]
CMA				−0.62				−0.37
				[−3.42]				[−2.35]

Table A3: Table 5 with Value-Weighting.

Description: Alphas and factor loadings of predicted liquidity risk factors. The table reports results from time-series regressions of the returns of the predicted liquidity risk factors onto various factor models. Predicted liquidity risk factors are long/short extreme deciles of value-weighted portfolios sorted on predicted liquidity betas, and are rebalanced annually at the end of December. In the first four specifications liquidity risk is predicted using all seven variables employed by PS; in the last four the liquidity risk prediction excludes past stock performance and share price as predictive variables.

Interpretation: Value-weighted predicted liquidity beta factor performs less impressively, and is still unrelated to momentum.

Explanatory Factors	α_MOM 1966 to 1999	α_MOM 1966 to 1982	α_MOM 1983 to 1999	β_LIQ 1966 to 1999	β_LIQ 1966 to 1982	β_LIQ 1983 to 1999
Panel A: Liquidity Risk Factors Based on Predictions Made Using Momentum Variables						
FF3	16.07	19.92	12.27			
	[4.90]	[4.29]	[2.57]			
FF3 + LIQ_{vw}	14.12	18.77	7.20	0.35	0.23	0.57
	[4.39]	[4.05]	[1.57]	[4.83]	[2.10]	[5.32]
FF3 + LIQ_{ew}	10.50	15.44	1.77	0.65	0.60	0.90
	[3.31]	[3.44]	[0.38]	[7.43]	[4.88]	[6.50]
Panel B: Liquidity Risk Factors Based on Predictions Made Excluding Momentum Variables						
FF3 + $\text{LIQ}_{\text{vw}}^{\text{w/o mom. var.}}$	16.05	20.03	10.53	0.01	−0.20	0.21
	[4.88]	[4.34]	[2.17]	[0.12]	[−1.66]	[1.87]
FF3 + $\text{LIQ}_{\text{ew}}^{\text{w/o mom. var.}}$	15.79	19.69	11.38	0.11	0.15	0.13
	[4.80]	[4.23]	[2.31]	[1.00]	[0.91]	[0.76]

Table A4: Table 6 Without Delisting Returns.

Description: The table reports annualized alphas from time-series regressions of momentum strategy returns onto the three Fama and French factors (FF3) and liquidity risk factors based on predicted liquidity betas. The momentum strategy is long/short the extreme deciles, using name breaks, of a sort on stock past performance over the first 11 months of the preceding year (i.e., prior year's performance ignoring the most recent month, which is commonly associated with short term reversals). Momentum portfolio returns are equal-weighted, and rebalanced monthly. Data *exclude* delisting returns. The explanatory factors include the Fama and French market, size, and value factors (MKT, SMB, and HML, respectively, which together make up the FF3), and value-weighted and equal-weighted predicted liquidity risk factors, constructed both including and excluding cumulative return and share price as predictors of liquidity risk (LIQ_{vw}, LIQ_{ew}, $\text{LIQ}_{\text{vw}}^{\text{w/o mom. var.}}$, and $\text{LIQ}_{\text{ew}}^{\text{w/o mom. var.}}$).

Interpretation: Excluding delisting returns has a material impact on PS momentum results.

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